THE EMISSIVITY OF METALS AND OXIDES

III. THE TOTAL EMISSIVITY OF PLATINUM AND THE RELATION BETWEEN TOTAL EMISSIVITY AND RESISTIVITY

By Paul D. Foote

On the basis of the Maxwell theory, Planck[1] has derived the following equation representing the relation between the reflection coefficient of a metallic surface, the volume resistivity of the metal, and the wave length of the incident radiation:

$$(1) \qquad R_\lambda = \frac{\sqrt{3600\lambda^2/r^2+1}+1-\sqrt{2(\sqrt{3600\lambda^2/r^2+1}-1)}}{\sqrt{3600\lambda^2/r^2+1}+1+\sqrt{2(\sqrt{3600\lambda^2/r^2+1}-1)}}$$

where R_λ is the ratio of the intensities of the reflected to the incident radiation, r the volume resistivity of the metal in ohms-cm, and λ the wave length of the radiation in cm. Defining the monochromatic emissivity as $E_\lambda = 1 - R_\lambda$ and expanding the above expression into a series of ascending powers of $\sqrt{r/\lambda}$, Hagen and Rubens[2] have obtained the following equation in which the terms of higher order than those indicated are usually negligible:

$$(2) \qquad E_\lambda = 0.365\sqrt{\frac{r}{\lambda}} - 0.667\left(\frac{r}{\lambda}\right) + \cdots - \cdots +$$

The spectral distribution of the energy radiated by a black body may be expressed by the Planck equation:

$$(3) \qquad J_\lambda = c_1\lambda^{-5}\left[e^{\frac{c_2}{\lambda T}} - 1\right]^{-1}$$

where T is the absolute temperature and $c_2 = 1.4450$ cm deg.

[1] Planck, Sitz. Königl. Ak. Wiss., Berlin, p. 278; 1903.

[2] Hagen and Rubens, Sitz. Königl. Ak. Wiss., Berlin, p. 468; 1910.

The total energy (any arbitrary unit) radiated by a black body is given by the integral of equation (3).

$$J=\int_0^\infty J_\lambda d\lambda \tag{4}$$

The total energy radiated by a nonblack metal may be expressed as the integral of the Planck relation multiplied by the monochromatic emissivity.

$$J'=\int_0^\infty J_\lambda E_\lambda d\lambda \tag{5}$$

If the total emissivity E of a metal is defined as the ratio of the energy emitted by the metal at an absolute temperature T to that emitted by a black body at the same temperature, one obtains the following relation:

$$E=\frac{J'}{J}=\frac{\int_0^\infty J_\lambda E_\lambda d\lambda}{\int_0^\infty J_\lambda d\lambda} \tag{6}$$

Equation (6) may be converted to the gamma function form, and is hence readily integrable. One thus obtains the following expression for the total emissivity of a metal:

$$E=0.5736\sqrt{rT}-0.1769rT \tag{7}$$

where T is the absolute temperature of the radiating metal and r the volume resistivity in ohms-cm at this temperature.

It is therefore apparent that the total emissivity of most pure metals should increase with increasing temperature, both because of the increasing value of T in equation (7) and because of, in general, the increasing volume resistivity. The increase in total emissivity with increasing temperature has been observed in nearly all of the extremely few instances where this quantity has been measured.[3]

The above derivation is essentially that of Aschkinass,[4] who, in an extensive paper, has discussed the energy emission of metals, the value of $\lambda_m T$ of the displacement law for metals, and the relation of various properties of the radiation from metals to similar

[3] Langmuir, Trans. Am. Elec. Chem. Soc., 23; p. 321; 1913. Randolph and Overholser, Phys. R., 2 (2) p. 144; 1913. Burgess and Foote, Bureau of Standards Scientific Paper No. 224.

[4] Aschkinass, Ann. d. Physik. (4), 17, p. 960; 1905.

properties of black body radiation. He used for the value of E_λ the first term only of equation (2), the one-term formula having been derived by Planck,[5] but inasmuch as Hagen and Rubens [6] have since found that the one-term relation was insufficient, it appeared of interest to derive the somewhat general relation (7) for total emissivity from the more accurate relation (2) for monochromatic emissivity. The first term of equation (7) may be obtained directly from Aschkinass's work by dividing his equation (10) by the integral of Planck's spectral equation and correcting the value of c_2 in (10) to the present accepted value of 1.445. The second term of equation (7) becomes equal to 11 per cent of the first term at 1700° C., and hence is of considerable importance, especially at the higher temperatures.

In order to check formula (7) quantitatively, measurements have been made, with Mr. Kellberg's assistance, upon the total emissivity of platinum. The apparent temperatures of thin platinum strips were measured by three radiation pyrometers of the Féry mirror type and the apparent temperatures for a wave length $\lambda = 0.65\mu$ were obtained with a Holborn-Kurlbaum optical pyrometer. The apparent temperatures measured by the optical pyrometer were converted in the usual manner to true temperatures using the value of $E_{0.65} = 0.33$. The emissivity of this metal for $\lambda = 0.65\mu$ is independent of the temperature.[7] This is of course in contradiction to the Maxwell relation (2) above. But it must be noted that, for metals, the visible spectrum is usually the region where resonance phenomena are taking place, and hence one would here expect that the general theoretical deductions might fail to apply, as is experimentally found to be the case. This fact does not materially alter the validity of the expression (7) for the total emissivity since by far the greater part of the radiant energy of glowing metals at temperatures below 1500° C is confined to a spectral region where resonance phenomena do not exist.

The relation between the true absolute temperature T of a material, the apparent absolute temperature, S, measured by a

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[5] See note 1, p. 607.

[6] See note 2, p. 607.

[7] Waidner and Burgess, Bureau of Standards Scientific Paper No. 55; Mendenhall Astrophys J., 33, p. 91, 1911; Spence, Astrophys J., 37, p. 194; 1913.

total radiation pyrometer, and the total emissivity is given by the following expression:

(8) $E=\frac{S^4-T_0^4}{T^4-T_0^4}$ where T_0 is the temperature of the receiver.

The temperature coefficient of mass resistivity of pure platinum from 0 to 1100° C. is very accurately given by the Callendar [8] parabolic equation where $\delta=1.49$ and C the fundamental coefficient is 0.0039.

It does not appear likely that serious error is introduced by the extrapolation of this equation to the melting point of platinum. Observations by Pirani [9], Holborn and Wien,[10] and Langmuir [11] above 1100° C. do not deviate seriously from the parabolic relation. Langmuir concluded from his determinations that the relation was linear above 1200°, but he uses the value 1710 instead of 1755 for the melting point of platinum. If his data are corrected for the error in the temperature scale the observations follow the Callendar formula quite satisfactorily.

The mass resistivity of a 0.6 mm × 100 cm wire of Heraeus platinum, presumably of the same degree of purity as that of the strips used in the emissivity determinations, was very kindly measured by C. F. Hanson, of this Bureau, with an accuracy of about 0.1 per cent.

Reduction to the volume resistivity at 0° C. gave the value of 9.77×10^{-6} ohms-cm, with possibly an error as great as 1 per cent. The ratio r_t/r_0 determined by resistances at constant mass is equal to the ratio determined at constant volume within the accuracy of the present work.

Fig. 1 represents the total emissivity of platinum as a function of the temperature in the range 0 to 1700° C.

The computed points are represented by crosses. Preliminary observations made with radiation pyrometers which were later found to be somewhat defective are shown by small dots and the final observations made with an instrument comparatively free from these defects are plotted as circles. The large dot at 15° C.

[8] Waidner and Burgess, Bureau of Standards Scientific Paper No. 124, p. 151.
[9] Pirani, Verh. D. Phys. Ges., **12**, p. 315; 1910.
[10] Holborn and Wien, Ann. d. Physik, **59**, p. 360, 1896.
[11] Langmuir, J. Am. Chem. Soc., **28**, p. 1357; 1906.

was obtained from the work of Hagen and Rubens and others on the reflection coefficients of platinum. These investigators have determined the dispersion of the reflection coefficient to 25μ. If the values of $E_\lambda = 1 - R_\lambda$ are substituted in equation (6) the total emissivity may be obtained. The integration was performed graphically. The triangles represent the observations of Lummer and Kurlbaum.[12]

The agreement between the theoretical and experimental values is excellent, when one considers the difficulties involved in an accurate determination of total emissivity. The deviations of

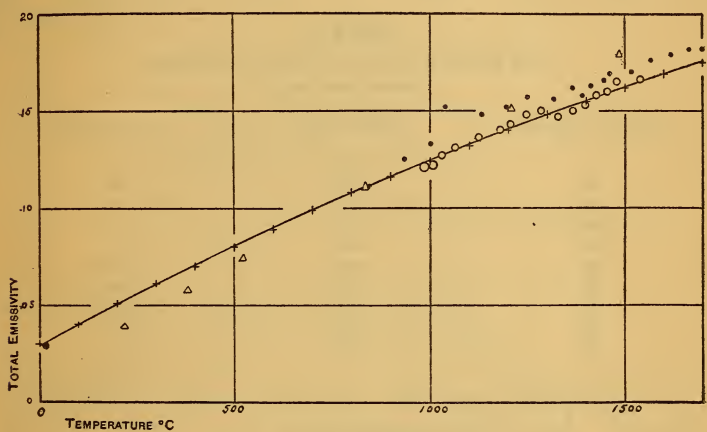

FIG. 1.—*Total emissivity of platinum vs. true temperature*

the computed values from the observed values are in general much less than ± 0.01, which is about the limit of accuracy of the experimental determinations.

Table 1 presents a summary of the values of the total emissivity of pure platinum for the temperature range 0 to 1700° C.

Table 2 shows the corrections which must be applied to the temperature observed with a radiation pyrometer, when sighted upon pure platinum, in order to obtain the true temperature of the radiating platinum. The temperature of the receiver is considered as 300° abs.

[12] Lummer and Kurlbaum, Verh. D. Phys. Ges., **17**, p. 106; 1898.

TABLE 1

Total Emissivity vs. Temperature

Degrees centigrade	Emissivity.	Degrees centigrade.	Emissivity
0	0.030	900	0.116
100	.040	1000	.124
200	.051	1100	.132
300	.061	1200	.140
400	.070	1300	.148
500	.080	1400	.155
600	.089	1500	.162
700	.099	1600	.169
800	.108	1700	.175

TABLE 2

Total Radiation Pyrometer Sighted on Platinum

[Receiver = 300° abs.]

Observed temperature	Correction to add	True temperature
°C	°C	°C
200	365	565
300	425	725
400	475	875
500	520	1020
600	560	1160
700	595	1295
800	630	1430
900	660	1560
1000	695	1695

In a subsequent paper the emissivity of other metals will be discussed. Approximate measurements have already indicated that equation (7) is a general relation, and is by no means restricted to the example cited in the present paper. This equation, however, is somewhat modified if one considers the resistance, r, of equation (1) to be a function of λ. This modification will be discussed elsewhere.

SUMMARY

A definite relation has been found to exist between the total emissivity and the volume resistivity of a metal. This relation follows directly from the Maxwell theory of reflection and absorption. Experimental determinations of the total emissivity of platinum have verified the derived relation.

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