MICROMETER MICROSCOPES

By Arthur W. Gray

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1. SOME ERRORS AFFECTING MEASURING MICROSCOPES

Accurate measurements of length made with micrometer microscopes require careful consideration of the errors to which such instruments are subject.

The most important corrections to the readings of a screw micrometer are necessitated by the deviations from the nominal value of a division, the progressive errors, and the periodic errors. The last named may proceed from periodic irregularities in the thread (so-called drunken thread), failure to have the thrust bearings perpendicular to the axis of rotation, an eccentric drum, or an irregular division of the drum. With proper use no errors will be introduced by lost motion or by an irregularly distributed oil film. Temperature changes may cause variations in magnification by thermal expansion of the tube, the screw, or the microscope lenses (changing the curvatures), or by altering the refractivity of the media traversed by the light in forming the image measured by the screw. Then there are other irregularities too little understood to be subject to calculation. While these variations in magnification are measurable, they are, however, small and generally negligible. They are best taken into account by frequently measuring some known length interval.[1] In certain cases it may

[1] Mr. E. D. Tillyer of this bureau, to whom I am indebted for many valuable suggestions and criticisms, called my attention to a paper by F. L. O. Wadsworth, "On the construction of telescopes whose relative or absolute focal length shall be invariable at all temperatures," Roy. Astron. Soc. Monthly Notices, 63, pp. 573–591; 1902. This discusses, with the aid of formulæ and illustrative examples, some of the sources of these variations of magnification, and it points out that when an ordinary flint and crown objective is mounted in a brass tube, the thermal expansion of the latter will almost compensate for the focal-length change of the former. Mr. Tillyer also tells me that it is a common practice in astronomical observatories to mount the objective so that its distance from the reticule can be varied sufficiently to permit adjustment of the magnification from time to time.

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be necessary to correct for distortion of the image near the edge of the field. Again, serious errors may sometimes be caused by failure to focus accurately. Neither absence of parallax nor sharpness of image is a sufficient test.[2]

Another possible trouble deserves to be mentioned, because if existent it may prove very exasperating and may long evade detection. Any holes in the surfaces of the thrust bearings, even if extremely small, may harbor minute chips or grains of dust, which in time work between the rubbing surfaces and cause large irregularities of a more or less periodic nature. When such trouble appears in a comparator, its periodic nature is apt to remain unnoticed, and the irregularities are likely to be ascribed to such things as a loose joint somewhere in the framework supporting the microscopes, a bad place in a micrometer screw, or something rubbing against one of the spider lines.[3]

The elusive nature of such troubles is well illustrated by an experience recorded by Hough,[4] who succeeded Sir David Gill in

[2] Curious anomalies, finally traced to inaccurate focusing, long puzzled Fizeau and Tresca of the French Section and Bosscha and Oudemans of the Dutch Commission while comparing line standards with the Mètre des Archives, an end standard. An explanation was given by Fizeau, and a method of reducing the trouble pointed out by Cornu and used in comparing the Mètre des Archives with the provisional international standard. (See A. Cornu and J. R. Benoît, "Détermination de l'étalon provisoire international." Trav. et Mém., **10**, p. 9, 1894; W. Foerster, "Ueber den Anschluss des älteren Urmaasses und der Copieen desselben an das neue Deutsche Prototyp für das Meter," Norm. Aich. Komm. Abhandl. **1**, p. 67, 1895; J. R. Benoît: "Mesures de dilatation et comparaisons des règles mètriques," Trav. et Mém., **2**, C. 26, 1883; A. Hirsch, "De l'influence de la mise au foyer sur la valeur du micromètre d'un microscope," Proc. verb., p. 255, 1877; and W. Foerster, "Note sur l'influence de la mise au foyer et de l'inclinaison des microscopes sur les mesures micrométriques," Proc. verb., p. 271, 1877.) That Cornu's procedure, however, merely reduced but did not eliminate the trouble unless aplanatic objectives (free from spherical aberration and coma) were used seems to have been recognized first by Benoît and Guillaume after obtaining systematic differences of nearly 4 μ in comparing length standards by two different methods. (See J. R. Benoît and C. E. Guillaume: "Mètres à bouts," Trav. et Mém., **12**, p. 35, 1902.) These experiences seem to have pretty generally escaped the notice of those who write treatises as well as of those who use micrometer microscopes. Errors of 10 μ or more may be easily introduced by inaccurate focusing even though repeated pointings may differ by less than 1 μ.

[3] In a case which caused the writer considerable annoyance the thrust bearings were found deeply scored and a grain of dirt embedded near the edge of a minute cavity due to porosity of the metal. Apparently in wiping the surfaces during the cleaning that preceded calibration, particles of dirt had been brushed into some of the cavities only to work out into the bearings when the micrometer was used. Removing the embedded dirt speck still left the periodic error about five times as great as when the microscope was calibrated. A careful grinding of the bearings reduced the error to about the value at the time of calibration. Attempts at further improvement by polishing only made matters worse. Of course an entirely new calibration was necessary after a thorough cleaning, with special attention to cavities. After this experience each micrometer drum and bearing was covered by a dust-excluding case with a celluloid window. This cover also prevents the silver of the drums from tarnishing and has the further advantage of eliminating blunders when reading the tenths of a turn, since the windows are only twelve divisions long and, therefore, show but one figure at a time except when the setting is close to an exact tenth.

[4] Report of His Majesty's Astronomer at Cape of Good Hope for the years 1906 and 1907, pp. 3–4.

charge of the Cape Observatory. After reporting the satisfactory behavior of some underground reference marks for the new meridian circle, Hough states "The discussion of azimuth determinations made during the year 1906 and the early part of 1907 indicated, however, that though the marks themselves seemed to possess all necessary stability, the azimuth determinations were affected by a persistent and apparently increasing systematic difference amounting at the time to $0^s.1$ in azimuth, depending on whether the observations were made with instrument in position Clamp East or in position Clamp West. The difference was eventually traced to a loose jewel in the end bearing of the micrometer screw, since the resetting of which the discordances seem to have entirely disappeared."

TABLE 1

	A	B	C	D	E
1	+151	−38	−1	−8	−15
2	+102	+27	0	+1	− 9
3	+101	−21	+1	+8	+12
4	− 40	−49	+1	+7	+ 9

Table 1 is given to show the magnitudes of the periodic errors found in some microscopes. On the assumption that the micronometers read correctly at the beginning and at the end of a turn, the corrections in ten-thousandths of a turn that must be added to the reading at the end of each fifth indicated in the first column are recorded in the other columns. Under A are the corrections for one turn of the microscope referred to in the footnote on page 376, after it had been repaired as well as possible. Under B are those found for a turn of a microscope used in making important length measurements. The instrument had never before been investigated. A curious feature of this microscope was the regular repetition of this series of corrections every other turn, with a somewhat different but equally regular series for the alternate turns. Column C represents a screw of unusual excellence, made by an American firm noted for high-grade astronomi-

cal instruments. D represents the average and E the worst of a group of 10 others made by this same firm. The irregularities which these 10 exhibit are of the same order of magnitude as those of the best microscopes belonging to the International Bureau of Weights and Measures.

As a final word of caution it should be noted that whenever high accuracy is demanded in measurements made by micrometer microscopes the screw errors should be investigated periodically. It is by no means safe to assume that screws in use will remain unchanged indefinitely. Not only can irregularities creep in without detection, but even the regular wear of the threads (especially when the screws are new) can produce marked changes in the errors. This seems to have been pointed out first by Christie.[5] Subsequently Gill investigated the effect of wear on gun-metal micrometer screws working in brass bearings.[6] Errors originally negligible had in the course of a little more than four years increased to such an extent that when different parts of the screws were used to measure the same length, covering about 4.8 turns, discordances as great as from 0.02 to 0.09 of a turn were obtained with the same micrometer, averaging as much as 0.04 turn for the six screws examined. In a later paper[7] he shows that "even when steel screws are employed the changes produced by wear in the non-periodic corrections are very marked," although "the wear of steel screws in brass bearings is very much less than that of gun-metal screws in brass bearings."

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[5] W. H. M. Christie: "On the effect of wear in the micrometer screws of the Greenwich transit circle," Roy. Astron. Soc. Monthly Notices, **37**, pp. 18–22; 1876.

[6] David Gill: "On systematic errors in the readings of the circle microscopes of the Cape transit circle," Roy. Astron. Soc. Monthly Notices, **45**, pp. 64–90; 1884. In this paper and also in a later one (Monthly Notices, **45**, pp. 432–444, 1885) Gill gives numerous reasons to show that what he considered serious errors in the Cape Catalogue of 1880 (embracing observations between 1870 and 1879), which had been attributed by the author, Mr. Stone, to constraint caused by imperfectly finished threads at the ends of the screws, were in reality caused by the wear to which the screws had been subjected since they were brought into use in 1855.

[7] David Gill: "Note on the effect of wear on the errors of micrometer screws," Roy. Astron. Soc. Monthly Notices, **59**, pp. 73–76; 1898.

2. METHODS OF APPLYING CORRECTIONS TO A GROUP OF MICROMETERS

Fig. 1 is presented for the purpose of recalling the relation between the length l to be measured and the observations made with the microscopes. The vertical arrows represent the optical axes of the two microscopes of a transverse comparator. The inclined arrows represent the axes of the pencils forming images of the lines defining the extremities of l. y is the distance between the points O_L and O_R on which the microscopes would be sighted

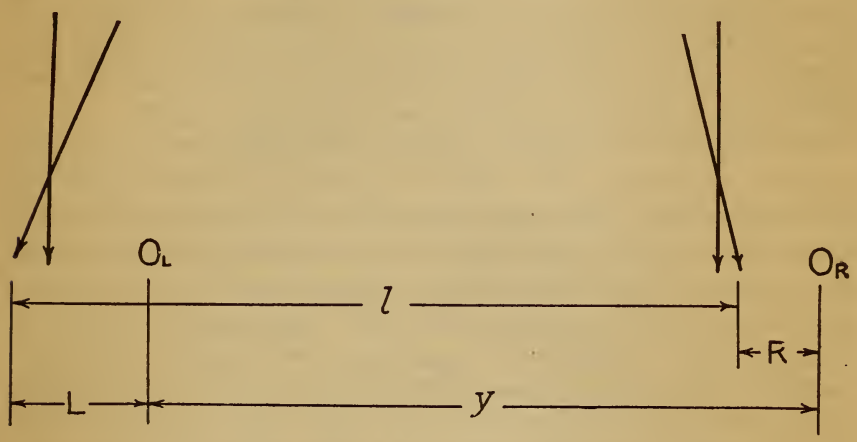

Fig. 1.—*Principle of microscope comparator*

when the micrometers read zero (if they could be moved that far). L and R are the distances from O_L and O_R, respectively, to the ends of l, and are given directly by the micrometer readings corrected to measure lengths in terms of the same unit. Then

$$l = y + (L - R) \tag{1}$$

The distance y must be determined or eliminated by comparing l with a known length. Let the subscripts A and B indicate measurements made on two bars to be compared. Then it follows that

$$l_A - l_B = (L_A - R_A) - (L_B - R_B) \equiv (L_A - L_B) - (R_A - R_B) \tag{2}$$

If the uncorrected readings of the micrometers are indicated by primes and the corrections by Δ's, then

$$L_A \equiv L'_A + \Delta L_A \qquad R_A \equiv R'_A + \Delta R_A$$
$$L_B \equiv L'_B + \Delta L_B \qquad R_B \equiv R'_B + \Delta R_B \tag{3}$$

If the progressive and the periodic errors are so small that it is sufficient to apply only the scale-value factor, the second of equations (2) is the more convenient. But in work demanding the highest accuracy it is by no means justifiable to omit the corrections for progressive and periodic errors unless investigation shows that the screws are of unusual excellence. However, the labor involved in applying, as well as in determining, these corrections by any of the methods ordinarily described is sufficient to discourage their use in all but very exceptional cases.

The writer has developed a method which takes into account all necessary corrections, but with an expenditure of labor after the microscopes have been calibrated once for all even less than that required to multiply the change in each micrometer reading by the mean scale-value and subtract. Essentially it is a simplification of the procedure followed by the International Bureau of Weights and Measures in measuring the national prototype meters.[8] The principal gain is effected by reducing to two tables (both of which can be mounted on a single card) all the information contained in the six tables used by the International Bureau. A simplified method of computing the tables will be discussed in the next section. The table for the left microscope gives at one inspection the total correction ΔL that must be added to any reading L' of the left microscope to give the corrected reading L that appears in equations (1), (2), and (3), above; that for the right microscope gives $-\Delta R$, the negative of the total correction to the reading R'. Reversing the sign of ΔR is merely a matter of convenience. It is usually sufficient to enter the corrections for every tenth of a turn and to obtain those for the hundredths by interpolation. A convenient arrangement is one similar to

[8] J. R. Benoît: "Mesures de dilatation et comparaisons des règles métriques," Trav. et Mém., 2, C. 131; 1883. The original observations are recorded on p. C. xix. and the tables for correcting the microscope readings are to be found on pp. C. xi.–xiv.

that followed in most mathematical tables—the whole turns are indicated at the side and the tenths at the top.

The displacement corresponding to one division (=0.01 turn) of the microscopes may differ so little from 1 μ that only small corrections will be required to reduce the readings to microns. It may, however, well happen that reduction to the desired length unit might require corrections so large as to be inconvenient. It then becomes advisable to select an auxiliary unit corresponding to the average screw value, and to express all corrections in terms of this unit, which should be the same for both microscopes.[9] It is further advantageous to minimize the magnitude of the corrections by adjusting both microscopes to give almost the same magnification—if possible one that permits a simple reduction factor. Then all computations can be made in terms of the auxiliary unit, multiplying by the reduction factor being deferred until the end.

By way of illustration Table 2 is presented, which contains all the corrections to the two microscopes discussed in the footnote on page 376. The arrangement is that indicated in the second paragraph before this. The auxiliary unit selected was 3/4 μ, since one drum division had nearly this value. Since the construction of the microscopes (with fixed tubes) made adjustment of the magnification inconvenient, it was omitted. The unit in which the corrections are expressed is 1/10 division = 3/40 μ. For convenience, each microscope was assumed to have no error at the beginning of turn number 30, which was the center of the range. Since these microscopes were used for measuring large elongations, the calibration was extended over 40 turns, embracing about four-fifths of the field of view. If each microscope had been adjusted to give the most convenient magnification, no number greater than 30 would have appeared in the table; and the corrections for readings near the middles of the screws would have been considerably smaller than this.

[9] All these considerations apply with increased force to a larger group of microscopes, for example, a group of four used to read a divided circle, or several used to step off a considerable length by means of a bar reaching from one microscope to the next.

TABLE 2

10	99	105	111	114	115	111	106	99	94	91	10	182	182	184	187	191	194	195	191	187	180	10
1	93	101	107	109	111	107	102	96	90	87	1	173	174	175	179	183	185	185	183	178	171	1
2	91	97	103	107	108	104	99	92	86	83	2	163	164	166	170	174	175	175	173	169	162	2
3	87	93	99	104	104	100	95	88	82	79	3	154	155	157	161	165	166	167	164	160	153	3
4	82	88	94	98	100	96	90	84	77	75	4	144	146	148	152	155	156	157	154	151	143	4
5	78	84	91	95	96	92	87	80	73	71	5	134	136	138	142	145	146	147	145	141	134	5
6	73	80	86	90	91	87	82	75	68	66	6	125	127	129	133	136	137	138	136	132	125	6
7	68	75	82	85	86	82	77	70	63	60	7	115	117	119	123	126	127	127	126	122	115	7
8	63	70	78	80	81	78	72	65	58	55	8	106	108	110	114	117	118	118	117	114	107	8
9	58	66	73	75	76	72	67	59	52	50	9	97	98	101	104	108	109	109	107	105	98	9
20	52	60	68	69	70	66	61	53	46	43	20	87	88	90	94	98	98	99	97	95	88	20
1	46	54	61	63	63	59	54	47	40	37	1	78	79	81	85	88	89	89	88	86	79	1
2	40	47	54	56	57	53	48	40	33	30	2	69	70	71	75	79	79	80	79	76	69	2
3	34	41	46	49	50	46	41	34	27	24	3	60	61	62	66	70	70	70	69	67	60	3
4	28	35	41	43	44	40	35	28	21	18	4	52	52	53	57	61	62	62	61	58	51	4
5	22	29	35	37	38	34	29	22	15	13	5	43	43	44	48	52	52	53	52	49	42	5
6	17	24	30	32	33	29	25	18	11	8	6	35	35	36	39	43	44	44	43	40	33	6
7	12	19	25	28	29	25	20	13	6	3	7	27	27	27	31	34	35	36	35	32	25	7
8	7	14	20	23	24	21	16	9	2	-1	8	18	18	18	21	25	26	26	25	23	16	8
9	3	10	16	19	20	17	12	5	-1	-4	9	9	9	9	12	15	16	17	15	13	6	9
		1	2	3	4	5	6	7	8	9			1	2	3	4	5	6	7	8	9	
30	0	7	13	17	18	15	10	3	-4	-7	30	0	-1	-1	2	5	6	7	6	3	-3	30
1	-3	4	10	14	15	12	7	0	-7	-10	1	-10	-11	-11	-9	-6	-5	-4	-5	-8	-14	1
2	-6	1	7	11	12	9	5	-3	-10	-13	2	-21	-22	-23	-21	-18	-16	-16	-17	-19	-25	2
3	-9	-2	4	8	9	6	2	-6	-13	-15	3	-33	-34	-35	-34	-32	-29	-29	-30	-32	-37	3
4	-12	-5	1	5	6	3	-1	-9	-15	-18	4	-44	-45	-46	-46	-44	-41	-41	-42	-44	-49	4
5	-15	-8	-2	2	3	0	-4	-11	-18	-21	5	-56	-57	-58	-58	-56	-54	-54	-55	-57	-61	5
6	-18	-11	-5	-1	0	-3	-7	-14	-21	-24	6	-67	-68	-69	-69	-68	-66	-65	-66	-68	-73	6
7	-21	-14	-8	-3	-2	-5	-10	-17	-24	-27	7	-78	-79	-80	-80	-79	-77	-77	-78	-80	-84	7
8	-23	-16	-10	-5	-4	-7	-12	-19	-26	-29	8	-90	-91	-92	-92	-91	-89	-89	-90	-92	-96	8
9	-26	-19	-13	-8	-7	-10	-14	-22	-29	-32	9	-101	-102	-103	-103	-102	-100	-100	-101	-104	-108	9
40	-28	-21	-15	-10	-9	-12	-16	-24	-31	-34	40	-114	-115	-116	-117	-115	-114	-114	-115	-117	-121	40
1	-31	-23	-17	-12	-11	-14	-18	-26	-33	-36	1	-126	-127	-128	-129	-127	-126	-126	-127	-129	-133	1
2	-32	-25	-19	-14	-13	-16	-20	-28	-34	-37	2	-138	-139	-141	-141	-139	-138	-138	-138	-142	-146	2
3	-34	-27	-21	-16	-15	-17	-22	-30	-36	-39	3	-152	-153	-155	-155	-154	-152	-152	-153	-156	-160	3
4	-36	-29	-23	-17	-16	-19	-24	-31	-38	-41	4	-169	-170	-172	-172	-171	-169	-169	-170	-173	-177	4
5	-38	-31	-25	-19	-18	-21	-25	-33	-40	-43	5	-177	-178	-180	-180	-179	-177	-177	-178	-181	-184	5
6	-40	-33	-26	-21	-20	-23	-27	-35	-42	-45	6	-191	-193	-195	-195	-193	-191	-191	-192	-195	-199	6
7	-41	-34	-27	-22	-21	-24	-28	-36	-43	-46	7	-204	-206	-208	-208	-206	-204	-203	-204	-208	-212	7
8	-43	-36	-29	-24	-23	-26	-30	-38	-45	-48	8	-218	-220	-223	-222	-220	-218	-216	-218	-222	-227	8
9	-45	-38	-31	-26	-25	-28	-32	-40	-47	-50	9	-233	-236	-238	-237	-235	-232	-231	-232	-236	-242	9

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Another gain is effected by recording observations in such a form that the corrections can be added directly without any copying of figures. For example, the complete record and computation for a single comparison of the bars A and B would appear about as follows:

	A	B
L'	3027.9	2431.9
R'	2938.1	3505.7
$L'-R'$	89.8	−1073.8
$\Delta L-\Delta R$	3.0	− 1.4
$L-R$	92.8	−1075.2

$\therefore\ l_A - l_B = 1168.0$ divisions or

$= 3/4$ of $1168.0 = 876.0\ \mu$.

The sum $(\Delta L - \Delta R)$ may be found mentally as the corrections are taken from the table, or the corrections may be entered separately in the record. In the above example the correction $\Delta L_A = 1.6$ is found by looking in the left half of Table 2 under 302 and interpolating for 8; similarly, $-\Delta R_A = 1.4$ is found in the right half. In the same way the table yields $\Delta L_B = 4.3$ and $-\Delta R_B = -5.7$.

Table 2 illustrates clearly the need of applying corrections, since pointings could be repeated to within a few tenths of a division. Examination of a column shows the steadily accumulating effect of progressive error and deviation of the screw from the nominal value of 3/4 μ per division. Examination of a row shows the effect of periodic error together with the slight amount of the other two errors that accumulate within one turn.

The accuracy that can be obtained in spite of such large errors is exhibited by measuring three times with each microscope the same interval of 999.05 $\mu \equiv 1332.0$ (3/4 μ). The results are recorded in Table 3, below. In each series the center of the field was located consecutively near the right edge, the center, and the left edge of the image, so as to obtain wide variations in the corrections, which were taken from Table 2. Part of the deviations from the averages are to be ascribed to inaccurate focusing and another part (perhaps) to linear interpolation, which is not altogether justifiable with such large and irregular periodic errors. Nevertheless, the agreement is even better than would have been predicted.

TABLE 3

Measurement of 999.05 $\mu = 1332.0$ (3/4 μ)

Uncorrected	Corrections		Corrected	Deviations
1341.5	−8.7	=	1332.8	+0.8
1333.1	−1.4	=	1331.7	−0.3
1337.1	−5.5	=	1331.6	−0.4
Averages with left microscope			1332.0	±0.5
1319.5	+13.1	=	1332.6	+0.5
1316.0	+15.5	=	1331.5	−0.6
1318.7	+13.6	=	1332.3	+0.2
Averages with right microscope			1332.1	±0.4

While the need of applying corrections to such microscopes as are described in Table 2 is perfectly evident, it may be well to point out in concluding this section that screw irregularities are of importance even in such micrometers as those of the International Bureau. An examination of the correction tables on pages C. xi and xii appended to Benoît's paper quoted on page 380 shows that an error of about 1.5 μ can be made in comparing two lengths if the only corrections applied are those for deviations of the mean screw-values from the nominal value of 100 μ per revolution; and further, that most of the error will be due to neglecting the *periodic* corrections.

3. DETERMINING THE CORRECTIONS OF MICROMETER MICROSCOPES

The calibration of a microscope by one of the processes usually described involves such laborious computations and least-square adjustments that it is no wonder to find it attempted only as a last resort when the demands of high accuracy compel. While, of course, the complete calibration of any divided scale requires considerable time, it is, however, possible to determine microscope corrections more accurately and with less labor than would appear from previous accounts. The whole procedure can be made one of direct measurement and simple graphical addition.

In determining the screw errors it is customary to use the ocular of the microscope under investigation for viewing the displacements of the spider lines. An enormous gain, however, is secured by removing the ocular and measuring the displacements by means of a second micrometer microscope.[10] The micrometer box may be detached from the microscope tube and mounted on the carriage of a dividing engine or other apparatus convenient for producing relative motion between the micrometer and the auxiliary microscope; but the writer prefers to remove the ocular only. The spider lines may be agreeably illuminated by placing

[10] The use of an auxiliary microscope for this purpose was first brought to my attention by Mr. Tillyer, who informed me that it was the regular practice at the United States Naval Observatory. (See J. C. Hammond: "Introduction to publications of U. S. Nav. Obs.," 6, Axiii.; 1911.) Sir David Gill (Roy. Astron. Soc. Monthly Notices, 45, p. 65; 1884) also employed a compound microscope in a simple apparatus he designed for rapidly and accurately investigating screw errors; but his method is in several respects inferior to that of the Naval Observatory.

beyond the objective a piece of milk glass or of thin white paper, behind which is mounted a small incandescent electric light. The auxiliary microscope is held by a slide that affords smooth motion in three directions mutually perpendicular.

We now measure: (1) The distance traveled by the spider lines as the screw under investigation is advanced five whole turns (a different number of turns may sometimes be more convenient); (2) the distance traveled during each single turn; (3) the distance traveled during each fifth (or tenth) of a turn. It is not necessary that the screw of the auxiliary microscope be very accurate, because all the pointings will be confined to the same portions of this screw, each only a fraction of a turn long. Besides, any irregularities here will be rendered negligible by choosing an objective that will magnify the interval to be measured sufficiently to make its image nearly fill the usable portion of the field of view. It is not necessary to use the same objective in measuring all three intervals, though it may be more convenient to do so. In obtaining the data for deriving Table 2 a 1-inch objective was used for the five-turn intervals, and a one-half-inch objective for the other intervals. All the lengths were then expressed in terms of the same unit, namely, one-drum division of the auxiliary microscope with the one-half-inch objective. The measurement of consecutive five-turn intervals for the entire length of the screw is not necessary, but it is advisable in order to reduce the accumulation of errors by addition in determining the progressive corrections; and the extra time consumed is a small item. Again, it is hardly worth while to carry out measurements (2) and (3) with every turn. It will usually be found sufficient to measure the first and the last turn of each five-turn interval. Since adjacent turns are not likely to differ much, this grouping in pairs will check blunders.

In measuring an interval the following procedure is observed: The auxiliary microscope (which we shall designate by A) is first focused on a transverse spider line attached to the screw (S) to be examined, and the image of this spider line is made parallel with the spider lines of A, care being taken to use always the same place

of the observed line, preferably close to the longitudinal line. S is now set accurately at the beginning of the interval to be measured, using for this purpose a lens to note coincidence between the line on the drum and that on the index. A is now pointed on the spider line of S and the reading is recorded. Then S is turned to the end of the interval and two pointings of A made. As a check against accidental displacements, S is returned to the beginning of the interval and the pointing of A repeated. The sensitiveness of the method is such that discrepancies exceeding the accidental error of a pointing can usually be traced to failure in setting S with sufficient accuracy at the boundaries of the interval. It is hardly worth while to increase the precision by averaging more than two pointings at each end. After measuring one interval, the screw S is advanced to the beginning of the next. Then the slide carrying A is moved until the spider line of S is again pointed at (the micrometer of A always being set to read the same at the beginning of each interval), and the procedure described for the first interval is repeated. When intervals of a different length are to be measured, the initial setting of A is so chosen that the same narrowly restricted portion of its screw will be used in pointing at the ends of all intervals. The pointings at the beginnings need not vary more than a few hundredths of a turn for all the intervals of approximately the same length.

After all the intervals have been measured, a length close to the average for one turn of S is selected as a basis of comparison. A multiple of 10 divisions of A is convenient. By simply adding the deviations of the measured five-turn intervals from five times this basis, we compute the progressive corrections at the beginning of each five turns, choosing the point of zero correction at the middle of the screw. These corrections will be expressed in divisions of A, and the same unit is retained in all stages of the computation until the end, when the correction curves are read off by a scale which automatically translates into the numbers to be entered in the final table.

We now plot a series of curves the abscissæ of which are the whole turns of S—that is, the comb readings of the micrometer to

which it belongs. Figure 2 is a reproduction of those used in computing Table 2. The ordinates of the first curve (C_0), measured from the dash dot line M N corresponding to 1170 divisions of A, are the progressive corrections mentioned in the preceding paragraph; those of the second (C_{100}) are the lengths of the one-turn intervals; those of the next five ($_0P_{20}$, $_{20}P_{40}$, $_{40}P_{60}$, $_{60}P_{80}$, $_{80}P_{100}$) are the

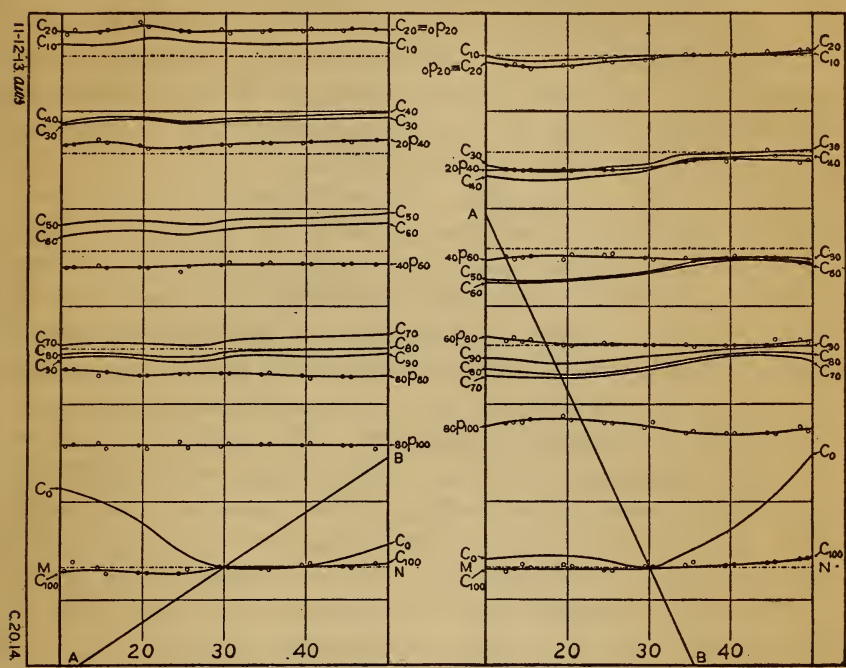

Fig. 2.—*Curves showing the corrections of a pair of micrometer microscopes used on a transverse comparator. The distance separating the light horizontal lines corresponds to a correction of 0.0513 turn, or 3.85 microns*

lengths of the one-fifth turn intervals in proper order. The next step is to adjust these curves so that they are mutually consistent; that is to say, the progressive corrections found by adding five consecutive ordinates of C_{100} measured from M N should agree with the corrections represented by C_0, and any ordinate of C_{100} should equal the sum of the corresponding ordinates of the curves $_0P_{20} \cdots\cdots {}_{80}P_{100}$. If the curves are plotted to a suitable scale, the

additions are quickly made by marking off the distances on strips of paper. These adjusted curves afford a fairly safe means of interpolation to obtain the corrections for intervals not actually measured. The curve plotted from the sums of the components of each interval is likely to lie above or below that plotted directly from the entire intervals. This divergence can proceed from progressive error or gradual change of scale-value in the screw of the auxiliary microscope or from thermal expansion of S, but it is of no consequence. The adjustment is first directed toward rendering the two curves parallel; then any distance separating them is divided among the curves representing the components of the interval by merely shifting the base lines from which the deviations would be measured. A small amount of such divergence was distributed among the curves for the right microscope.[11]

After the adjustments have been completed, the curves for the one-fifth turn intervals are added graphically to form a new set (C_{20}, C_{40}, C_{60}, C_{80}) yielding the corrections at the beginning of each fifth by measuring ordinates from the dash-dot lines corresponding to 1/5 of $1170 = 234$ divisions of A. If interpolation is necessary, it is readily accomplished graphically with sufficient accuracy. The interpolating curve for any selected turn will exhibit the corrections at every drum reading within this turn. Five points beside the initial zero are obtained from the correction curves just mentioned. If interpolating curves are plotted for several turns evenly distributed along the course of the screw (Fig. 3), sufficient data can be obtained for plotting curves (C_{10}, C_{30}, C_{50}, C_{70}, C_{90} of Fig. 2), to give the corrections at every tenth of a turn, or oftener, as functions of the comb readings. All that now remains is to determine the magnification and to translate these curves into the figures that form the final table.

It is, of course, desirable that the numbers in the correction table, and also the tabular differences, be small. Tabular differences arising from periodic irregularities can not be reduced except

[11] Although adjustments of this kind made by different computers would probably differ slightly, a least-square solution would be a sheer waste of effort, since the differences in almost any case would affect the final corrections by considerably less than the accidental error of a pointing in using the micrometer of which S forms a part.

by improving the screw. Progressive errors and deviations from the nominal screw-value, however, even if small, lead to the accumulation of large corrections when many turns must be used. But if the magnification is adjustable, these corrections can often be materially reduced. The first of the series of curves plotted, namely, that representing the progressive errors, was based on deviations

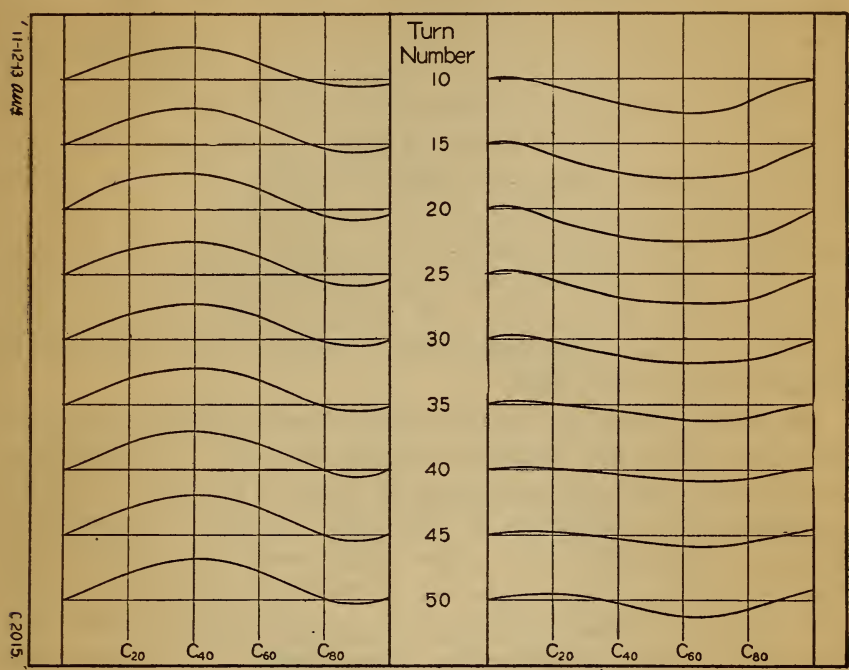

Fig. 3.—*Interpolation curves showing the corrections necessitated by periodic errors in the micrometer screws. The distance separating the light horizontal lines corresponds to a correction of 0.0342 turn, or 2.56 microns*

of the measured lengths of individual turns from the length represented by the basis selected for comparison. If a known length, such as a standard millimeter, be measured with the microscope to which the screw S belongs, and the readings corrected by using the curves described above, the distance measured that would correspond to the basis of comparison is likely to

differ somewhat from the distance to be represented by 100 divisions (one turn) in the correction table. A little reflection will make it apparent that corrections for this divergence can be added to the progressive corrections by the simple device of drawing a line (AB) at the proper angle through the point selected for zero correction. The distance parallel to the axis of ordinates between this line and the progressive curve (C_0) measures the total correction for any comb reading. It is, therefore, evident that these corrections can be reduced to a minimum for any given part of the screw by adjusting the magnification of the microscope so as to give this line the most favorable slope. A moment's inspection of the progressive curve will decide what this should be. After the magnification has been adjusted with sufficient closeness, a standard length interval should be carefully measured, and the correcting line accurately located. Several measurements using different parts of the screw will afford valuable checks on the accuracy of the correction curves. Table 3 above represents measurements of this kind.

We are now ready for the final step of reading off the corrections from the curves and entering them in the table. A translating scale is used which reads the plots in terms of the unit chosen for expressing the corrections in the table. A piece of coordinate paper cut diagonally at the proper angle forms a convenient scale. The corrections at the beginning of each turn are measured by the distances between the curve of progressive corrections and the inclined line correcting for magnification. The corrections at the tenths of a turn are read from the corresponding curves, the base line of each being shifted if necessary to correct for magnification within a turn. (Progressive errors within a turn as well as periodic errors were cared for in deriving the curves.) These internal corrections must, of course, be added to the correction at the beginning of the turn before entering in the table.

WASHINGTON, November 5, 1913.