CHARACTERISTIC EQUATIONS OF TUNGSTEN FILAMENT LAMPS AND THEIR APPLICATION IN HETEROCHROMATIC PHOTOMETRY[1]

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[1] A preliminary paper on this subject was read at the Eighth Annual Convention of the Illuminating Engineering Society, Cleveland, Ohio, Sept. 21–24, 1914.

I. INTRODUCTION

This investigation was carried out at the Bureau of Standards with a view to finding a more satisfactory method than those at present in vogue for overcoming the difficulties due to color difference when lamps of higher or lower efficiency are to be compared with the carbon primary standards. In order that the application of characteristic equations in this connection may be fully understood, it is deemed advisable, before discussing the main problem at issue, to first give a brief description of the character of the standard photometric work of the Bureau, the methods employed, and the principal difficulties encountered.

II. STANDARD PHOTOMETRIC WORK AT THE BUREAU OF STANDARDS

1. THE PRIMARY PHOTOMETRIC STANDARDS

The photometric standards, by means of which the international candle is maintained at the Bureau of Standards, are a group of carbon filament incandescent lamps operated at an efficiency of

4 watts per mean horizontal candle. On account of photometric difficulties which are involved in comparing lights of different color, and which increase as the color difference increases, it is fortunate that these primary standards occupy a position at about the middle of the range of color of lamps now used as secondary and working standards, being blue in comparison with the Hefner and other flame standards and red in comparison with tungsten lamps when operated at or near normal efficiency. Even with the primary standards in this intermediate position, the color difference met with in many cases is considerable and very troublesome in standard photometry where the highest accuracy is required.

2. CLASSES OF LAMPS SUBMITTED FOR STANDARDIZATION

In general there are two classes of lamps submitted to the Bureau of Standards for certification, (a) flame lamps, of which practically all are pentane and (b) electric incandescent lamps, of which during the past year one-third were carbon and two-thirds tungsten.

(a) FLAME LAMPS

Although different forms of flame lamps have been recognized as primary standards (for example, the Carcel in France, the Hefner in Germany, and the pentane in England) none of them is now considered entirely satisfactory for this purpose.[2] Though made and used according to official specifications, individual lamps differ among themselves by amounts considerably greater than the errors of photometric measurement. Incandescent lamps, on the other hand, when operated at a definite voltage at which they have been standardized in terms of a light unit agreed upon, will maintain that unit constant with an accuracy far above that which is possible with flame standards. With so much in favor of incandescent lamps, it is perfectly reasonable that they, in preference to any of the present so-called primary-flame standards, should be chosen to maintain the international candle constant, at least until a lamp appears the value of which can be satisfactorily reproduced from specifications. Accordingly in this country, the real primary standard is a group of incandescent lamps, and the various flame standards which are assigned values by comparison

[2] Rosa and Crittenden, "Flame Standards in Photometry." This Bulletin, 10, p. 557; 1914. (Scientific Paper No. 222.)

with the primary carbon standards have become secondary standards, and their specifications are practically ignored except by the makers.

In the photometry of pentane lamps the measurements are made directly in terms of carbon working standards operating at a low efficiency (about 7 watts per candle) so as to match the pentane flame in color. As this color is practically the same for all pentane lamps and remains constant under normal working conditions, only one group of electric working standards is required, and the color difference involved in checking them in terms of the primary standards is the same from time to time. In so far as the preparation and maintenance of working standards are concerned, the standardization of pentane lamps is therefore simple in comparison with that of electric incandescent lamps.

(b) ELECTRIC INCANDESCENT LAMPS

On account of the great flexibility, as to color, of the incandescent lamp, especially of the tungsten lamp, when supplied with current at different voltages, the range of color through which it may be used as a standard extends from the dull red of initial incandescence to comparative blue at normal efficiency. Even normal efficiency is not a fixed value but has been and is still steadily increasing with improvements in manufacture. The Bureau, therefore, as custodian of the primary standards, must be in a position to calibrate incandescent lamps at a large number of efficiencies, and hence a single group, or even a number of groups, of working standards, each group of some definite color, can not meet the desired condition of color match with any given lamp that may be submitted for test.

It is obvious, therefore, that, with the exception of carbon lamps at 4 watts per candle, the standardization of practically every lamp submitted involves, either directly or indirectly, a color difference in the photometric measurements.

3. DIFFICULTIES DUE TO COLOR DIFFERENCE

Comparison of lamps of the same color involves no special difficulties and comparatively little experience is required to enable an observer to produce results of considerable accuracy. But when a color difference exists, even observers of experience do not agree in their photometric measurements, the disagreement being due principally to two causes, (1) difference in color vision and (2) difference in judgment as to what constitutes a match in intensity, thus rendering it difficult for an individual to repeat even his own observations with the precision possible when there is a color match. Hence, to establish standards, it is not only necessary to have observers of normal color vision but also to obtain a very large number of readings by experienced observers who have learned to fix and maintain a suitable criterion so that they can obtain consistent results at different times.

III. METHODS OF HETEROCHROMATIC PHOTOMETRY

In the photometry of differently colored lights at least four methods have been used, viz, visual acuity, critical frequency, equality of brightness, and flicker. Of these, only the last two mentioned are sufficiently sensitive for the accuracy required in precision work.[3] In the experience of the Bureau of Standards more consistent results have been obtained by the equality of brightness than by the flicker method, and for this reason the Lummer-Brodhun contrast photometer, which makes use of the equality of brightness principle, has been used for all photometric work, both for lights of the same color and of different color.

IV. ELIMINATION OF COLOR DIFFERENCE IN ROUTINE WORK

In the Bureau's regular routine work of standardizing lamps of high efficiency (i. e., blue in comparison with carbon standards) the measurements, so far as is possible, are made with color match obtained either by the use of blue-glass screens placed in the path of the light from the carbon working standards or by the use of tungsten working standards of approximately the color of the lamps to be measured. The latter may be a group of standards each operated at a single voltage (or color) or a group whose candlepower values are known through a considerable range of color. However, it is the common practice, especially when important standards are submitted, to make measurements in terms of all three groups.

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[3] Ives, H. E.; Phil. Mag., 24, p. 156; 1912.

1. USE OF COLOR SCREENS

Obviously either red or blue screens of the proper spectral absorption may be used to eliminate color difference. Up to the present, however, red screens of sufficient transparency, for most purposes, have not been obtainable, hence all color matching of test lamps with the primary standards has been done with blue glasses, which can be obtained with almost any desired degree of color absorption and very uniform throughout.

2. USE OF SECONDARY STANDARDS

Evidently the best way of eliminating color difference in the photometry of any light source is by the use of a standard of the same kind. Hence it is an advantage to use electric incandescent standards when measuring lamps of this kind. Fortunately, the tungsten lamp is so flexible as to color adjustment that it can be made to match all incandescent lamps, regardless of the material of the filament. Further than this, it can be made to match, at least very approximately, the color of every flame lamp used as a standard. It is, therefore, an ideal working standard in heterochromatic photometry. It has an advantage over the use of color screens in that it will match the test lamps at whatever efficiency they may fall, but it has the disadvantage of being less durable.

Although both methods are satisfactory in the elimination of color difference in routine work, both transfer rather than dispose of the difficulties which must be met in the calibration of the screens and the standardization of the lamps. As secondary tungsten standards have usually been standardized at the Bureau by the use of glass screens, the problem of stepping from one color to another has been reduced to the calibration of the screens.

V. CALIBRATION OF COLOR SCREENS

The calibration of color screens as done heretofore introduces deviations in observed values fully as great as those which obtain in the direct comparison of lamps of different color. This renders the derivation of reliable values a more difficult matter than would at first appear. Even after a reliable value is obtained, the advantage of measurements at a color match is not fully realized, because the range in efficiency through which a given screen eliminates color difference is rather narrow. For the best results a different screen should be used for each efficiency, and the calibration of each screen means that the same difficulties must again be encountered. It was primarily with a view of finding a method of reducing both the errors and the amount of labor involved in these calibrations that this investigation was undertaken.

1. OLD METHOD, INVOLVING COLOR DIFFERENCE

The most direct method of determining the percentage transmission (or transmission coefficient) of a color screen for use with a source of 4-wpc-carbon color is as follows: The photometer is arranged as for the ordinary comparison of two lamps A and B, of which A is adjusted to 4-wpc-carbon color. The screen is placed at the side of the photometer head in the path of the light from this lamp, and B is adjusted to match the transmitted light in color. A photometric setting is then made with the screen in position and another with it removed. The ratio of the first setting to the second gives the coefficient of the screen independently of the values of A and B. The test may be varied by adjusting B to color match A when the screen is removed, or B may be adjusted to a color anywhere between those of the direct and transmitted light from A. In every case, however, at least one of the two settings required must be made with a color difference, thus involving all the difficulties and uncertainties of the photometry of differently colored lights.

In the calibration of the screen it is important that lamp A be of the same color as the light source with which the screen is to be used, otherwise the coefficient will have a different value depending upon the color of the incident light.

2. PROPOSED METHOD, USING VOLTAGE CANDLEPOWER RELATION

In order to avoid measurements involving color difference when making such calibrations the following method is proposed: Both sources A and B are tungsten lamps. A is first adjusted in voltage (to about 3.1 wpc) to match 4-wpc-carbon color, and B is adjusted as before—that is, to a color match with the light transmitted by the screen. After making a photometric setting, the screen is removed and, with B constant, A is increased in voltage until it matches B in color, and a second setting is made. Calling the settings S_1 and S_2 and the two candle power values of A, C_1 and C_2, respectively, the coefficient of transmission is $T=\frac{S_1}{S_2}\cdot\frac{C_2}{C_1}$. As both settings are made with a color match, all the difficulties involved in the method first given above are eliminated, close agreement among different observers results, and T is determined with comparatively little labor.

This method, however, assumes that the ratio $\frac{C_2}{C_1}$ has been accurately determined, which can be easily done in terms of the corresponding observed voltages, provided the voltage-candle-power relation for lamp A is known with sufficient accuracy. The method, therefore, transfers the special difficulties of the direct comparison of lights of different color, not to the calibration of the proper screens for color match but to the problem of determining accurately the variation of candlepower with voltage. As this variation must follow some regular law, there was hope of finding an equation which would represent it over a considerable range. As the results of this paper will show, this hope was more than realized, as there was found a simple equation so general in form as to express the variation with voltage, not only of candlepower but also of wattage, current, and wpc, with a high degree of precision over a much wider range than was at first anticipated.

By the use of this equation values observed at different voltages (or wpc) can be so adjusted as to bring them into harmony with one another, and, hence, there is obtained at all points of observation an accuracy greater than would be possible if independent measurements were made at the various points on unrelated quantities like the transmission of different color screens.

The method also makes it possible to obtain accurate values at any point in terms of values at any other point within the range through which the equation is known to hold. Hence, within this range, a tungsten lamp, the value of which at any given voltage (or wpc) is known, may be used as a standard at color match with any light source to be measured, the value of the standard at that color being accurately determined by computation from the corresponding observed voltage. All measurements are thus made at color match without the use of color screens, which may be entirely discarded except only as auxiliaries in making check measurements.

In case screens are to be used, their calibration is accomplished as outlined above, the value of the ratio $\frac{C_2}{C_1}$ being accurately determined by means of the equation.

VI. MEASUREMENT OF LAMPS FOR CHARACTERISTIC RELATIONS

1. THE LAMPS INVESTIGATED

The group first investigated was composed of seven 60-watt, 110-volt tungsten standards whose constancy had been well established. The filaments of these lamps were "formed" drawn wire comprising four hairpin loops, with legs welded in series to the bottom radial anchors. The forming process having removed the elasticity at the bend, the loop retains its form without varying the pressure on the top supports. The latter were light molybdenum wires, each with a helical coil between the loop supporting the filament and the end fused into the central hub. This arrangement gives an elastic mount resulting in ready adaptation to expansion and contraction of filament without sag, and a steady value of current is reached very soon after the lamp is placed in circuit.

Other groups of lamps investigated are described later.

2. THE STANDARDS EMPLOYED

As the above lamps were to be run at 12 voltages corresponding to efficiencies ranging from about 3.8 to 1.1 wpc, it was decided to divide the measurements so as to secure values in terms of carbon and tungsten working standards, with an approximately equal range of color against each group of standards.

Consequently, from the lowest voltage to that corresponding to about 2.0 wpc, carbon working standards were used. Measurements at higher voltages were made against tungsten working standards. It is evident, therefore, that there was a considerable range in color between standards and test lamps running from reddish (relatively low efficiency) to bluish (relatively high efficiency) against each group of standards. But as the average results of the five observers who made the measurements on these lamps had been found to be the same as that of a much larger group of equally experienced observers when a color difference was involved in the photometric measurements, it is believed that the results obtained in these measurements are such as would be obtained by an average or normal eye.

(a) CARBON STANDARDS

These standards operate at the same color as the primary standards with which they are kept in check by frequent intercomparisons. When matching the carbon standards in color the tungsten lamps operate at about 3.1 wpc.

(b) TUNGSTEN STANDARDS

The normal operating efficiency of these standards is approximately 1.5 wpc. Their candlepower and current values have been indirectly verified by the National Physical Laboratory of England, as agreement in candlepower and current values on several groups of lamps (which the tungsten working standards reproduce) was obtained between the National Physical Laboratory and the Bureau as the result of careful measurements by each laboratory, several different methods having been employed.

3. APPARATUS

All measurements of candlepower were made on a double photometer,[4] thus affording a valuable check of one observer against the other. The individual settings were recorded on a chart [5] by a special printing device auxiliary to the photometer. By this method of recording the observer is entirely unprejudiced in his settings. Voltage and current were measured simultaneously by means of two potentiometers. The chart referred to contains a printed candlepower scale from which the values of lamps under

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[4] Described by Rosa and Middlekauff, Pro. A. I. E. E., 29, p. 1191; 1910.

[5] Described by G. W. Middlekauff, this Bulletin, 7, p. 11; 1911. Scientific Paper No. 144.

test may be read off directly in terms of the standards. Thus are avoided the labor and possible errors involved in computing the candlepower values from readings made from a centimeter or other divided scale on the photometer bar.

4. OBSERVED VALUES

In order to avoid a repetition of tables, the observed and computed values for these lamps are given together in Tables 5 and 6, to which reference should be made at this point. (See pp. 503–504.) The two lamps (Nos. 2658 and 2660) exhibited in Table 5 were selected as best representing the group of seven. The observed values given for each lamp are the means of all values obtained by the various observers, there being an average of about seven at each voltage. Each observed value is the average of from 10 to 40 individual photometric settings represented by a corresponding number of points on the record chart.

5. ACCURACY OF THE MEASUREMENTS

Of a total of about 680 individual observed values, about 5 per cent, having comparatively large deviations from the mean, were discarded for reasons considered justifiable. After this procedure, the mean of the remaining observations at each voltage for each lamp was accepted as the observed value and is so designated in what follows.

At the individual voltages the deviation from the accepted mean observed value ranged from 0.7 to 0.2 per cent, the mean deviation being 0.45 per cent. Deviations at color match with carbon and with tungsten standards were approximately equal and were very close to the mean value of 0.45 per cent.

VII. CHARACTERISTIC EQUATIONS

1. FORM OF EQUATIONS PREVIOUSLY USED

Characteristic equations most commonly used heretofore have been of the constant exponent form,[6] $y = ax^b$, the range of application having been usually admitted to be comparatively limited;

[6] A. S. Merrill, Trans. A. I. E. E., 29, p. 959; 1910.

or of the form[7] $y=ax^b[1+cx+dx^2]$ in which x is a variable ratio (*e. g.*, voltage ratio) and a, b, c, and d are constants. The second equation, though considerably more general than the first, does not fit the observed values herein given with the accuracy required for the present purpose. In actual practice, however, the method most generally followed has been to draw curves through observed values without reference to their equations. A modification of this method[8] involves the use of the slope of the characteristics as exponents.

2. FORM OF EQUATION ADOPTED BY THE AUTHORS

As a logarithmic graph of the observed values of voltage and candlepower obtained in this investigation indicated a smooth curve for each lamp, an empirical equation of the form $y=Ax^2+Bx+C$ was assumed, in which $y=\log$ candlepower and $x=\log$ volts, A, B, and C being constants. This is equivalent to the assumption that the rate of change of the slope is constant and equal to $2A$, *i. e.*, $\frac{d^2y}{dx^2}=2A$. This is the first proposal of an equation of this form, although it is the form logically required to produce a smooth curve on logarithmic paper.

The slope, $\frac{dy}{dx}=2Ax+B$, corresponds to the differential coefficient mentioned by Cady[9] and its evaluation gives a factor which may be used for correction from observed values to other values within a small range. A discussion of this equation and its application to certain problems will be given later (p. 514).

The equation expressing the relation of volts to watts was found to be of the same form, y indicating log watts in this case.

Since log wpc = log watts − log cp, the constants in the equation expressing the relation of volts to wpc, are obtained by subtracting those of the voltage-candlepower equation from those of the voltage-wattage equation. Similarly, since log amperes = log watts − log volts, the equation expressing the relation of volts to amperes is of the form $y=Ax^2+(B-1)x+C$, the constants A, B,

[7] F. E. Cady, Elec. Rev. & West. Elec., **59**, p. 1092; 1911.
[8] E. J. Edwards, Gen. Elec. Review, **17**, p. 283; 1914.
[9] Elec. Rev. & West Elec., **59**, p. 1091; 1911.

C having the same values, respectively, as in the voltage-wattage equation. It is, therefore, necessary to solve only two equations expressing the relations, voltage to candlepower, and voltage to wattage (or voltage to current).

The equations expressing these two relations were then solved for each lamp of the group by the method of least squares, Gauss's method of substitution and successive reduction being employed, the results giving A, B, and C, respectively, for each equation, and from these equations all the others were derived.

As the slope of the curve represented by these equations is a function of the wpc, it is evident that either to compare similar curves of the various lamps or to determine a mean value of each variable for the group, it is necessary to choose some value of wpc as a basis and to express the values of the dependent variables in terms of (or percentages of) the values corresponding to the chosen basis. For the sake of brevity, the wpc basis and the corresponding values of all the variables will be hereafter referred to as normal values.

It is entirely immaterial what efficiency is chosen as normal, but for this group of curves 1.20 wpc was found most convenient, and for this reason it was selected. As will be shown later, it is a simple matter to change to any other normal wpc, if it is so desired.

By assuming the values of all variables, except wpc, to be unity at normal efficiency (1.20 wpc) the constant C disappears from every equation except the one for wpc evaluation, where it has the value 0.07918 (i. e., log 1.20).

As all the equations expressing the relation of x to the other variables are of the same form, they may be written together in the general equation

$$y_n = A_n x^2 + B_n x + C$$

where $C = 0$ except when $n = 1$, the quantities signified by $n = 1$, 2, 3, and 4, respectively, being as given in the following section.

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3. THE FUNDAMENTAL EQUATIONS

The fundamental equations and the most important equations derived from them are given in Table 1. In these equations,

x = log voltage ratio,
y_1 = log actual wpc,
y_2 = log candlepower ratio,
y_3 = log wattage ratio,
y_4 = log current ratio,

and the conditional relation among these quantities is that

$$x = y_2 = y_3 = y_4 = \log 1 = 0, \text{ when } y_1 = \log 1.20 = 0.07918.$$

TABLE 1

Characteristic Equations

$$y_1 = 0.918x^2 - 2.009x + 0.07918 \quad (1)$$

$$y_2 = -0.946x^2 + 3.592x \quad (2)$$

$$y_3 = -0.028x^2 + 1.583x \quad (3)$$

$$y_4 = -0.028x^2 + 0.583x \quad (4)$$

Equations derived from the above:

$$x = 1.09490 - 1.0439\sqrt{1.02073 + y_1} \quad (1a)$$

$$x = 1.89870 - 1.0282\sqrt{3.41000 - y_2} \quad (2a)$$

$$y_1 = 2.88028 - 1.5169\sqrt{3.41000 - y_2} - 0.970y_2 \quad (5)$$

$$y_2 = 1.74682 - 1.5878\sqrt{1.02073 + y_1} - 1.031y_1 \quad (5a)$$

These derived equations are obtained as follows: 1a and 2a from 1 and 2, respectively, by expressing x in terms of the corresponding y; 5 from 1 and 2 by the elimination of x; and 5a from 5 by expressing y_2 in terms of y_1. In many problems the equations expressed in this form will be found more convenient than in the original form.

Since equations 1 to 4 are quadratics, there are two values of x which satisfy each. For example, in equation 1a, $x = 1.09490 \pm 1.0439\sqrt{1.02073 + y_1}$. The conditional relation that $x = y = 0$ when $y_1 = 0.07918$ leads to the selection of the value in which the last term is negative. Similar reasoning applies to the other three equations also.

4. METHOD OF REDUCING TO ANY WPC BASIS

(a) REDUCTION OF THE FUNDAMENTAL EQUATIONS

In passing from 1.20 wpc to any other wpc as a basis, no change occurs in the form of the general equation

$$y_n = A_n x^2 + B_n x + C. \tag{6}$$

Even the constant A_n remains unchanged, because, by the fundamental assumption made in choosing this form of equation, the rate of change of slope, $\frac{d^2y}{dx^2}$, is constant and equal to $2A_n$, irrespective of the value of the wpc chosen as normal. The conditional relation among the variables determines the value of the constant C, that is, the log of the normal wpc. Hence, in passing to the new basis, a new value C' (=log new basis) is at once assigned to this constant. The constant B_n is the only one, therefore, requiring any adjustment and this is readily done in the following manner:

Differentiating general equation (6) with respect to x, we have

$$\frac{dy_n}{dx} = 2A_n x + B_n \tag{7}$$

which represents the slope at the wpc corresponding to x, and which at normal wpc is equal in value to the constant B_n. Hence, if x', corresponding to a chosen wpc basis, is found (the value of x' being determined by the substitution of log new basis in equation 1a) and this value be put for x in the above differential equation, we obtain for the slope at the new normal wpc a value

$$B'_n = 2A_n x' + B_n.$$

Therefore, the general equation on the new basis is

$$y'_n = A_n x^2 + (2A_n x' + B_n)x + C' \tag{8}$$

or,

$$y'_n = A_n x^2 + B'_n x + C'. \tag{9}$$

To apply this method of reduction to a particular problem, suppose 1.10 wpc, which is now used as a basis in modern practice, be assumed as normal. The substitution of 0.04139, i. e., log 1.10, in equation 1a gives $x' = 0.01895 = \log 1.0448$, the voltage ratio corresponding to 1.10 wpc. Substituting in general equation (8) this value of x', the values of A_n and B_n as taken from Table 1 (for $n=1$, 2, 3, and 4, respectively), and log 1.10 for C', we have the following fundamental equations based on the new wpc, the conditional relation among the various quantities being that $x=y'_2=y'_3=y'_4=\log 1=0$ when $y'_1=\log 1.10=0.04139$.

TABLE 2

$$y'_1 = 0.918x^2 - 1.974x + 0.04139 \qquad (1')$$

$$y'_2 = -0.946x^2 + 3.556x \qquad (2')$$

$$y'_3 = -0.028x^2 + 1.582x \qquad (3')$$

$$y'_4 = -0.028x^2 + 0.582x \qquad (4')$$

(b) REDUCTION OF KNOWN VALUES

If Y_n is the value of any variable on a basis of 1.20 wpc and Y'_n the corresponding value on any other basis, it is evident, on comparing equations (6) and (9) that

$$\log\left(\frac{Y'_n}{Y_n}\right) = y'_n - y_n = (B'_n - B_n)x + (C' - C)$$
$$= 2A_n x' \cdot x + (C' - C) \qquad (10)$$

C' and C being $=0$ except when $n=1$.

Hence, if a curve, or table, of values of the variables Y_n is given, other curves, or tables, of values based on any chosen wpc may be derived at once by applying the antilogarithm of the second member of the equation (10) as a multiplying factor to the given values of Y_n. This method of reduction is exemplified in the Appendix (p. 525).

5. EXAMPLES ILLUSTRATING USE OF THE EQUATIONS

The first step in the solution of every problem by the present method is to determine from the observed values of voltage, candlepower, and wpc the corresponding values at normal efficiency (1.20 wpc).[10] This is done by substituting the values of log observed wpc in equations 1a and 5a, obtaining from 1a the log of the ratio of the observed voltage to the voltage corresponding to normal wpc, and from 5a the log of the ratio of the observed candlepower to the candlepower corresponding to normal. The numeri-

[10] This is the normal wpc used throughout the remainder of the paper.

cal values of voltage and candlepower at normal are obtained by dividing the observed values by the ratios just found. The value of watts at normal equals the product of normal candlepower by 1.20.

As an example to illustrate the use of the equations, let us take the observed values corresponding to the first and second voltages of lamp No. 2658, as given in Table 5, *viz.*,

1	65.0 volts,	6.64 cp.,	3.694 wpc.
2	70.7 "	9.28 "	3.021 "

Substituting in equation 1a the logs of these two values of wpc, in succession, we obtain values for normal voltage as follows:

1. log. voltage ratio $=1.09490-1.0439\sqrt{1.02073+0.56750}$.
$=1.09490-1.31557=9.77933-10$.
Or voltage ratio=0.6016=60.16 per cent.
That is, 65.0 volts=60.16 per cent of normal voltage.
Therefore, normal voltage=65÷0.6016=108.04.

2. log. voltage ratio $=1.09490-1.0439\sqrt{1.02073+0.48015}$.
$=9.81602-10$.
Or voltage ratio=0.6546=65.46 per cent.
Therefore, normal voltage=70.7÷0.6546=108.00.

Now, substituting the values of the logs of the two values of wpc in equation 5a we obtain values for normal candlepower as follows:

1. log cp ratio $=1.74682-1.5878\sqrt{1.02073+0.56750}-1.031\times0.56750$.
$=1.74682-2.00101-0.58510$.
$=9.16071-10$.
Or cp ratio =14.478 per cent.
Therefore, normal cp=6.64÷0.14478=45.87.

2. log cp ratio $=1.74682-1.5878\sqrt{1.02073+0.48015}-1.031\times0.48015$.
$=9.30658-10$.
Or cp ratio =20.256 per cent.
Therefore, normal cp=9.28÷0.20256=45.81.

The means of these results give the following normal values: 108.02 volts, 45.84 candles, and 55.01 watts (=45.84×1.20). With the above value of voltage (108.02) as basis, computed values for all the variables corresponding to any voltage ratio to normal are found by substituting the log of this ratio in equations 1–4. Values for this lamp corresponding to 65 volts and 70.7 volts are computed as follows. The ratios of these voltages to mean normal (108.02) are 0.6017 and 0.6545, respectively. Substituting the log of these ratios in equation 2, we have

1. log cp ratio $=-0.946\,(9.77938-10)^2+3.592\,(9.77938-10)$.
$=-0.04604-0.79247=-0.83851$
$=9.16149-10$.
Or cp ratio=0.14504=14.504 per cent.
Therefore, computed cp at 65 volts=45.84×0.14504=6.65.

2. log cp ratio $=-0.946\,(9.81591-10)^2+3.592\,(9.81591-10)$.
$=-0.03206-0.66126=-0.69332$
$=9.30668-10$.
Or cp ratio=20.262 per cent.
Therefore, computed cp at 70.7 volts=45.84×0.20262=9.29.

These examples are sufficient to show the method of solution. Although the equations are simple and easy to handle, their use involves comparatively long and tedious computations.

6. TABLES OF CHARACTERISTIC RELATIONS

In order to avoid the computations just referred to, a set of tables (19–22) has been calculated from which, with the aid of an ordinary slide rule, the various factors may be read off directly.

It should be stated that the constants as given in the equations of Table 1 do not include as many figures as were actually used in computing the tables of characteristic relations (Tables 19 to 22), but the differences, when differences occur, between the tabulated values and those obtained by means of the equations of Table 1 are, in all cases, so small as to be entirely negligible.

As these tables will be employed, instead of the equations, in what follows, a brief explanation of their use is given here.

Normal values for voltage and candlepower are obtained from the double Table 19 which takes the place of equations 1a and 5a. Observed wpc in steps of 0.1 and intermediate steps of 0.01 are given, respectively, at the top and left margin of the table, and corresponding to these, in the body of the table, are given percentage factors by which observed values of voltage and candlepower, respectively, are to be multiplied to reduce them to values they would have at normal efficiency. These percentage factors are the reciprocals of those which are obtained in a similar reduction by means of equations 1a and 5a. In other words, the equations referred to give *divisors*, while Table 19 gives *multipliers* to be applied to the observed values to reduce them to normal. The table was thus constructed in order to make it consistent with the other three tables, the figures in the body of each table being used as multipliers in every case when per cent values are given.

Having obtained the value for normal candlepower, the value for normal watts is derived by multiplying by 1.20. Knowing the normal values for all the variables, the values corresponding to any percentage of normal volts may then be read from one of the other tables depending upon the variable considered. Practical use of the tables is made in the following section.

VIII. COMPARISON OF COMPUTED AND OBSERVED VALUES

1. METHOD OF DERIVING COMPUTED VALUES

In order to show the method of using the tables in computing values of the several variables, which are later compared with the observed values, computations are made by this method and use of data obtained on lamp No. 2658 are here continued. In Table 19, corresponding to the last seven values of observed wpc, the following percentage factors are found:

No.	Observed wpc	Percentage factors	
		Voltage	Candlepower
1	1.921	125.13	228.4
2	1.754	120.03	195.1
3	1.556	113.44	158.4
4	1.402	107.88	131.8
5	1.274	102.96	111.2
6	1.204	100.16	100.6
7	1.118	96.51	88.08

Applying these factors to the corresponding values of observed voltage and candlepower, respectively, we obtain the following values for normal voltage and normal candlepower, respectively:

TABLE 3

No.	Observed volts	Percentage factor	Normal volts	Observed cp	Percentage factor	Normal cp
1	86.3	×125.13	=107.99	20.06	×228.4	=45.82
2	90.0	×120.03	=108.03	23.49	×195.1	=45.83
3	95.0	×113.44	=107.77	28.84	×158.4	=45.68
4	100.0	×107.88	=107.88	34.74	×131.8	=45.80
5	105.0	×102.96	=108.11	41.28	×111.2	=45.90
6	108.0	×100.16	=108.17	45.67	×100.6	=45.94
7	112.0	× 96.51	=108.09	52.12	× 88.08	=45.91
Mean			=108.01	Mean		=45.84
Av. dev.			= 0.10%	Av. dev.		= 0.15%

Normal watts=45.84×1.20=55.01

Therefore the mean normal values for this lamp are: 108.01 volts, 45.84 candles, 55.01 watts.

It will be noticed that the mean normal values for this lamp, as found (p. 499) by the equations from two observed values at and near color match with 4-wpc carbon standards, are identical with those found here from seven observed values. While identical results are not to be expected, the above results show that agreement to about 0.2 per cent in candlepower and 0.1 per cent in voltage on the average would be obtained by using any single set of observed values as basis.

The results of a similar computation on lamp No. 2660 are as follows:

TABLE 4

Observed volts	Normal volts	Normal candles
86.4	107.59	45.35
90.0	107.60	45.34
95.0	107.54	45.35
100.0	107.59	45.38
105.0	107.77	45.56
108.0	107.96	45.64
111.5	107.46	45.31
Means	107.65	45.42
Av. dev.	0.12%	0.23%

Normal watts = 45.42 × 1.20 = 54.50

With 108.01 and 107.65, respectively, as the values of normal volts for the two lamps considered, computed values for candlepower, watts, and watts per candle at each observed voltage are obtained from Tables 20, 21, and 22, respectively. For example, the computed values for lamp No. 2658 at 65.0 volts are found as follows:

Since 65.0 = 60.18 per cent of 108.01 volts, we find in the tables corresponding to this per cent volts the following values:

From Table 22, 3.690 actual wpc, the computed value at 65 volts.

From Table 20, 14.52 per cent candles.

From Table 21, 44.620 per cent watts.

$$45.84 \times 0.1452 = 6.66, \text{ computed cp at 65 volts.}$$

$$55.01 \times 0.44620 = 24.55, \text{ computed watts at 65 volts.}$$

In this manner computed values corresponding to all the observed voltages are obtained. If the above values are found by means of the equations, the particular ones used are 1, 2, and 3.

2. TABLES OF COMPUTED AND OBSERVED VALUES

TABLE 5

Comparison of Computed and Observed Values

Lamp No. B. S. 2658

Volts	Candles		Watts		Watts per candle	
	Computed	Observed	Computed	Observed	Computed	Observed
65.0	6.66	6.64	24.55	24.53	3.690	3.694
70.7	9.30	9.28	28.07	28.04	3.020	3.021
75.0	11.71	11.76	30.83	30.81	2.634	2.620
80.0	15.03	15.01	34.17	34.15	2.273	2.275
82.0	16.52	16.56	35.53	35.52	2.151	2.145
86.3	20.06	20.06	38.54	38.53	1.921	1.921
90.0	23.49	23.49	41.20	41.19	1.754	1.754
95.0	28.71	28.84	44.88	44.88	1.564	1.556
100.0	34.67	34.74	48.69	48.70	1.404	1.402
105.0	41.40	41.28	52.60	52.61	1.271	1.274
108.0	45.82	45.67	55.00	55.01	1.200	1.204
112.0	52.21	52.12	58.27	58.29	1.116	1.118
	Av. dev., 0.23%		0.04%		0.19%	

Lamp No. B. S. 2660

Volts	Candles, Computed	Candles, Observed	Watts, Computed	Watts, Observed	Watts per candle, Computed	Watts per candle, Observed
65.0	6.68	6.67	24.45	24.43	3.661	3.662
70.9	9.43	9.38	28.08	28.06	2.977	2.992
75.0	11.75	11.78	30.71	30.68	2.614	2.605
80.0	15.08	15.05	34.03	34.01	2.257	2.260
82.0	16.57	16.56	35.39	35.37	2.136	2.136
86.4	20.21	20.22	38.46	38.44	1.903	1.901
90.0	23.56	23.59	41.03	41.02	1.742	1.739
95.0	28.81	28.83	44.71	44.70	1.553	1.550
100.0	34.77	34.82	48.49	48.50	1.395	1.393
105.0	41.53	41.41	52.39	52.42	1.262	1.266
108.0	45.96	45.71	54.78	54.79	1.192	1.199
111.5	51.53	51.69	57.62	57.63	1.119	1.115
	Av. dev., 0.23%		0.05%		0.24%	

TABLE 6

Comparison of Computed and Observed Values

Mean of the Group of Seven Lamps

Volts	Candles		Deviation in per cent observed from computed value
	Computed	Observed	
65.0	6.63	6.62	−0.15
70.6	9.18	9.16	− .22
75.0	11.66	11.70	+ .34
80.0	14.96	14.98	+ .13
82.0	16.45	16.44	− .06
86.5	20.13	20.17	+ .20
90.0	23.39	23.41	+ .09
95.0	28.61	28.67	+ .21
100.0	34.55	34.62	+ .20
105.0	41.27	41.20	− .17
108.0	45.68	45.43	− .55
111.8	51.72	51.84	+ .21
			Mean... .21%

It should be stated that every lamp of the group was measured at all the voltages given in column 1 of the table with the exception of the second, sixth, and last voltages. These are the means of voltages of the individual lamps, the differences from the mean in every case being less than half a volt. Likewise, the corresponding values of computed and observed candlepower at these voltages are the means of the values for the individual lamps.

3. A SECOND METHOD OF COMPUTING CANDLEPOWER

Another method of computing candlepower values is based on the assumption that computed watts equal observed watts. After obtaining values of voltage and candlepower at normal efficiency the voltage ratios are determined and wpc is read directly from Table 22. Then computed candlepower equals observed watts divided by computed wpc. This method is illustrated also by the use of lamp No. 2658, observed values being taken from Table 5.

TABLE 7

Voltage	Voltage ratios	1 Watts computed from Table 21	2 Watts determined from observations of current and voltage	3 Wpc computed from Table 22	4 Candles (Col. 2÷Col. 3)	5 Observed candles
65.0	60.18	24.55	24.53	3.690	6.65	6.64
70.7	65.46	28.07	28.04	3.020	9.30	9.28
75.0	69.44	30.83	30.81	2.634	11.71	11.76
80.0	74.07	34.17	34.15	2.273	15.03	15.01
82.0	75.92	35.53	35.52	2.151	16.52	16.56
86.3	79.90	38.54	38.53	1.921	20.06	20.06
90.0	83.33	41.20	41.19	1.754	23.49	23.49
95.0	87.95	44.88	44.88	1.564	28.70	28.84
100.0	92.58	48.69	48.70	1.404	34.68	34.74
105.0	97.21	52.60	52.61	1.271	41.39	41.28
108.0	99.99	55.00	55.01	1.200	45.83	45.67
112.0	103.69	58.27	58.29	1.116	52.21	52.12
		Av. dev. 0.04%			0.22%	

It is seen from the above that the computed watts are of a degree of accuracy sufficiently high that we may employ the values determined from the observations, computing wpc by Table 22, and from the two obtain candlepower by division, *i. e.*, cp = W ÷ wpc. Consequently, Tables 21 and 22 are the only ones which need to be used in adjustments of this kind.

IX. FURTHER VERIFICATION OF THE EQUATIONS

1. APPLICATION TO A GROUP OF DRAWN-WIRE LAMPS OF RECENT MANUFACTURE

Although the lamps used in the determination of the constants of the equations were operated at voltages corresponding to efficiencies no higher than 1.1 wpc (because of their value as standards), the results obtained seemed to justify extrapolation to 0.7 wpc. This value was chosen as a limit because it is often used in life testing of tungsten lamps. However, there arose an opportunity to verify the extrapolated values by a group of lamps submitted for test over this range.

These lamps were manufactured early in 1914 and were not intended for standard purposes. Though there was a slight variation in current, they were remarkably steady for commercial lamps, especially at higher voltages. The computed and observed candlepower values are given in Table 8.

TABLE 8

Comparison of Computed and Observed Values. Group of Seven 40-Watt Drawn Wire Lamps

Volts	Candles		Deviation in per cent observed from computed value
	Computed	Observed	
96	18.26	18.20	−0.33
104	24.51	24.48	− .12
112	32.05	32.15	+ .31
120	40.97	41.07	+ .24
128	51.37	51.46	+ .18
136	63.33	63.48	− .24
144	76.91	76.71	− .26
Wpc range 1.6 to 0.72			Mean .24

The maximum deviation of observed from computed candlepower values of a single lamp of this group at one voltage point was 0.56 per cent.

A comparison of the deviations in Tables 6 and 8 (96 volts in the latter corresponding approximately, in wpc, to 95 volts in the former) shows that the voltage-candlepower curve fits the observed values with practically the same degree of accuracy throughout the whole range from 3.7 to 0.7 wpc, and that the + and — deviations are about equal in number.

In order to test fully the accuracy of equation 4, Table 1, as applied to these lamps, the following method was used in obtaining ampere readings on three lamps of the group:

A ballast lamp of 40-watt size was set to a desired voltage, and this voltage was applied suddenly to the lamp under test while rotating approximately 120 rpm.

The test lamp, still rotating, was then thrown out of circuit, the next voltage being determined and applied in the same manner, and so on through all of the voltages. On changing over from ballast to test lamp a small adjustment of voltage (not exceeding 0.2 per cent) was necessary, since the resistances of test and ballast lamps were not quite equal.

Previous measurements had been made as follows:

The lamps while rotating were kept continuously in circuit and the voltages given applied in regular order ascending and descending by adjusting resistance, thus causing the filament to burn at all temperatures included in the voltage range.

Results on one lamp are given in Table 9. The differences on the other lamps were of about the same magnitude.

TABLE 9

Current by Up and Down Method

Volts	Ampere		
	Up	Down	Mean
72	0.26200	0.26248	0.26224
80	.27905	.27940	.27922
88	.29527	.29556	.29542
96	.31083	.31103	.31093
104	.32574	.32599	.32586
112	.34022	.34035	.34028
120	.35413	.35430	.35422
128	.36781	.36782	.36782
136	.38083	.38087	.38085
144	.39365	.39368	.39366

Because of differences between up and down readings at voltages below 112, the method of direct voltage application described above was employed at each voltage from 72 to 112, inclusive, the order of application and observed ampere values being as shown in Table 10.

TABLE 10

Current, Direct Voltage Application Method

Volts	Ampere	Mean ampere	Mean ampere from up and down method
72	0. 26231	0. 26229	0. 26224
112	. 34025	. 34023	. 34028
80	. 27924	. 27920	. 27922
96	. 31092	. 31090	. 31093
88	. 29538	. 29536	. 29542
104	. 32582	. 32582	. 32586
112	. 34021		
72	. 26227		
104	. 32581		
96	. 31087		
80	. 27915		
88	. 29533		

The up and down method had shown values which were very close to those obtained from equation 4 between 112 and 144 volts, but at the lower voltages the residuals were of greater magnitude. The probability of these residuals being greater is indicated by increasing deviations from the mean as the lower voltages are reached. That the method of direct voltage application obviates this difficulty is shown in the following table, in which observed values of current from 72 to 112 volts are those found by this method.

TABLE 11

Comparison of Computed and Observed Values of Current

MEAN OF THREE 40-WATT LAMPS.

Volts	Ampere		Deviation of observed from computed values
	Computed	Observed	
72	0. 26183	0. 26176	0. 00007
80	. 27874	. 27865	. 00009
88	. 29481	. 29478	. 00003
96	. 31028	. 31027	. 00001
104	. 32522	. 32516	. 00006
112	. 33956	. 33956	. 00000
120	. 35346	. 35346	. 00000
128	. 36696	. 36696	. 00000
136	. 38007	. 38001	. 00006
144	. 39281	. 39280	. 00001
Average	....	....	. 011%

2. APPLICATION TO "GETTER" LAMPS OF VARIOUS SIZES AND MANUFACTURE

The following are lamps of recent manufacture with "getter" material placed on the anchor wires. The filaments were each a single piece of drawn wire mounted on an arbor having five top anchors in some lamps and six in the others.

TABLE 12

Size of lamp	Volts	Candles: Computed	Candles: Observed	Deviation, per cent observed from computed value
25-watt	88	8.49	8.50	+0.12
	99	13.18	13.14	− .30
	110	19.37	19.37	.00
	121	27.22	27.11	− .41
	Wpc 1.97 to 1.02			Mean .21
40-watt	88	15.34	15.29	−0.33
	99	23.74	23.78	+ .17
	110	34.73	34.78	+ .14
	121	48.63	48.68	+ .10
	Wpc 1.80 to 0.94			Mean .14
60-watt	88	22.44	22.43	−0.05
	99	34.71	34.75	+ .12
	110	50.78	50.82	+ .08
	121	71.09	71.12	+ .04
	Wpc 1.78 to 0.93			Mean .07
100-watt	88	40.34	40.41	+0.17
	99	62.21	62.20	− .02
	110	90.74	90.49	− .28
	121	126.70	126.80	+ .08
	Wpc 1.66 to 0.88			Mean .14

X. USE OF CHARACTERISTIC EQUATIONS IN STANDARDIZING LAMPS

1. LAMPS STANDARDIZED FOR VOLTAGE AT A SPECIFIED CANDLEPOWER

The following illustrates the application of the characteristic equations in standardizing work. The lamps considered were of recent manufacture and were to be standardized as follows:

Lamp No.	Size	Required
1 and 2	25-watt	Volts for 20 candles
3 and 4	40-watt	Volts for 34 candles
5, 6, and 7	60-watt	Volts for 52 candles

The lamps were photometered at five voltages corresponding to the following and against the standards mentioned: (1) At color match with 4-wpc carbon standards; (2) at color match with 1.5-wpc tungsten standards; (3) at two voltages corresponding as nearly as possible to, but on either side of, the required candlepowers, these observations involving color difference with tungsten standards; (4) at color match with tungsten standards whose candlepower values are known through a wide range in voltage, this group including three of the lamps first investigated and three lamps calibrated as working standards in terms of the original group.

The observed values of candlepower and the values of wpc obtained from observed values of voltage and amperes are given in Table 13. Values obtained by the third method (see above) are not given in this table, as they involved measurements at two voltages, one above and the other below the candlepower values required.

TABLE 13

Observed Values

Lamp No.	First method			Second method			Fourth method		
	Volts	Cp	Wpc	Volts	Cp	Wpc	Volts	Cp	Wpc
1	68.7	3.83	3.006	94.0	12.66	1.508	106.8	20.04	1.158
2	69.6	3.87	3.001	96.0	13.15	1.474	107.9	20.00	1.166
3	68.5	6.31	3.036	95.0	21.98	1.467	107.2	34.15	1.144
4	68.2	6.17	3.077	95.0	21.87	1.473	107.3	34.11	1.144
5	68.5	8.94	3.069	95.0	31.10	1.485	109.6	52.20	1.109
6	70.8	9.19	3.071	97.3	31.12	1.506	112.1	52.04	1.128
7	70.8	9.20	3.060	97.6	31.14	1.508	112.3	51.66	1.136

From these observed values for each lamp the voltage for the candlepower required was computed, the value obtained for each being as given in the following table:

TABLE 14

Computed Voltage for Required Candlepower

Lamp No.	First method	Second method	Third method	Fourth method	Means
1	106.7	106.65	106.75	106.8	106.7
2	107.7	107.75	107.9	107.9	107.8
1	107.2	107.1	107.2	107.1	107.2
4	107.3	107.3	107.25	107.2	107.3
5	109.5	109.5	109.55	109.5	109.5
6	112.2	112.1	112.15	112.1	112.1
7	112.3	112.4	112.3	112.5	112.4
Means	109.0	109.0	109.0	109.0	109.0

In Table 15 the observed candlepower values at the voltages corresponding to color match with tungsten standards (see Table 13) are compared with the values obtained by computation from observed values at carbon color.

TABLE 15

Lamp No.	Computed cp	Observed cp	Deviation in per cent observed from computed value
1	12.63	12.66	+0.24
2	13.17	13.15	−.15
3	21.92	21.98	+.27
4	21.87	21.87	.00
5	31.14	31.10	−.13
6	30.95	31.12	+.55
7	31.32	31.14	−.57
Means	23.29	23.29	.27

It should be noted that the above lamps were not adjusted to a mean of 23.29 cp, but that the mean of the seven lamps *is actually identical.* This agreement is undoubtedly accidental, as a deviation of at least 0.1 per cent would be expected from a consideration of the mean deviation.

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2. LAMPS STANDARDIZED AT GIVEN VOLTAGE BY COMPUTATION FROM VALUES OBSERVED AT COLOR MATCH WITH 4-WPC CARBON STANDARDS

Table 16 illustrates what may be accomplished by computation of candlepower from values determined at a single voltage. These lamps are 60-watt Osram standards (sintered filament), which were measured about two years ago by the Bureau directly in terms of the 4-wpc primary carbon standards, with which they were matched in color by the use of calibrated blue glass screens. These lamps had been previously measured at the National Physical Laboratory, England, in terms of a corresponding group of carbon standards. In both laboratories all measurements were made with the lamps operating at 102.0 volts, corresponding to about 1.5 wpc.

In order to determine how well the equations apply, the lamps were remeasured a short time ago, not at 1.5 wpc, but at color match with 4-wpc carbon standards. At this color their efficiency was about 3.1 wpc, the corresponding voltages of the various lamps being approximately 71. From the values thus obtained those given in column (a) of the table were computed. The values in columns (b) and (c) are, respectively, the certified values of the laboratory indicated.

TABLE 16

Lamp No.	Candles			Deviations in per cent	
	(a) Computed values	(b) B. S. observed values	(c) N. P. L. observed values	(b-a)	(c-a)
57-i	31.9	32.0	31.95	+0.31	+0.16
57-j	27.7	27.85	27.7	+ .54	± .00
57-k	32.85	32.85	32.85	± .00	± .00
57-l	32.45	32.35	32.35	− .31	− .31
57-m	32.25	32.05	32.1	− .62	− .47
57-n	31.75	31.55	31.5	− .63	− .79
Means	31.48	31.44	31.41	.40	.29

Evidently, measurements at color match with either carbon or tungsten standards will suffice to determine the values for candlepower; further, values of candlepower or voltage at any intermediate point can be computed from the color match observations with no sacrifice in precision, the great advantage being the elimination of personal errors due to color difference.

However, the following points should receive careful attention in this connection: (1) In adjusting the voltage of tungsten lamps for color match with carbon standards great care should be exercised, because if a color match is not really obtained, two observers of like color perception may introduce errors which do not counterbalance even though their observations may appear consistent and conclusive; (2) candlepower values of the smaller sizes of lamps being low, small differences in distance from the test lamps for photometric balance result in considerable differences in observed candlepower values, so that determinations at this point should be made with the highest degree of accuracy attainable, and all confusing features should be eliminated.

XI. REDUCTION OF THE GENERAL EQUATION TO OTHER FORMS

1. EXPONENTIAL FORM

General equation (6) may be readily changed to the form $y = x^u$, which is frequently used in factory and laboratory practice. If we assume Y_n, X, and c as the values of which y_n, x, and C are the logs, then (6) may be written in the form,

$$Y_n = cX^{A_n x + B_n} \qquad (11)$$

This becomes evident when logs are taken of both members of equation (11). We then have

$$\log Y_n = A_n (\log X)^2 + B_n (\log X) + \log c$$

or

$$y_n = A_n x^2 + B_n x + C$$

Now, if we assume $c = 1$, C becomes O, and equation (11) reduces to

$$Y_n = X^{A_n x + B_n} \equiv X^{u_n} \qquad (12)$$

where u_n $(= A_n x + B_n)$ is an exponent which may be applied to the voltage ratio, to obtain the corresponding ratios of all the other variables, wpc included, normal values being at 1.20 wpc. On any other wpc basis this exponent, in accordance with the nomenclature in equation (9), has the value $u_n = A_n x + B'_n$.

With the constants A_n and B'_n known for different normal bases, it is a simple matter to evaluate u_n for any value of x, and to construct a family of curves from which at any observed wpc may be read the proper exponent to apply to any ratio of the observed voltage to obtain the corresponding ratios of the other variables. A family of curves for u_2, the voltage-candlepower exponent, and another for u_4, the voltage-current exponent, constructed in this manner, are shown in Figures 1 and 2, respectively. These two families of curves are sufficient for the determination of all values of u_n because the voltage-wattage exponent $u_3 = u_4 + 1$, and the voltage-wpc exponent $u_1 = u_3 - u_2$.

2. DIFFERENTIAL FORM

General equation (9), which is based on a voltage ratio corresponding to any chosen wpc, by differentiation reduces to

$$\frac{dy_n}{dx} = 2A_n x + B_n' \tag{13}$$

which at normal

$$= B_n' \tag{14}$$

For a small finite change in voltage this may be written

$$\Delta y_n = B_n' \Delta x$$

or as a close approximation (using the same nomenclature as in the preceding section)

$$\Delta Y_n = B_n' \Delta X$$

it being assumed that the change in log Y_n is proportional to the change in Y_n itself. Hence, at normal wpc, the constant B_n' may be used as a multiplying factor for corrections of all the other variables through a small range in voltage. For precision work this range can not exceed 2 per cent, since beyond this limit errors greater than 0.2 per cent in candlepower (not negligible in accurate measurement) will result. As B_n' is the value of the exponent u_n at normal ($x = 0$), the values appearing on the 100 per cent curves of

Figures 1 and 2 may be used as differential coefficients to apply to voltage change to obtain corresponding changes in the other variables.

Cady,[10] who has investigated the voltage-candlepower relation, gives a table of coefficients based on 1.25 normal wpc. These values are given below in comparison with values of B_n' as determined by equation (14), normal being taken at 1.25 wpc.

TABLE 17

Comparison of Differential Coefficients

Per cent of normal voltage	Coefficients		
	Cady		M. & S.
	Former	New	
60	4.06	4.06	4.028
70	3.91	3.90	3.901
80	3.79	3.77	3.790
85	3.74	3.71	3.758
90	3.69	3.66	3.695
95	3.64	3.60	3.650
100	3.59	3.56	3.608
105	3.54	3.51	3.568
110	3.49	3.46	3.530
115		3.42	3.493
120		3.39	3.458
130		3.32	3.393
140		3.25	3.332

If we take the mean of two coefficients as determined by equation (13), one for $x=0$, the other for $x=x_1$, we obtain $A_n x_1 + B_n'$, which, by last section, is the value of u_n for the voltage ratio, the log of which is x_1. Hence, with a table of differential coefficients as that given above, the exponent corresponding to any voltage ratio within the limits of the table is at once obtained by taking the mean of the coefficients corresponding to normal and to the given ratio. For example, using values in the last column of the table, the mean of the values at 70 per cent and 100 per cent volts (viz, 3.901 and 3.608) equals 3.754, the exponent corresponding to 70 per cent volts. Compare with this the value read from Figure 1 at 1.25 wpc and 70 per cent volts.

[10] Elec. Rev. and West. Elec., 59, p. 1091; 1911.

Example illustrating the use of exponents (see Figs. 1 and 2):

Given 100 per cent voltage at 1.10 wpc, to find ratios of candlepower, current, wattage, and wpc at 115 per cent voltage.

Solution: At 1.10 wpc and 115 per cent voltage in Figure 1, we find voltage-candlepower exponent $u_2=3.499$, and in Figure 2 voltage-current exponent $u_4=0.5802$. Then, voltage-wattage exponent $u_3=u_4+1=1.5802$, and voltage-wpc exponent $u_1=u_3-u_2=-1.919$. Therefore,

$$\text{candlepower ratio}=(1.15)^{3.499}=1.631 \quad =163.1 \text{ per cent.}$$
$$\text{current ratio}=(1.15)^{0.5802}=1.0845 \quad =108.45 \text{ per cent.}$$
$$\text{wattage ratio}=(1.15)^{1.5802}=1.2471 \quad =124.71 \text{ per cent.}$$
$$\text{wpc ratio}=(1.15)^{-1.919}=0.76476=76.476 \text{ per cent.}$$
$$\text{actual wpc}=1.10\times .76476=0.8412$$

A check upon these results may be had by reference to p. 525 where the same problem is solved by means of the tables.

Example illustrating the use of differential coefficients (see Figs. 1 and 2):

Given as observed values, volts 104, candles 32.5, and wpc 1.15, to find volts for 34 candles.

Solution: At 1.15 wpc and 100 per cent voltage in Figure 1 we find $u_2=3.575$.

$$Cp=34-32.5=1.5. \quad \text{Since } \frac{\Delta cp}{cp}=3.575\frac{\Delta \text{volts}}{\text{volts}} \quad \Delta V=\frac{V\Delta cp}{3.575cp}=\frac{104\times 1.5}{3.575\times 32.5}=1.3 \text{ volts.}$$

Hence, voltage for 34 candles $=V+\Delta V=104+1.3=105.3$, the voltage required.

The use of exponents has been described quite fully by Edwards,[11] who also gives two families of curves similar to those shown in Figs. 1 and 2. As a method of reduction the use of exponents is convenient in certain cases, but it requires the use of a table of logarithms, or an exponential slide rule, auxiliary to the curves, and at best the method is merely a special application of the equations of Table 1.

Curves of this kind could, of course, be drawn for any two variables chosen, but all would be based on the equations just referred to. Such curves are useful principally in computations based on one observed value, considered as normal, and of necessity assumed on the characteristic curve.

The more general method described in this paper determines the characteristic curve which most nearly approaches any desired number of observed values, none of which may fall exactly on the curve. Further, reduction to normal values from each of several observations furnishes a valuable preliminary

[11] General Electric Review, **17**, p. 283; 1914.

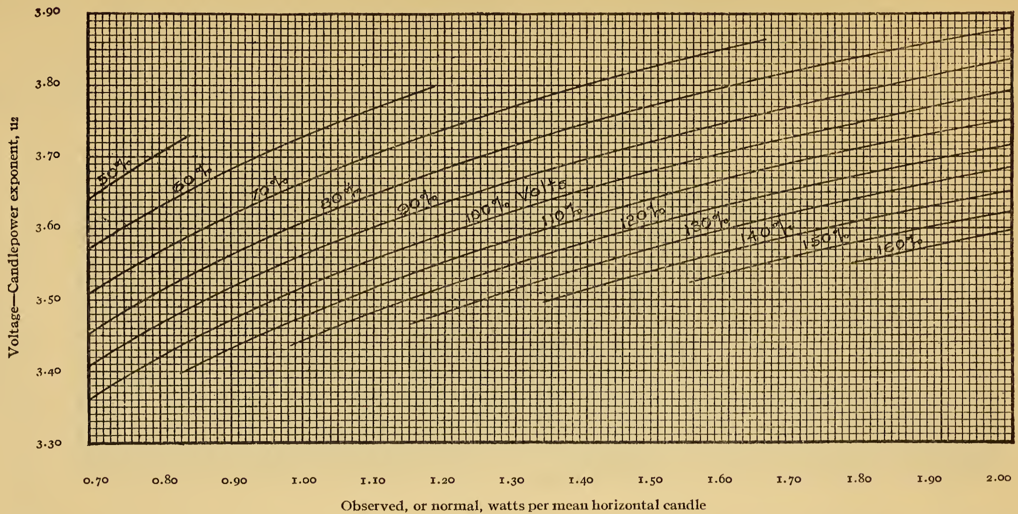

FIG. 1.—*From this family of curves may be read, at any observed or normal wpc and per cent of the corresponding voltage, the exponent u_2 which, when applied to the voltage ratio, gives the corresponding candlepower ratio.* *See example page 516*

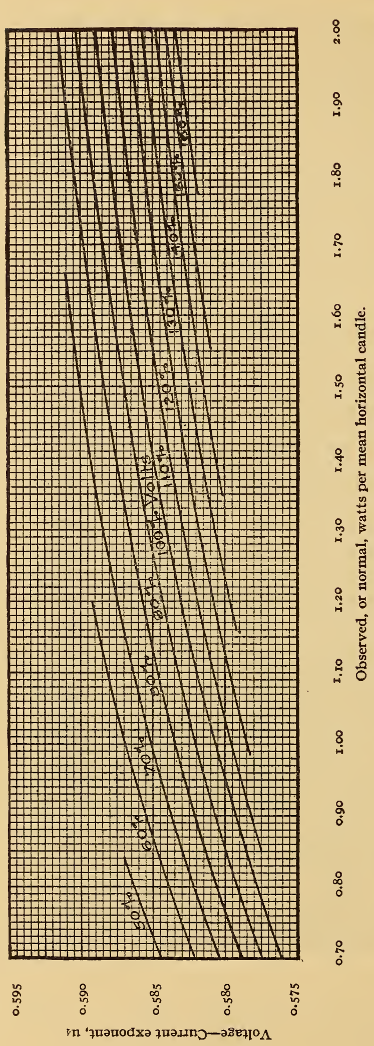

FIG. 2.—*From this family of curves may be read, at any observed or normal wpc and per cent of the corresponding voltage, the exponent u_4 which, when applied to the voltage ratio, gives the corresponding current ratio. See example, page 516*

check upon the accuracy of the observations. The problem of reducing observations at any given wpc to values they would have at any other wpc can certainly be most simply solved by the use of the factors read directly from Table 19. (See problem 3, p. 524.)

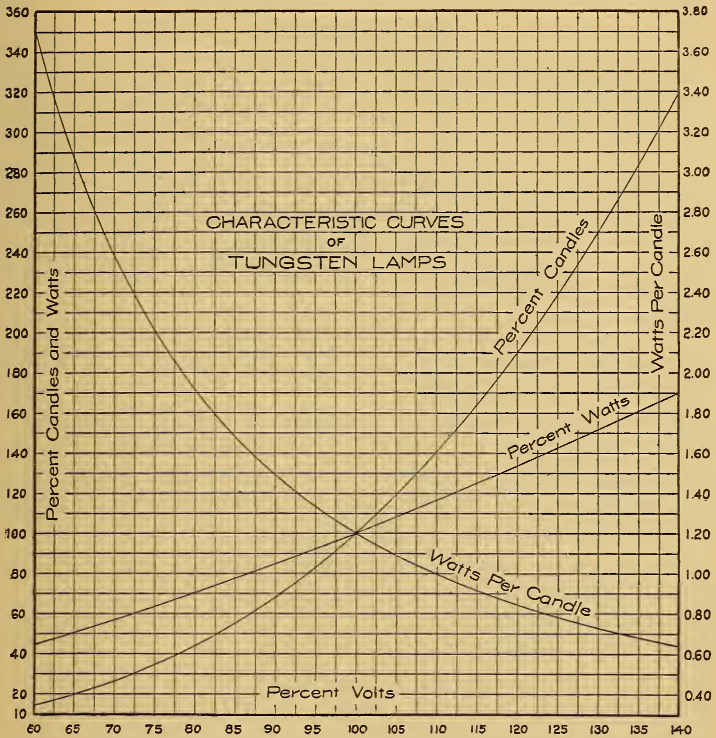

FIG. 3.—*Characteristic curves of 100-130-volt vacuum tungsten filament lamps of sizes 25 to 100 watts for the range included between 0.7 and 3.7 wpc, 100 per cent values being at 1.20 wpc*

Any derived method based on the fundamental equations of Table 1 can, of course, be made to give results of the same degree of accuracy as those obtained by substitution in the equations themselves. For example, a family of curves of percentage values could be drawn, each curve being designated by the normal wpc on which it is based. If curves are to be employed, this method would appear to be the most direct, and ordinary sizes of plotting paper could be used, if the range in wpc is not too great, or the data could be divided into sections, each with its own family of curves.

XII. SUMMARY.

The results found in this investigation show that for 100 to 130 volt vacuum tungsten lamps of the ordinary sizes an equation of the form $y_n = A_n x^2 + B_n x + C$ expresses the voltage-candlepower and the voltage-wpc relations to well within 0.3 per cent, and the voltage-wattage relation to well within 0.05 per cent of the observed values over the whole range investigated, viz, from 0.7 wpc to 3.3 wpc, the latter limit extending somewhat beyond the wpc corresponding to color match with 4-wpc carbon lamps.

It is therefore possible, after carefully standardizing a tungsten lamp at color match with the 4-wpc carbon primary standards which maintain the international candle unit, to calculate with a high degree of precision its candlepower, voltage, and current at any desired wpc (or color) within the range mentioned. In this manner groups of standards at different values of wpc may be established in terms of the primary standards without a single measurement made with a color difference.

Further, any standardized tungsten lamp of the size and voltage investigated, together with the set of tables appended, is virtually a standard for use at any wpc (or color) within the range above specified, because its candlepower and current at that color can be completely determined from a knowledge of the corresponding observed voltage. By this method of standardizing lamps, all measurements are made at color match without the use of color screens, which may be entirely discarded, except in so far as they may be of use as auxiliaries in making check measurements.

Although the fundamental equations as given and the tables derived from them are computed on a basis of 1.20 wpc, they are readily reduced to any other desired basis by any one of several different methods which are fully described.

It is shown that, from the general equation given above, the more important equations heretofore employed may be derived. Curves used in other methods are constructed and the limitations of their use are discussed.

The reduction to normal values, which is the foundation of all computations by the method outlined in this paper, seems to be the simplest and most direct, as it does not directly involve the use of exponents, differentials, or other terms of mathematics beyond those of simple arithmetic.

The wpc employed throughout the paper is watts per mean horizontal candle. Variations in reduction factor among lamps of the same size, or among lamps of different sizes, have not been of sufficient magnitude to affect relative values.

If the relation of life to any one of the variables is known its relation to any other variable may be readily expressed by substitution in the fundamental equations.

As to the effects of a radical change in filament material on lamp characteristics, no prediction is made.

The investigation is being extended to include the gas-filled tungsten lamp, and the results, though yet incomplete, indicate that, provided due consideration is given to certain new variables introduced by this type, the same equations, with little, if any, change in the constants, apply to this lamp also.

The authors are under obligations to their collaborators in the laboratory for assistance in the photometric measurements, especially to Mr. D. H. Tuck; and to Mr. H. B. Sinelnick, who also drew the curves and checked most of the computations.

Washington, October 10, 1914.

XIV. TABLES OF CHARACTERISTIC RELATIONS

1. EXPLANATION OF USE OF TABLES

The first step in the solution of every problem involving characteristic equations is to determine from the observed values of voltage, candlepower, and wpc the corresponding values at normal efficiency, which for these tables was chosen at 1.20 wpc.

This is done by reference to Table 19, in which observed wpc in steps of 0.1 and intermediate steps of 0.01 are given at the top and left margin, respectively. In the body of the table under "volts" and "cp," respectively, are given the corresponding percentage factors by which the observed voltage and observed candlepower, respectively, are to be multiplied to reduce them to normal values. Normal wattage is found by multiplying normal candlepower by 1.20.

For example, if the observed values are 110 volts, 25 candles, 1.35 wpc, the corresponding normal values are found as follows: Corresponding to 1.35 wpc, find 106.0 under volts and 123.3 under cp. Then, $110 \times 1.060 = 116.6$ volts, $25 \times 1.233 = 30.82$ candles, and $30.82 \times 1.20 = 36.98$ watts, these being the normal values.

With these values known, we are in a position to read from one of the other three tables (viz, 20–22) values corresponding to any desired percentage value of any one of the variables given.

The simplest problem is when values corresponding to a given voltage are required, because all three tables are arranged for voltage considered as the independent variable, and the other variables are given in the body of the table.

For example, assuming the normal values just found, suppose values for candlepower, wattage, and wpc corresponding to 125 volts are required. The voltage ratio $= 125 \div 1.166 = 107.2$ per cent. Corresponding to 107.2 per cent volts in Tables 20, 21, and 22, find 128.1 per cent cp, 111.63 per cent watts, and 1.045 actual wpc, respectively. The numerical values corresponding to the two percentage values are found by multiplying each by the corresponding normal value as follows:

$1.281 \times 30.82 = 39.49$ candles, and $1.1163 \times 36.98 = 41.28$ watts.

Hence the corresponding values of all the variables are 125.0 volts, 39.48 candles, 41.28 watts, and 1.045 wpc.

As a second problem, suppose that the values of voltage, wattage, and wpc, corresponding to 20 candles, are required, the same normal values being assumed. This candlepower value is $20 \div 30.82 = 64.9$ per cent of normal. From 64.9 per cent cp in the body of Table 20 find the corresponding voltage per cent at the top and margin—that is, $88.0 + (2.14 \div 2.67) = 88.8$ per cent volts. With this value known, find 82.846 per cent watts in Table 21 and 1.532 actual wpc in Table 22. Multiplying percentage values by corresponding normal values, we have $0.888 \times 116.6 = 103.5$ volts, and $0.82846 \times 36.98 = 30.64$ watts. The variables are therefore 20.0 candles, 103.5 volts, 30.64 watts, and 1.532 wpc.

In the same manner values for all the variables corresponding to a given value of wattage or of wpc may be found also.

A third problem of importance to the testing laboratory involves the reduction of voltage, candlepower, and wattage from observed values to values they would have at some given wpc. (For example, a wpc at which the lamps are to be run on life test.) The calculation of voltage is the one of most importance, the other variables being usually neglected in life-test calculations.

Example: Given 110 volts, 88 candles, 1.05 wpc; required volts, candles, and watts at 0.7 wpc.

Solution: In Table 19 find at 1.05 wpc 93.49 per cent volts and 78.66 per cent candles, and at 0.7 wpc 75.26 per cent volts and 37.23 per cent candles. Then at 0.7 wpc

$$\text{volts} = \frac{110 \times 93.49}{75.26} = 136.6$$

$$\text{candles} = \frac{88 \times 78.66}{37.23} = 185.93$$

$$\text{watts} = 185.93 \times 0.7 = 130.15$$

2. REDUCTION OF VALUES TO A WPC BASIS OTHER THAN 1.20

If some other wpc than 1.20 be chosen as normal, tables of values can be readily determined from these tables by any of the three following methods:

(*a*) Suppose, for example, that 1.10 wpc is chosen as normal. Corresponding to 1.10 in Table 22 find 104.48 per cent volts. Corresponding to 104.48 per cent volts in Tables 20 and 21 find 116.9 per cent cp and 107.18 per cent watts, respectively. Therefore, the values in the present tables corresponding to normal in the new tables are as follows: 104.48 per cent volts, 116.9 per cent cp, 107.18 per cent watts, and 1.10 actual wpc.

Now, suppose, for example, that values at 115.0 per cent volts on the new basis are required. Voltage ratio is then 115.0 × 1.0448 = 120.15 per cent. Corresponding to 120.15 per cent volts in Tables 20, 21, and 22 find 190.7 per cent cp, 133.66 per cent watts, and 0.8412 actual wpc, respectively.

Hence, corresponding to 115.0 per cent volts, we have for the new tables:

$$\begin{aligned} 120.15 \div 1.0448 &= 115.0 \text{ per cent volts,} \\ 190.7 \div 1.169 &= 163.1 \text{ per cent candles,} \\ 133.66 \div 1.0718 &= 124.71 \text{ per cent watts,} \\ \text{and } 0.8412 &= \text{actual wpc.} \end{aligned}$$

In the same manner, values corresponding to other percentage values of voltage may be found, and a complete set of tables corresponding to Tables 20, 21, and 22, on the new basis, may be constructed.

Values for Table 19 are obtained by dividing the tabulated values of the factors designated "volts" by 0.9573 and those designated "cp" by 0.8554, these being the values at 1.10 wpc. For example, the tabulated values of "volts" and "cp" at 1.00 wpc in Table 19 are 91.17 and 72.00, respectively. Values for the new table are then

$$\frac{91.17}{0.9573} = 95.24 \text{ "volts" and } \frac{72.00}{0.8554} = 84.17 \text{ "cp."}$$

(*b*) Values corresponding to each point in Tables 20, 21, and 22 need not necessarily be computed as given above. A simple method is to compute values at points, say, 5 or 10 per cent apart in voltage (for example, 80, 85, 90, 95, etc.) and taking differences between the values obtained and those given in the tables. Then, with per cent volts as ordinates and differences as abscissas, a smooth curve may be drawn through the points found, and from it the difference at any per cent volts may be read. These differences added to the tabulated values give values for the new tables.

(*c*) The ratio of the values based on any normal wpc to the values given in Tables 20 to 22, inclusive, may be accurately computed from equation (10), page 498. Designating this ratio by R_n and its logarithm by r_n, equation (10) becomes

$$r_n = 2A_n x'.x + (C' - C) \equiv K_n x + L,$$

C' and C being $= O$ except when $n = 1$, x representing any voltage ratio on the new normal wpc basis, and x' the voltage ratio on the 1.20-wpc basis corresponding to the new normal wpc. The following table gives values for R_n for a normal 1.10-wpc basis:

TABLE 18

Factors by Which Tabulated Values May Be Multiplied to Obtain Corresponding Values for a Normal 1.10 Wpc Basis

Per cent volts	R_2 Table 20	R_3 Table 21	R_1 Table 22
60	1.019	1.0005	0.9004
70	1.013	1.0004	.9053
80	1.008	1.0002	.9095
90	1.004	1.0001	.9133
100	1.000	1.0000	.9167
110	0.9966	0.99989	.9197
120	.9935	.99982	.9225
125	.9920	.99977	.9238

From a smooth curve drawn through the above values of per cent volts as ordinates and R_n as abscissas, the values of R_n may be read for any per cent volts, and the new tables may be quickly and accurately constructed.

Values for Table 19 are most easily obtained by method (*a*) first outlined above.

3. TABLES OF CHARACTERISTIC RELATIONS

TABLE 19

Table of percentage multiplying factors for reducing observed values of voltage and observed values of candlepower at known wpc to values they would have at 1.20 wpc. Voltage factors are indicated by "Volts"; candlepower factors by "Cp."

Example: Given as observed values, 112.0 volts, 16.0 candles, 1.450 wpc, to find volts and candles at 1.20 wpc.

Solution: Corresponding to 1.450 wpc find 109.7, the voltage percentage multiplier, and 139.9, the candlepower percentage multiplier. The values corresponding to 1.20 wpc are, therefore, 112.0×1.097=122.86 volts, and 16.0×1.399=22.38 candles.

Obs. wpc	0.70				0.80				0.90			
	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.
0.00	75.26	0.60	37.23	1.01	81.01	0.55	47.79	1.11	86.29	0.50	59.40	1.21
.01	75.86	.59	38.24	1.02	81.56	.54	48.90	1.13	86.79	.50	60.61	1.23
.02	76.45	.59	39.26	1.03	82.10	.54	50.03	1.13	87.29	.50	61.84	1.23
.03	77.04	.58	40.29	1.04	82.64	.53	51.16	1.15	87.79	.49	63.07	1.25
.04	77.62	.58	41.33	1.05	83.17	.53	52.31	1.16	88.28	.49	64.32	1.25
.05	78.20	.57	42.38	1.06	83.70	.53	53.47	1.17	88.77	.49	65.57	1.27
.06	78.77	.57	43.44	1.07	84.23	.52	54.64	1.17	89.26	.48	66.84	1.27
.07	79.34	.56	44.51	1.09	84.75	.52	55.81	1.19	89.74	.48	68.11	1.29
.08	79.90	.56	45.60	1.09	85.27	.51	57.00	1.19	90.22	.48	69.40	1.29
.09	80.46	.55	46.69	1.10	85.78	.51	58.19	1.21	90.70	.47	70.69	1.31
.10	81.01		47.79		86.29		59.40		91.17		72.00	

Obs. wpc	1.00				1.10				1.20			
0.00	91.17	0.47	72.00	1.31	95.73	0.44	85.54	1.41	100.0	0.4	100.0	1.5
.01	91.64	.47	73.31	1.32	96.17	.43	86.95	1.41	100.4	.4	101.5	1.5
.02	92.11	.46	74.63	1.33	96.60	.44	88.36	1.42	100.8	.4	103.0	1.5
.03	92.57	.46	75.96	1.35	97.04	.43	89.78	1.44	101.2	.4	104.5	1.5
.04	93.03	.46	77.31	1.35	97.47	.43	91.22	1.44	101.6	.4	106.0	1.6
.05	93.49	.45	78.66	1.36	97.90	.42	92.66	1.45	102.0	.4	107.6	1.5
.06	93.94	.45	80.02	1.36	98.32	.43	94.11	1.46	102.4	.4	109.1	1.5
.07	94.39	.45	81.38	1.38	98.75	.42	95.57	1.47	102.8	.4	110.6	1.6
.08	94.84	.45	82.76	1.38	99.17	.42	97.04	1.47	103.2	.4	112.2	1.6
.09	95.29	.44	84.14	1.40	99.59	.41	98.51	1.49	103.6	.4	113.8	1.5
.10	95.73		85.54		100.00		100.00		104.0		115.3	

TABLE 19—Continued

Obs. wpc	1.30				1.40				1.50			
	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.
0.00	104.0		115.3		107.8		131.5		111.5		148.5	
		0.4		1.6		0.4		1.6		0.3		1.7
.01	104.4		116.9		108.2		133.1		111.8		150.2	
		.4		1.6		.4		1.7		.4		1.8
.02	104.8		118.5		108.6		134.8		112.2		152.0	
		.4		1.6		.4		1.7		.3		1.7
.03	105.2		120.1		109.0		136.5		112.5		153.7	
		.4		1.6		.3		1.7		.4		1.8
.04	105.6		121.7		109.3		138.2		112.9		155.5	
		.4		1.6		.4		1.7		.3		1.8
.05	106.0		123.3		109.7		139.9		113.2		157.3	
		.4		1.6		.3		1.7		.4		1.8
.06	106.4		124.9		110.0		141.6		113.6		159.1	
		.4		1.6		.4		1.7		.3		1.8
.07	106.8		126.5		110.4		143.3		113.9		160.9	
		.3		1.7		.4		1.7		.4		1.8
.08	107.1		128.2		110.8		145.0		114.3		162.7	
		.4		1.6		.3		1.8		.3		1.8
.09	107.5		129.8		111.1		146.8		114.6		164.5	
		.3		1.7		.4		1.7		.4		1.8
.10	107.8		131.5		111.5		148.5		115.0		166.3	

Obs. wpc	1.60				1.70				1.80			
0.00	115.0		166.3		118.3		184.8		121.4		204.1	
		0.3		1.8		0.3		1.9		0.4		2.0
.01	115.3		168.1		118.6		186.7		121.8		206.1	
		.3		1.8		.3		1.9		.3		1.9
.02	115.6		169.9		118.9		188.6		122.1		208.0	
		.4		1.9		.3		1.9		.3		2.0
.03	116.0		171.8		119.2		190.5		122.4		210.0	
		.3		1.8		.4		1.9		.3		2.0
.04	116.3		173.6		119.6		192.4		122.7		212.0	
		.3		1.8		.3		2.0		.3		2.0
.05	116.6		175.4		119.9		194.4		123.0		214.0	
		.4		1.9		.3		1.9		.3		2.0
.06	117.0		177.3		120.2		196.3		123.3		216.0	
		.3		1.9		.3		1.9		.3		2.0
.07	117.3		179.2		120.5		198.2		123.6		218.0	
		.3		1.8		.3		2.0		.3		2.0
.08	117.6		181.0		120.8		200.2		123.9		220.0	
		.3		1.9		.3		2.0		.3		2.1
.09	117.9		182.9		121.1		202.2		124.2		222.1	
		.4		1.9		.3		1.9		.3		2.0
.10	118.3		184.8		121.4		204.1		124.5		224.1	

TABLE 19—Continued

Obs. wpc	1.90 Volts	Dif.	Cp	Dif.	2.00 Volts	Dif.	Cp	Dif.	2.10 Volts	Dif.	Cp	Dif.
0.00	124.5		224.1		127.4		244.9		130.3		266.3	
		0.3		2.1		0.3		2.1		0.3		2.2
.01	124.8		226.2		127.7		247.0		130.6		268.5	
		.3		2.0		.3		2.2		.2		2.2
.02	125.1		228.2		128.0		249.2		130.8		270.7	
		.3		2.1		.3		2.1		.3		2.2
.03	125.4		230.3		128.3		251.3		131.1		272.9	
		.3		2.0		.3		2.2		.3		2.2
.04	125.7		232.3		128.6		253.5		131.4		275.1	
		.3		2.1		.3		2.1		.2		2.3
.05	126.0		234.4		128.9		255.6		131.6		277.4	
		.3		2.1		.2		2.2		.3		2.2
.06	126.3		236.5		129.1		257.8		131.9		279.6	
		.3		2.1		.3		2.1		.3		2.2
.07	126.6		238.6		129.4		259.9		132.2		281.8	
		.3		2.1		.3		2.1		.3		2.2
.08	126.9		240.7		129.7		262.0		132.5		284.0	
		.3		2.1		.3		2.2		.2		2.2
.09	127.2		242.8		130.0		264.2		132.7		286.2	
		.2		2.1		.3		2.1		.3		2.2
.10	127.4		244.9		130.3		266.3		133.0		288.4	

Obs. wpc	2.20 Volts	Dif.	Cp	Dif.	2.30 Volts	Dif.	Cp	Dif.	2.40 Volts	Dif.	Cp	Dif.
0.00	133.0		288.4		135.7		311.1		138.3		334.5	
		0.3		2.2		0.2		2.3		0.2		2.4
.01	133.3		290.6		135.9		313.4		138.5		336.9	
		.3		2.3		.3		2.4		.3		2.4
.02	133.6		292.9		136.2		315.8		138.8		339.3	
		.2		2.3		.3		2.4		.2		2.4
.03	133.8		295.2		136.5		318.2		139.0		341.7	
		.3		2.3		.2		2.3		.3		2.4
.04	134.1		297.5		136.7		320.5		139.3		344.1	
		.3		2.3		.3		2.3		.2		2.4
.05	134.4		299.8		137.0		322.8		139.5		346.5	
		.2		2.2		.2		2.4		.3		2.4
.06	134.6		302.0		137.2		325.2		139.8		348.9	
		.3		2.3		.3		2.3		.2		2.4
.07	134.9		304.3		137.5		327.5		140.0		351.3	
		.3		2.3		.3		2.3		.3		2.4
.08	135.2		306.6		137.8		329.8		140.3		353.7	
		.2		2.3		.2		2.4		.2		2.4
.09	135.4		308.9		138.0		332.2		140.5		356.1	
		.3		2.2		.3		2.3		.3		2.4
.10	135.7		311.1		138.3		334.5		140.8		358.5	

TABLE 19—Continued

Obs. wpc	2.50				2.60				2.70			
	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.
0.00	140.8		358.5		143.2		383.2		145.6		408.5	
		0.2		2.5		0.2		2.5		0.2		2.6
.01	141.0		361.0		143.4		385.7		145.8		411.1	
		.3		2.5		.3		2.5		.2		2.5
.02	141.3		363.5		143.7		388.2		146.0		413.6	
		.2		2.4		.2		2.6		.3		2.6
.03	141.5		365.9		143.9		390.8		146.3		416.2	
		.2		2.5		.2		2.5		.2		2.6
.04	141.7		368.4		144.1		393.3		146.5		418.8	
		.3		2.5		.3		2.5		.2		2.6
.05	142.0		370.9		144.4		395.8		146.7		421.4	
		.2		2.4		.2		2.6		.3		2.6
.06	142.2		373.3		144.6		398.4		147.0		424.0	
		.3		2.5		.3		2.5		.2		2.6
.07	142.5		375.8		144.9		400.9		147.2		426.6	
		.2		2.5		.2		2.5		.2		2.6
.08	142.7		378.3		145.1		403.4		147.4		429.2	
		.3		2.4		.2		2.5		.2		2.6
.09	143.0		380.7		145.3		405.9		147.6		431.8	
		.2		2.5		.3		2.6		.3		2.6
.10	143.2		383.2		145.6		408.5		147.9		434.4	
	2.80				2.90				3.00			
0.00	147.9		434.4		150.1		460.8		152.3		487.8	
		0.2		2.6		0.2		2.7		0.3		2.8
.01	148.1		437.0		150.3		463.5		152.6		490.6	
		.2		2.6		.3		2.7		.2		2.7
.02	148.3		439.6		150.6		466.2		152.8		493.3	
		.3		2.7		.2		2.7		.2		2.7
.03	148.6		442.3		150.8		468.9		153.0		496.0	
		.2		2.6		.2		2.7		.2		2.8
.04	148.8		444.9		151.0		471.6		153.2		498.8	
		.2		2.7		.2		2.7		.2		2.7
.05	149.0		447.6		151.2		474.3		153.4		501.5	
		.2		2.6		.2		2.7		.2		2.8
.06	149.2		450.2		151.4		477.0		153.6		504.3	
		.2		2.6		.3		2.7		.2		2.7
.07	149.4		452.8		151.7		479.7		153.8		507.0	
		.3		2.7		.2		2.7		.2		2.8
.08	149.7		455.5		151.9		482.4		154.0		509.8	
		.2		2.6		.2		2.7		.2		2.8
.09	149.9		458.1		152.1		485.1		154.2		512.6	
		.2		2.7		.2		2.7		.2		2.8
.10	150.1		460.8		152.3		487.8		154.4		515.4	

TABLE 19—Continued

Obs. wpc	3.10				3.20			
	Volts	Dif.	Cp	Dif.	Volts	Dif.	Cp	Dif.
0.00	154.4		515.4		156.5		543.5	
		0.2		2.7		0.2		2.8
.01	154.6		518.1		156.7		546.3	
		.2		2.8		.2		2.9
.02	154.8		520.9		156.9		549.2	
		.2		2.8		.2		2.8
.03	155.0		523.7		157.1		552.0	
		.3		2.8		.2		2.9
.04	155.3		526.5		157.3		554.9	
		.2		2.8		.3		2.9
.05	155.5		529.3		157.6		557.8	
		.2		2.9		.2		2.9
.06	155.7		532.2		157.8		560.7	
		.2		2.8		.2		2.8
.07	155.9		535.0		158.0		563.5	
		.2		2.9		.2		2.9
.08	156.1		537.9		158.2		566.4	
		.2		2.8		.2		2.9
.09	156.3		540.7		158.4		569.3	
		.2		2.8		.2		2.9
.10	156.5		543.5		158.6		572.2	

TABLE 20

Table for determining values of candlepower corresponding to observed values of voltage, when the values of both candlepower and voltage at 1.20 wpc are known. All values in this table are expressed in per cent.

Example: Given 125.0 volts and 34.0 candles, both at 1.20 wpc, to find candles at 100.0 volts.

Solution: 100.0 volts=80 per cent of 125.0 volts. Corresponding to 80 per cent volts find in the table 43.95 per cent candles. Therefore, candles at 100.0 volts=43.95 per cent of 34.0=14.94.

Obs. volts	60		70		80		90	
	Cp	Dif.	Cp	Dif.	Cp	Dif.	Cp	Dif.
0	14.34	0.98	26.35	1.49	43.95	2.11	68.18	2.83
1	15.32	1.03	27.84	1.55	46.06	2.18	71.01	2.90
2	16.35	1.07	29.39	1.61	48.24	2.24	73.91	2.98
3	17.42	1.12	31.00	1.66	50.48	2.31	76.89	3.05
4	18.54	1.18	32.66	1.73	52.79	2.39	79.94	3.14
5	19.72	1.22	34.39	1.79	55.18	2.45	83.08	3.22
6	20.94	1.28	36.18	1.85	57.63	2.53	86.30	3.30
7	22.22	1.33	38.03	1.91	60.16	2.60	89.60	3.38
8	23.54	1.38	39.94	1.97	62.76	2.67	92.98	3.47
9	24.92	1.43	41.91	2.04	65.43	2.75	96.45	3.55
10	26.35		43.95		68.18		100.00	

Obs. volts	100		110		120		130	
	Cp	Dif.	Cp	Dif.	Cp	Dif.	Cp	Dif.
0	100.0	3.6	140.3	4.5	189.9	5.5	249.5	6.5
1	103.6	3.8	144.8	4.6	195.4	5.6	256.0	6.6
2	107.4	3.8	149.4	4.8	201.0	5.7	262.6	6.8
3	111.2	3.9	154.2	4.8	206.7	5.8	269.4	
4	115.1	3.9	159.0	4.9	212.5	5.9		
5	119.0	4.1	163.9	5.0	218.4	6.0		
6	123.1	4.2	168.9	5.1	224.4	6.1		
7	127.3	4.2	174.0	5.2	230.5	6.2		
8	131.5	4.4	179.2	5.3	236.7	6.3		
9	135.9	4.4	184.5	5.4	243.0	6.5		
10	140.3		189.9		249.5			

TABLE 21

Table for determining values of wattage corresponding to observed values of voltage when the values of both wattage and voltage at 1.20 wpc are known. All values in this table are expressed in per cent.

Example: Given 98.0 watts and 110.0 volts, both at 1.20 wpc, to find watts at 90.2 volts.

Solution: 90.2 volts=82 per cent of 110.0 volts. Corresponding to 82 per cent, find in the table 73.007 per cent watts. Therefore, watts at 90.2 volts=73.007 per cent of 98.0=71.55 watts.

Obs volts	60		70		80		90	
	Watts	Dif.	Watts	Dif.	Watts	Dif.	Watts	Dif.
0	44.406		56.772		70.200		84.628	
		1.187		1.295		1.399		1.495
1	45.593		58.067		71.599		86.123	
		1.198		1.307		1.408		1.505
2	46.791		59.374		73.007		87.628	
		1.209		1.317		1.419		1.514
3	48.000		60.691		74.426		89.142	
		1.220		1.328		1.428		1.524
4	49.220		62.019		75.854		90.666	
		1.231		1.338		1.438		1.533
5	50.451		63.357		77.292		92.199	
		1.243		1.348		1.448		1.542
6	51.694		64.705		78.740		93.741	
		1.253		1.359		1.458		1.551
7	52.947		66.064		80.198		95.292	
		1.264		1.368		1.467		1.560
8	54.211		67.432		81.665		96.852	
		1.275		1.379		1.477		1.570
9	55.486		68.811		83.142		98.422	
		1.286		1.389		1.486		1.578
10	56.772		70.200		84.628		100.000	

Obs volts	100		110		120		130	
	Watts	Dif.	Watts	Dif.	Watts	Dif.	Watts	Dif.
0	100.00		116.27		133.40		151.35	
		1.59		1.68		1.76		1.84
1	101.59		117.95		135.16		153.19	
		1.60		1.68		1.77		1.85
2	103.19		119.63		136.93		155.04	
		1.60		1.69		1.77		1.85
3	104.79		121.32		138.70		156.89	
		1.61		1.70		1.79		
4	106.40		123.02		140.49			
		1.63		1.71		1.79		
5	108.03		124.73		142.28			
		1.63		1.72		1.80		
6	109.66		126.45		144.08			
		1.64		1.72		1.80		
7	111.30		128.17		145,88			
		1.65		1.74		1.82		
8	112.95		129.91		147.70			
		1.65		1.74		1.82		
9	114.60		131.65		149.52			
		1.67		1.75		1.83		
10	116.27		133.40		151.35			

TABLE 22

Table for determining watts per candle corresponding to observed voltage when the latter is expressed in per cent of the voltage at 1.20 wpc.

Example: Given 115.0 volts at 1.20 wpc, to find watts per candle corresponding to 96.6 volts.

Solution: 96.6 volts=84.0 per cent of 115.0 volts. Corresponding to 84 per cent volts, find in the table 1.724, the wpc required.

Obs. volts	60		70		80		90	
	Wpc	Dif.	Wpc	Dif.	Wpc	Dif.	Wpc	Dif.
0	3.716	0.145	2.585	0.083	1.916	0.051	1.490	0.034
1	3.571	.136	2.502	.078	1.865	.049	1.456	.033
2	3.435	.129	2.424	.075	1.816	.047	1.423	.032
3	3.306	.121	2.349	.071	1.769	.045	1.391	.030
4	3.185	.115	2.278	.067	1.724	.043	1.361	.029
5	3.070	.108	2.211	.065	1.681	.041	1.332	.028
6	2.962	.102	2.146	.061	1.640	.040	1.304	.028
7	2.860	.097	2.085	.059	1.600	.038	1.276	.026
8	2.763	.091	2.026	.056	1.562	.037	1.250	.026
9	2.672	.087	1.970	.054	1.525	.035	1.224	.024
10	2.585		1.916		1.490		1.200	

Obs. volts	100		110		120		130	
	Wpc	Dif.	Wpc	Dif.	Wpc	Dif.	Wpc	Dif.
0	1.200	0.024	0.9945	0.0172	0.8431	0.0130	0.7280	0.0098
1	1.176	.023	.9773	.0167	.8301	.0126	.7182	.0098
2	1.153	.022	.9606	.0163	.8175	.0122	.7084	.0095
3	1.131	.021	.9443	.0157	.8053	.0119	.6989	
4	1.110	.021	.9286	.0153	.7934	.0116		
5	1.089	.020	.9133	.0149	.7818	.0113		
6	1.069	.020	.8984	.0144	.7705	.0110		
7	1.049	.018	.8840	.0140	.7595	.0108		
8	1.031	.019	.8700	.0136	.7487	.0105		
9	1.012	.0175	.8564	.0133	.7382	.0102		
10	0.9945		.8431		.7280			