WINDAGE RESISTANCE OF STEAM-TURBINE WHEELS

A CRITICAL STUDY OF THE EXPERIMENTAL DATA WHICH HAVE BEEN PUBLISHED AND OF EQUATIONS FOR REPRESENTING THEM

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1. *General Considerations.*—The power dissipated in driving a given wheel against the resistance of the surrounding medium, depends on the speed of rotation and the mechanical properties of the medium.

If the medium is an homogeneous fluid such as water, air, or dry steam, its mechanical behavior is determined by its density, viscosity, and compressibility. If the linear speeds involved are all small compared with the speed of sound in the medium, the energy of the acoustic waves generated is insignificant, the drag on the wheel due to the generation of these waves is negligible, and the medium acts sensibly as if they did not exist, i. e., as if it were incompressible. Except possibly in extreme cases, it can hardly be doubted that treating the medium as incompressible is allowable to the degree of approximation needed in steam-turbine calculations or justified by the accuracy of published determinations of windage losses. The density and viscosity of the medium in which the wheel rotates are therefore the only ones of its physical properties which we need to take into account, so long as the medium is homogeneous.

If instead of confining our attention to a particular wheel and casing we extend it to other wheels of various sizes which are, together with their casings, geometrically similar to the wheel and casing first considered, we see that the power dissipated may, further, depend on the diameter of the wheel, this single magnitude sufficing for the complete geometrical specification of the system when the shape is once given. The shape itself may be

specified by the values of the ratios of a number of lengths to some one length, such as the diameter of the wheel. It is to be understood, in what follows, that when we speak of wheels of the same shape, or "geometrically similar wheels," the similarity is to extend to the casings as well as to the wheels proper.

Let P be the power dissipated in driving a wheel of given shape and of diameter D, at a speed of n revolutions per unit time in an homogeneous medium of density ρ and viscosity μ. Then since the power can depend only on the size of the wheel, its speed of rotation, and the properties of the medium we have

$$P = f\ (\rho,\ n,\ D,\ \mu)$$

This physical equation must consist of terms which are all of the same dimensions, and this can be true only if the equation is of the general form

$$P = \Sigma N \rho^{\alpha} n^{\beta} D^{\gamma} \mu^{\delta} \tag{1}$$

in which the N's are pure numbers and the α, β, γ, δ, of each term have a set of values which give that term the same dimensions as P. There may be any number of terms, but for each of them a dimensional equation,

$$[P] = [\rho^{\alpha} n^{\beta} D^{\gamma} \mu^{\delta}] \tag{2}$$

must be satisfied.

It is most convenient to use the ordinary m, l, t system of fundamental units. If we do so and derive our other units from them, we have the following dimensional equations for the quantities which appear in equation (1):

$$[P] = [ml^2t^{-3}]$$
$$[\rho] = [ml^{-3}]$$
$$[n] = [t^{-1}]$$
$$[D] = [l]$$
$$[\mu] = [ml^{-1}t^{-1}]$$

The exponents of each term in equation (1) must therefore be such as to satisfy a dimensional equation

$$[ml^2t^{-3}] = [m^{\alpha}l^{-3\alpha}t^{-\beta}l^{\gamma}m^{\delta}l^{-\delta}t^{-\delta}]$$

The units of m, l, and t being independent, this equation must be homogeneous in each of them, so that we have

$$\left.\begin{array}{l} 1 = \quad \alpha + \delta \\ 2 = -3\alpha + \gamma - \delta \\ 3 = \quad \beta \ + \delta \end{array}\right\} \text{whence} \left\{\begin{array}{l} \alpha = 1 - \delta \\ \beta = 3 - \delta \\ \gamma = 5 - 2\delta \end{array}\right.$$

as the relations which must exist between the exponents of each term of equation (1).

If we substitute the foregoing values of α, β, and γ, in equation (1) and take out the common factor $(\rho n^3 D^5)$, we have

$$P = \rho n^3 D^5 \Sigma N \left(\frac{\mu}{\rho n D^2}\right)^{\delta} \tag{3}$$

The number of terms, the coefficient N of each term, and the exponent δ with which the single variable $\left(\frac{\mu}{\rho n D^2}\right)$ appears in that term, all remain indeterminate, i. e., they may have any values whatever without violating the dimensional requirements. We may also write equation (3) in the simpler form

$$P = \rho n^3 D^5 \varphi\left(\frac{\mu}{\rho n D^2}\right) \tag{4}$$

where φ is an unknown function of $\left(\frac{\mu}{\rho n D^2}\right)$.

For wheels of any given shape, the form of the function φ is fixed, but for any other shape the form of φ will or may be different. It depends on the values of a number of ratios of lengths sufficient to fix the shape of the wheel and casing, including the closeness of the casing to the wheel as part of the "shape." These ratios determine the values of the N's and δ's of equation (3). The variable $\left(\frac{\mu}{\rho n D^2}\right)$ has no dimensions, so that its numerical value in any absolute units is fixed by the physical magnitudes of μ, ρ, n, and D, and is independent of the magnitudes of the fundamental units.

2. *Remarks on the Function* φ.—Equations (3) and (4) contain all the information obtainable from the principle of dimensional homogeneity, and further information must be sought elsewhere.

The resistance offered by a fluid to a solid body in steady motion through it may be looked upon as a retarding drag due to transverse communication of momentum between currents or layers of the fluid which are moving over each other with different average velocities and are maintained by the motion of the solid body. In quiet stream-line motion, this transfer of momentum takes place by intermixing on a molecular scale, i. e., by interdiffusion of the different streams of fluid; and the coefficient of viscosity, μ, is a measure of the activity of this molecular intermixing. But whenever, by reason of high speed or of roughness or irregularity of the solid, the motion of the fluid becomes very turbulent, the lateral transfer of momentum between different streams occurs mainly by motions of relatively large masses of fluid in all sorts of irregular cross-currents and eddies. For any given geometrical arrangement of such a state of turbulent motion, this molar transfer of momentum is evidently proportional to the masses involved, or in other words, to the density of the fluid.

We must therefore expect that when the motion of the fluid about the solid body is perfectly quiet and regular, which will usually mean that all the motions are slow, the resistance encountered by the solid will be directly proportional to the viscosity of the fluid. But if, on the other hand, the circumstances are such that the motion of the fluid is very turbulent, it is to be expected that density will play the determining part in the phenomena of resistance, viscosity being of relatively small or even vanishing importance. The substantial correctness of this general qualitative reasoning is established by well-known facts relating to skin friction, aeroplane and ship resistance, and the flow of liquids and gases through pipes. Let us now see what it leads to in connection with equation (3).

If we have to deal with a smooth disk at a low speed of rotation, we must expect the retarding torque and therefore the power absorbed at any given speed to be directly proportional to the viscosity of the medium about the disk. This requires that the second member of equation (3) shall consist of only a single term and

that its exponent shall be $\delta = 1$. The equation for the power thus reduces to the very simple form

$$P = Nn^2D^3\mu \tag{5}$$

and the retarding torque is therefore independent of the density and proportional to the speed of rotation, the viscosity of the medium, and the cube of the diameter of the disk.

If the speed of rotation is increased, the stream-line motion will become unstable and will break up, at or near a certain definite critical speed, into a quite different, turbulent motion. At all events, this is what we should expect from our knowledge of the flow of fluids through pipes, and our expectation is confirmed by experiments on disks. After this abrupt change in the character of the fluid motion, the density of the fluid must appear in any equation which is to describe the facts, and if the viscosity μ appears at all it must be, in each term, with a smaller exponent than that of ρ in the same term. In the limiting case, the viscosity might be of altogether vanishing importance and then equation (3) would necessarily have the form

$$P = N\rho n^3D^5 \tag{6}$$

the retarding torque being proportional, for a given disk, to the density and the square of the speed, while for disks of different diameters but geometrically similar—in regard to roughness as well as general shape—the torque would be proportional to the fifth power instead of the cube of the linear dimensions. In practice we should expect, rather, that the relation would not reach this very simple limiting form and that μ would still appear, though only in terms with small exponents.

If it is found that the dependence of P on ρ, n, D, or μ can be represented with a degree of approximation sufficient for the experimental accuracy by giving the independent variable a single fixed exponent, it follows that the second member of equation (3) may be represented by a single term, the others being negligible. Equation (3) then reduces to

$$P = N\rho^{1-\delta}n^{3-\delta}D^{5-2\delta}\mu^{\delta} \tag{7}$$

It appears that such an equation is adequate to the representation of the experimental data which are available—the experimental accuracy not having been high. Equations (5) and (6) correspond to the limiting values $\delta = 1$ and $\delta = 0$.

The value of N depends on the form of the wheel and casing. For a bladeless disk running in the open, it depends on the profile and the roughness. For a thin flat disk it depends on the roughness alone; but the roughness is to be measured in terms of the diameter, so that for a surface with granulations or irregularities of a certain general absolute size, a large disk is "smoother" than a small one, in the present sense of the term smooth.

For smooth plane disks of small thickness, the critical speed, at which stream line motion breaks up into turbulent motion and the exponent δ drops suddenly from nearly unity to a much smaller value, is fairly definite and the change in the law of resistance is sharp. For ordinary turbine wheels, consisting of a disk and blades, the change is less abrupt and there is no definite critical speed; for the motion about the blades must always be turbulent at any speeds which are high enough to be of practical interest. The smaller the ratio, $\frac{l}{D}$, of blade length to disk diameter, the more nearly the behavior of the wheel approaches that of a simple disk, while with long blades, the additional resistance due to the blades is so much greater than that which would be encountered by the disk alone, that although the law of resistance is different at high and at low speeds the change is gradual.

In an experimental study of the form of the function φ we have first to work with wheels of some one shape, preferably a simple one, to start with. If we then find, as in fact we do for considerable ranges of the variables, that the dissipation P is proportional to a fixed power of ρ, or of n, or of D, or of μ, we know that equation (7) is adequate for this shape of wheel and our experiments give us the value of the coefficient N for this shape as well as the value of δ, no matter which of the four variables we may have selected for independent variation during the experiments. There are thus, in principle, four different modes of attacking the problem which must lead to the same result and may be used for checking one another. In practice we can not always conven-

iently vary the density and viscosity of the medium independently, so that there are often only three modes of attack which are practicable.

The general procedure is obvious; keeping any three of the variables at fixed values, we vary the fourth and observe the corresponding values of P. If the phenomena can be described by equation (7), the values of log P when plotted against the logarithm of the independent variable will give points which lie on a straight line, within the experimental errors. The slope of this line is the exponent with which the independent variable appears in equation (7); it determines the value of δ. The position of the line, taken in connection with the fixed values of the other three variables, determines the value of N. We may now proceed to examine the experimental data which illustrate the foregoing statements. Readers who are interested only in the final result of this somewhat laborious examination may proceed at once to section 6, page 214.

3. *The Relation of Power Dissipated to Speed of Rotation.*—A. Disks Without Blades: As the simplest sort of wheel we may take a flat bladeless disk rotating in the open air. We have data on such disks from Stodola's [1] experiments on a smooth but unmachined disk of thin boiler plate and from Odell's [2] experiments on paper disks.

Stodola's disk had a diameter of 537 mm or 21.1 inches, and he gives points on a plot of log P against log n only for 1500, 1800 and 2000 r. p. m. These three observations very nearly satisfy the relation $P \propto n^{2.92}$. The retarding torque was thus proportional to the 1.92 power of the speed n, which is about what might have been expected from the work of Reynolds,[3] Froude,[4] Zahm,[5] and others.

Odell's experiments covered a wider range of speed and he used four disks with diameters of 15.0, 21.8, 26.8, and 47.1 inches, recording the torque needed to drive each disk at measured speeds. When log *torque* is plotted against log n it is found that for any given disk, the points for all the higher speeds lie close to a straight line. Upon decreasing the speed, a critical speed, n_c, is reached and the law of resistance changes abruptly.

The results, so far as they concern us at present are collected in Table 1.

TABLE 1

Odell's Experiments on Paper Disks Rotating in Air

	A	B	C	D
Diameter in inches, $D=$	15.0	21.8	26.8	47.1
Range of speeds, (r. p. m.)	113 2100	200 900	100 525	250 740
Critical speed, $n_c=$	918	376	243	(100)
Value of β for $n>n_c$	3.32	3.55	3.54	3.08
$10^{-5}n_cD^2$	2.07	1.79	1.75	

The values of n_c and of the speed exponent $\beta=3-\delta$ were found by replotting Odell's observations; they do not differ much from Odell's values. The value of β is, of course, obtained by adding unity to the slope of the line on the plot of log *torque* against log n.

Upon inspection of the logarithmic plot there is no doubt that above the critical speed, β is very nearly constant, and there is also no doubt that the values of β are distinctly different for the different disks, the uncertainty in finding β from the published observations being not over 0.1. If the disks had all been geometrically similar, the form of φ in equation (4) would have been the same for all and the exponent β would therefore have had the same value. Since the values of β are not the same, we are sure that the disks were sensibly dissimilar. They were, in fact, made of four different kinds of paper and it could not be expected that they should be either similarly rough or similarly stiff.

When thin paper disks are driven at high speeds they become covered with fluttering waves which must have an important effect in increasing the resistance, so that it is not safe to draw conclusions regarding rigid disks from results obtained with flexible ones. Disk D made of "canvas-backed diagram paper" was no doubt much the stiffest, the others being of two kinds of drawing paper and of cartridge paper. It gave the value $\beta=3.1$ which is not far from the value 2.92 for Stodola's disk.

Disks B and C gave nearly the same value of β at the high speeds and seem to have behaved as if geometrically similar; the wave systems in them had similar effects on the resistance.

When two geometrically similar wheels or disks are compared, any singular point on the curve $P = f(n)$, such as the critical speed, must by equation (4) occur at the same value of $\frac{\mu}{\rho n D^2}$ for both. In Odell's and Stodola's experiments the medium (air) was always approximately unchanged, so that the value of $n_c D^2$ should be the same for geometrically similar disks. The values in the last line of Table 1 show that this was nearly true for Odell's disks B and C, which gave the same value of β at speeds above the critical. The value of n_c given in the table for disk D was computed from that observed for A which was most nearly like D; it is only a rough approximation, and being far below the lowest speed used with this disk, it was not observed. The critical speed for Stodola's disk, computed in the same way, would be in the vicinity of 500 r. p. m., which is also below the lowest speed recorded.

Disk C, of cartridge paper, shows the interesting phenomenon of two critical speeds, the lower, not mentioned in Table 1, being at about 150 r. p. m. Below this, the torque is proportional to $n^{1.1}$ or, within the experimental errors, directly to n. We have thus the limiting case described by equation (5) or by equation (7) with $\delta = 1$. At the first critical speed it appears that the stream-line motion of the air broke up into turbulent motion, the torque became proportional to $n^{1.7}$ and the power dissipated to $n^{2.7}$. For the range between the two critical speeds we therefore have $\beta = 3 - \delta = 2.7$, $\delta = 0.3$. At the second critical speed of about 243 r. p. m. the disk suddenly began to flutter * and the law of resistance changed to $P \propto n^{3.5}$, a new element being introduced by the presence of waves on the disk. The middle range, between the two critical speeds corresponds to Stodola's observations on his rigid disk. The value $\beta = 2.7$ which is somewhat lower than Stodola's $\beta = 2.92$ indicates that the cartridge paper, as long as it stayed plane, was smoother than the boiler plate, which is not surprising.

* This is the writer's inference; Odell is not responsible for it.

Disk B shows only the upper critical speed, the observations below this point being very few and the lowest coming about where the lower critical speed would probably have been found. Since the upper critical speed and the phenomena at still higher speeds depend on the stiffness of the disk, which has not appeared at all in our equations, the close agreement of β and of $n_c D^2$, for disks B and C, must be regarded as in some degree accidental. It happened that the disks were similarly stiff or, at all events, developed wave systems which had similar effects on the resistance.

The few observations on disk B below the upper critical speed give $P \propto n^{2.5}$. We can not say that this is different from the value $\beta = 2.7$ obtained for disk C, the evidence not sufficing to decide the point. But the difference, if real, is in the direction indicated by what we can judge of the smoothness of the two disks as described by Odell. For B was of a smoother thinner drawing paper than A, i. e., probably a rather hard smooth-surfaced paper, while the "cartridge paper" of disk C probably had a somewhat fuzzy surface.

If we compute the critical speeds of the rigid disk D and of Stodola's steel disk from the lower critical speed of disk C, we get 49 and 244 r. p. m., respectively, values about half those computed from disk A and probably nearer the truth.

In the case of disk A, which was relatively stiffer and rougher than B and C, the observations below the single critical speed observed are too scattered for us to detect any lower critical speed, if one occurred, or to distinguish the effects of turbulence of the air and fluttering of the paper. The effect of the fluttering at the high speed was less than with B and C as is shown by the fact that the value $\beta = 3.3$ is nearer the values for rigid disks, e. g., Stodola's disk ($\beta = 2.92$), disk D ($\beta = 3.14$) and disk C between its critical speeds ($\beta = 2.7$). The greater stiffness is also shown by the larger value of $n_c D^2$; the stiffer disk requires a higher speed to make it break into a flutter.

Odell's experiments were only preliminary and his own opinion of their accuracy is indicated by his saying that $P = const \times n^3 D^5$ "agrees pretty well with the experimental results." But in view of the very good agreement which they show with the predictions of the general theory in all cases where the range of speed is

large enough, the present writer forms a higher estimate of the interest and relative accuracy of these experimental results, even though the absolute values of the power dissipated may be considerably in error, and though we can evidently not safely predict the behavior of rigid disks from experiments on flexible ones.

B. Ordinary Single-Row Steam-Turbine Wheels: Besides the bladeless disk already mentioned, Stodola experimented on five single-row turbine wheels with various blade lengths and with disk diameters, measured at the root of the blades, of 20 to 45 inches. The wheels were run in air; all of them in the open, and three of them also inclosed in casings. At all the higher speeds the results, as Stodola shows them on a logarithmic plot, satisfy the relation $P \propto n^{\beta}$ quite closely. For the ten series shown, the value of β is from 2.82 to 2.97 with a mean of 2.89 and a mean residual of ± 0.04. The wheels all acted very nearly like the bladeless disk as regards the variation of power with speed.

Lewicki [6] gives the results of a few observations on the windage losses of a small Laval turbine at 14 000 to 20 000 r. p. m. The wheel had a disk diameter of 180 mm or 7.09 inches, and the blades were 20 mm or 0.79 inch long. Three observations in air at atmospheric pressure and 30° C give $P \propto n^{2.43}$ nearly. Four observations in saturated steam at atmospheric pressure lie fairly close to $P \propto n^{3.06}$ while three of them give almost exactly $P \propto n^{3.0}$. There is no doubt that β was nearly constant in each case, but the observations are so few and the range of speeds is so small that no great weight can be given to the numerical values of β.

We have, finally, the results published by Holzwarth.[7] Admitting that for a given wheel running at a given speed in steam, the power dissipated in windage is proportional to the density of the steam, and admitting, further, that for practical purposes the density may be treated as proportional to the absolute pressure p, Holzwarth sets $P = Kp$, and gives the values of K deduced from his experiments, by means of curves on a three-coordinate diagram. Each curve shows the relation of K to n for a given diameter, the diameters being 10, 20, 30, 40, and 50 inches. The lowest speed is 750 r. p. m. and the highest runs up to 4000 r. p. m. for the small wheels. There is a separate diagram for each of the five blade lengths 0.5, 1.0, 1.5, 2.0, and 2.5 inches. The wheels

were inclosed but had somewhat more clearance than is usual in practice. No information is given as to how nearly geometrically similar the wheels were, except the mere statement of disk diameter and blade length. The results *appear* to be the most comprehensive we have; but all details are omitted so that we have no means of estimating the accuracy of the experiments, of the reduction of the observations, or of the construction and reproduction of the diagrams as published. Though the curves may be quite adequate to the practical end for which their author intended them, it is evident at a glance that they can not all be represented by a single equation containing only n, D, and l as independent variables, and that readings of K from them are liable to rather large percentage errors.

With the exception of a few curves for the largest diameters and blade lengths, each curve shows two distinct forms and may be divided, though not sharply, into a low-speed and a high-speed part. The relation $K=f(n)$ is different for the two regions and the transition corresponds to the definite critical speed in the case of a bladeless disk. The 10-inch curves fall almost entirely in the low-speed range which is in general of little practical interest. The results for this diameter are therefore omitted from further consideration. For each of the other four diameters the writer made readings of the value of the coefficient K* for each of the five blade lengths and at the various speeds shown on the diagrams. The value of log K was then plotted against that of log n, and 20 series of points were thus obtained, each referring to a fixed diameter and blade length but varying speed.

For the higher speeds, the points of each series lie fairly close to straight lines, sometimes quite close. These lines were drawn in by inspection and their slopes, which represent β in the equation $P=const\times n^{\beta}$ were read off. For the small wheels with short blades there is a fairly definite critical speed at which the relation $P=f(n)$ changes; but the points for the "low" speeds are not exact or numerous enough to show what the relation is at these speeds. For larger wheels or longer blades the transition from "high" to "low" speeds when shown at all is gradual and

* Holzwarth used the symbol m for this coefficient.

not sharp, the lowest points on the logarithmic plot being above the straight lines drawn to represent the high-speed points.

The values obtained for the speed exponent β, are from 2.3 to 3.6; the mean is 3.0 and the mean residual is ± 0.3. The uncertainty of each value, due to doubt as to just where the straight line should be drawn, is generally about ± 0.1 but may be ± 0.2. For most of the series there is no doubt that $P \propto n^{\beta}$ is an approximate representation of Holzwarth's published curves for the higher speeds. There is also no doubt that the values of β, are distinctly different in different cases, but there is no evident systematic variation of the values with either diameter or blade length.

To sum up the conclusions which may legitimately be drawn from the data discussed in the foregoing section, we may say that both for flat bladeless disks and for single-row turbine wheels of ordinary forms, running either inclosed or in the open and in either air or steam, the power dissipated in windage by a given wheel is very nearly proportional to a fixed power of the speed of rotation throughout the range of speeds actually used with the diameters in question, except possibly at the lowest speeds, where the windage losses are economically insignificant. It follows that only a single term need be used in the second member of equation (3), so that an equation of the form (7) or

$$P = N\rho^{1-\partial} n^{3-\partial} D^{5-2\partial} \mu^{\partial} \tag{7}$$

is adequate to representing the facts, over the range mentioned, as closely as we know what the facts are.

The observed values of $3 - \delta = \beta$ are grouped about a general mean $\beta = 3$ and as a first approximation we have

$$P = N\rho n^3 D^5 \tag{8}$$

the coefficient N having a value determined by the shape of the wheel, and the viscosity not appearing at all. In individual cases the observed values of β vary from 2.5 to 3.5 without our being able to decide, from the rather meagre accounts published, whether the variations are genuine, i. e., not due to errors of measurement, and if they are genuine, what causes them.

The best experiments we have, Stodola's, give $\beta = 2.9$ very nearly, for all of his wheels and all conditions, and this value is quite consistent with the results by other experimenters in similar fields. It is probable that in general the equation

$$P = N\rho^{0.9}n^{2.9}D^{4.8}\mu^{0.1} \tag{9}$$

obtained from (7) by setting $3 - \delta = 2.9$ will give a better approximation than the simpler equation above, in which $\delta = 0$.

At a speed of rotation of 20 000 r. p. m. the peripheral speed of the tips of the blades of Lewicki's wheel was 755 ft. sec. The speed of sound in air at 30° C. is about 1150 ft. sec., and in saturated steam at one atmosphere it is about 1350 ft. sec. The linear speeds which occurred in Lewicki's experiments were therefore by no means all what would commonly be thought of as "small" in comparison with the acoustic speed. Nevertheless, an equation developed on the assumption that compressibility is negligible appears to hold for these high speeds as well as for lower ones. It would have been interesting to have data on the windage of Lewicki's wheel at even higher speeds, for it seems probable that if the speed had been run up to 30 000 the equation $P = const \times n^{\beta}$ would have failed completely. Experiments on the resistance of projectiles [8] indicate that the effects of compressibility begin to be sensible at about three-fourths of the acoustic speed and increase rapidly for some distance beyond that point, so that within a range of from three-fourths to one and one-half times the acoustic speed, the law of resistance is rather complicated. The speeds in Lewicki's experiments appear to have been nearly but not quite high enough to necessitate the consideration of compressibility in making up the theoretical equations. We may conclude that the simple theory as given in section 1 is probably always sufficiently exact when the peripheral speeds are not over one-half the acoustic speed in the medium in question.

4. *The Relation of Power Dissipated to Diameter.*—The experimental results discussed in section 3 suffice to show than an equation of the form (7) describes the facts, and they give us the value of δ in some specific cases. The same information might be got from experiments on geometrically similar wheels, by

making D the only independent variable and comparing the dissipation for wheels of various diameters run at the same speed in the same medium; but no experimental results on exactly similar wheels have been published, so that an investigation by this method can not be based on any published data. Nevertheless, as a check on the previous work it is interesting to compare such imperfect data as we have with equation (7) and the values of δ already found in section 3.

A. Disks Without Blades: We have to select for comparison, disks which gave nearly the same value of $\beta = 3 - \delta$, since disks which do not satisfy this condition can not possibly be geometrically similar. Odell's paper disks B and C gave the same value of β above the fluttering point, but since the rigidity of the disk is here an essential element which has not been allowed for in our theory, we must not expect equation (7) to hold at all exactly. If we compare these disks at 500 r. p. m., the observed power ratio is 4.1 as against 3.5, computed from the diameter ratio and the value $\beta = 3.54$ obtained from the experiments at varying speed. The discrepancy of 15 per cent may be due to experimental error but it seems quite as likely to be due to the fact that the flexibility of the disks has not appeared in the equations.

We may next compare these two disks in the region just below their upper critical speeds, where they were still behaving as if rigid. From the writer's plot of Odell's observations he finds

$$\text{for disk B} \qquad P = 2.51 \times 10^{-10} n^{2.470}$$
$$\text{for disk C} \qquad P = 2.28 \times 10^{-10} n^{2.654}$$

The observations on B in this range are so few that the exponent $\beta = 2.47$ is very uncertain and the equation could not safely be used for extrapolation, but it represents the actual observations reasonably well. At $n = 200$ r. p. m., which is within the range of speeds for both disks, the power computed from these equations is 1.21×10^{-4} hp for B and 2.92×10^{-4} hp for C. Comparing the power ratio with the diameter ratio we have

$$\frac{2.92}{1.21} = \left(\frac{26.8}{21.8}\right)^{4.26}$$

If we regard the disks as similar we have $5-2\delta=4.26$, whence $3-\delta=\beta=2.63$, which is sensibly identical with the value 2.65 obtained directly for disk C.

As a more severe test of the theory we may compare Odell's stiffest disk D, of 47.1 inches diameter, with Stodola's boiler-plate disk of 21.1 inches diameter. At $n=2000$ Stodola observed $P=0.147$ hp and his speed exponent was $\beta=2.92$; hence we have for this disk

$$P=3.38\times 10^{-11}n^{2.92}$$

From Odell's measurements on his disk D we have approximately

$$P=48.\times 10^{-11}n^{3.08}$$

Odell regarded his experiments on this disk, especially at the lower speeds, as less accurate than his other work, so that it seems only fair to make the comparison at one of the higher speeds within the range used by Odell. Taking $n=700$, we have by the above equations; for Stodola's disk $P=0.00679$ hp, and for Odell's disk D, $P=0.278$ hp. Comparing the power ratio with the diameter ratio we have

$$\frac{0.278}{0.00679}=\left(\frac{47.1}{21.1}\right)^{4.62}=40.9$$

Regarding the disks as similar, we have $5-2\delta=4.62$, whence $3-\delta=2.81$ as compared with $\beta=2.92$, obtained directly for Stodola's disk.

The value $\beta=2.92$ or $5-2\delta=4.84$ would give a power ratio of 48.7. Hence if we use equation (7), with the values of N and δ found for Stodola's disk, to compute the power for Odell's disk at 700 r. p. m. we get within 20 per cent of the observed value. The equation given above for Odell's disk would give, at 2000 r. p. m. and on a diameter of 21.1 inches, $P=0.112$ hp while Stodola's observed value at this speed was 0.147 hp. In view of all the circumstances and the long extrapolation from observations which Odell himself regarded with suspicion, the agreement must be considered quite satisfactory.

B. Single-Row Turbine Wheels: When we look for experiments on geometrically similar turbine wheels we find only a very few cases where we can assume that the wheels were even roughly similar.

One of Stodola's wheels had a disk diameter of 504 mm, or 19.84 inches and a blade length $l=60$ mm, so that the blade-length ratio was $\frac{l}{D}=0.119$. This wheel was run in air at atmospheric pressure and temperature, in a casing which allowed an axial clearance of 4 mm at the edge of the blades. Lewicki,[6] as already mentioned, made observations on a Laval wheel which had a disk diameter of 180 mm or 7.089 inches and a blade-length ratio $\frac{l}{D}=0.111$. These two wheels were therefore nearly similar in respect to the important element $\frac{l}{D}$. From the small scale drawing it appears that Lewicki's wheel had an axial clearance at the blade edges of about 2 mm. For geometrical similarity, the clearance on Stodola's wheel would therefore have had to be about 6 mm instead of the actual 4 mm. Lewicki's wheel thus had relatively more clearance; and since it is known experimentally that reducing the clearance round the blades reduces the windage, we must expect that in making comparisons between the two wheels, Lewicki's observed values will be somewhat larger than values computed for the same speed and diameter from the data obtained by Stodola.

Lewicki's experiments on windage were only a subsidiary part of a larger investigation and are not comprehensive enough to furnish satisfactory values of the constants N and δ. We shall therefore deduce an equation from Stodola's observations and compare values computed from this equation with averages from Lewicki's observations.

The speed exponent given by Stodola for this wheel is 2.87, but it appears from the plot that a larger value might be taken. We can probably do no better than to set $\beta=2.9$ and use equation (9). At 2100 r. p. m. the dissipation was found to be 0.704 hp.

The disk diameter was 19.84 inches and the density of the air, if we take the value given for the bladeless disk, was $\rho = 1.12$ kgm/m^3 or 0.070 lb./ft.^{3} From these values we get $N\mu^{0.1} = 1.058 \times 10^{-15}$, whence

$$P = 1.06 \times 10^{-15} \rho^{0.9} n^{2.9} D^{4.8} \left(\frac{\mu}{\mu_0}\right)^{0.1} \tag{10}$$

in which

P = the horsepower dissipated.

ρ = the density of the medium in lb./ft.^{3}

n = the speed of rotation in r. p. m.

D = the disk diameter in inches.

μ_0 = the viscosity of the air during Stodola's experiments.

μ = the viscosity of the medium in any other case for which we wish to compute the value of P.

Since most of Lewicki's work was done at a speed of 20 000 r. p. m. we may confine our attention to this speed. If we then set $n = 20\,000$ and $D = 7.089$, we have

$$P = 38.0 \rho^{0.9} \left(\frac{\mu}{\mu_0}\right)^{0.1} \tag{11}$$

This equation would give us the power dissipated by a wheel geometrically similar to Stodola's wheel but of the same disk diameter as Lewicki's, when running at 20 000 r. p. m. in any homogeneous medium of density ρ and viscosity μ.

To find the correction for the lack of exact similarity, we first make the computation for air because in this case the value of $\left(\frac{\mu}{\mu_0}\right)^{0.1}$ will certainly be sensibly equal to unity. The mean of Lewicki's three observations in air, which do not differ enough to make it worth while to treat them separately, was P(obs) = 4.37 hp at a mean density $\rho = 0.0647$ lb./ft.^{3} Substituting in equation (11) we have P(calc) = 3.24 hp. The ratio $\frac{4.37}{3.24} = 1.35$ indicates that the combined effect of the relatively greater clearance and the slightly smaller blade length of Lewicki's wheel, was to increase the windage loss by about 35 per cent. We therefore modify

equation (11) by introducing this correction factor and so obtain, for further use, the equation

$$P = 51.4\rho^{0.9}\left(\frac{\mu}{\mu_0}\right)^{0.1} \tag{12}$$

To compare the values computed by equation (12) with Lewicki's observations in steam we must form an estimate of the value of $\left(\frac{\mu}{\mu_0}\right)$ and it is fortunate that this appears with so small an exponent since we know almost nothing about the viscosity of steam. According to measurements by L. Meyer and O. Schumann (see Landolt and Börnstein's tables) the viscosity of saturated steam at 100° is about 0.72 times that of air at room temperature. We may assume as a sufficient approximation, under the circumstances, that μ is proportional to the square root of the absolute temperature and is independent of the density. In applying equation (12) to steam we therefore have

$$\left(\frac{\mu}{\mu_0}\right)^{0.1} = 0.968\left(\frac{460+t}{672}\right)^{0.05} \tag{13}$$

where t is the temperature of the steam on the Fahrenheit scale. Equation (12) then takes the form

$$P = 49.7\rho^{0.9}\left(\frac{460+t}{672}\right)^{0.05} \tag{14}$$

applicable to dry steam of density ρ lb./ft.^{3} and temperature t °F.

We may first consider the 15 observations made by Lewicki in superheated steam, which was presumably dry and homogeneous though at the lower temperatures this may not have been quite true. The data which concern us are collected in Table 2, reduced to English units.

TABLE 2

Lewicki's Observations in Superheated Steam; n = 20 000 r. p. m.

Pressure p [lb./in.²]	Temperature t [°F.]	Density ρ [lb./ft.³]	Power P [U. S. hp.]	$\frac{P}{\rho}$	$\frac{P}{\rho^{0.9}}$
14.7	221	0.0368	3.02	82	59
14.7	253	0.0350	2.77	79	57
14.7	406	0.0286	2.33	81	57
14.7	462	0.0268	2.30	86	60
14.7	478	0.0263	2.00	76	53
14.7	531	0.0249	1.98	80	55
14.7	556	0.0243	1.87	77	53
14.7	574	0.0239	1.83	77	53
9.56	586	0.0153	1.08	70	46
5.69	462	0.0100	0.88	88	56
5.69	489	0.0097	0.81	84	52
5.68	259	0.0127	0.94	74	48
5.63	561	0.0089	0.60	69	42
5.39	295	0.0115	0.90	78	50
5.39	590	0.0083	0.58	70	43
Means	448	0.0202		78	52
Mean residuals				±5.8%	±8.5%

The values of the density are not very exact, partly because for the first eight observations the pressure is merely stated to have been atmospheric, and partly because the densities at high superheats are not very accurately known.

In view of the unavoidable errors we can probably do no better than to average the values and set $P = 78\rho$, the mean density being $\rho = 0.0202$ and the mean temperature 448° F. Under these circumstances we therefore have $P(\text{obs}) = 1.58$ hp. Equation (14) with these values of ρ and t gives us $P(\text{calc}) = 1.51$ hp. The agreement of the observed and calculated values to within 5 per cent must be regarded as very satisfactory.

We may also represent the observations fairly well by setting $P(\text{obs}) = 52 \times \rho^{0.9}$, while equation (14) gives us $P(\text{calc}) = 50.5 \times \rho^{0.9}$. The agreement is a trifle closer but there is no great difference. The result of the comparison of Stodola's and Lewicki's wheels is to show that as nearly as we can tell, equation (9) represents the facts and that dry steam is entirely comparable with air when the proper physical constants are used.

We may now consider Lewicki's five observations in saturated steam, the results of which are shown in Table 3.

TABLE 3

Lewicki's Observations in Saturated Steam; n = 20 000 r. p. m.

Pressure p [lb./in.²]	Density ρ [lb./ft.³]	Power P [U. S. hp.]	$\frac{P}{\rho}$	$\frac{P}{\rho^{0.9}}$
14.7	0.03732	3.22	86.2	62.0
10.50	0.02727	2.69	98.3	68.9
8.56	0.02253	2.05	91.1	63.7
6.44	0.01726	1.66	96.0	64.0
5.69	0.01538	1.49	96.8	63.7
Mean	0.02395		93.7	64.5
Mean residuals			±4.3%	±2.8%

As for superheated steam, we can represent the results approximately by $P \propto \rho$ or $P \propto \rho^{0.9}$, the second being in this case distinctly the better, as is shown by the fact that the mean residual is only 2.8 per cent as compared with 4.3 per cent.

Not knowing how the viscosity varies with temperature when the steam is kept saturated, we ignore the last factor of equation (14) which is certainly nearly unity, and we then have $P(\text{calc}) = 49.7\rho^{0.9}$ as compared with $P(\text{obs}) = 64.5\rho^{0.9}$. The observed value is thus 1.30 times the calculated. Making the computation by the mean density we have $P = 93.7\rho$, $\rho = 0.02395$ $P(\text{obs}) = 2.23$ hp while equation (14) gives us $P(\text{calc}) = 1.73$, the ratio being now 1.29 in place of 1.30.

The cause of the discrepancy of 30 per cent is clear. To obtain dry saturated steam is a difficult matter, requiring elaborate precautions, though this was not so well known at the date of Lewicki's experiments. The increase of 30 per cent in the resistance in passing from air or dry steam to saturated steam, was due to the wetness of the steam. Not knowing how wet the steam may have been, we have, perforce, used the density of the steam alone and not the mean density of the mixture. The mean density would have been larger and so would, if used in the computations, have reduced the discrepancy between the observed and calcu-

lated values. But it is most unlikely that the steam was so wet that the whole 30 per cent could be accounted for in this way, even supposing that the water remained completely suspended. And under such conditions only a very small amount of water remains in suspension as fog; most of it is deposited and that is undoubtedly what happened here. Just how wet the steam was and why the deposition of water on the wheel should have increased the resistance 30 per cent it is of course impossible to say. But it is clear that the steam did not act like an *homogeneous* medium, so that equations developed for homogeneous media, which, as we have seen, describe the facts satisfactorily for both air and dry steam, are not strictly applicable. All we can say at present is that the resistance to the rotation of a wheel in steam increases considerably if the steam changes from dry to wet; but how the amount of the increase depends on the wetness or other circumstances can only be decided by further experiments.

It remains to examine Holzwarth's results for wheels which had the same ratio of blade height to disk diameter and so had at least this one element of geometrical similarity. We have the figures given in Table 4, for $\frac{l}{D}=0.05$.

TABLE 4

Holzwarth's Results on Wheels with the same Blade Length Ratio

Disk diameter D (ins.)	Blade length l (ins.)	Speed exponent β	$\frac{10^{16}K}{n^3D^5}$
20	1.0	3.6	2.5
30	1.5	3.0	3.5
40	2.0	2.9	2.5
50	2.5	3.1	2.3

The values of β, taken from the straight lines drawn on the plot of log K against log n vary considerably and are rather uncertain so that the round values $\beta=3$ and $\gamma=5$ have been adopted. The values in the last column were got by averaging over all the points which lay distinctly above any indication of a rapid change in the exponent of n, omitting a few doubtful readings. If these

wheels were, together with their casings, all geometrically similar, and if we had $\delta = 0$, the values of $\frac{10^{16}K}{n^3D^5}$ should all be the same; it is seen that they are nearly so. The high value for the 30-inch wheel is consistent with other readings which make it appear that the 30-inch disks were much rougher than those of the other diameters.

We have also the following figures for $\frac{l}{D} = 0.025$

TABLE 5

D	l	β	$\frac{10^{16}K}{n^3D^5}$
20	0.5	3.6	1.31
40	1.0	2.5	1.29

If the readings of K from the published curves really represent the facts as Holzwarth observed them, the great difference in the two values of β proves that these two wheels were far from geometrically similar. The agreement of the values in the last column is therefore little better than accidental, though it shows that the facts may be represented, at least roughly, by the equation $P = N\rho n^3 D^5$.

5. *The Effect of the Density of the Medium.*—All writers on the subject appear to agree that when a given wheel runs at a given speed, the power dissipated is directly proportional to the density of the medium. Holzwarth says that this relation holds for steam "within limits accurate enough for practical purposes," and Lasche, in an equation quoted by Stodola,[1] sets $P \propto \rho n^3$. Stodola experimented on a multi-disk impulse turbine driven in stagnant steam of densities of 0.1 to 1.7 kgm/m^3, which correspond, if the steam was dry saturated, to pressures of 2.5 to 45 lb/in^2 absolute. His observations as plotted on a small scale diagram indicate the existence of a linear relation between P and ρ, and he sets $P \propto \rho$ and uses this relation without further question. Lewicki's observations as given in Tables 2 and 3 also indicate that the resistance is approximately proportional to the density. We may therefore say that within the range

and the accuracy of the published experimental data on this point, the windage of turbine wheels in a given medium is proportional to the density of the medium, but the evidence is not at all sufficient to show how nearly exact the relation is. We can, in fact, only regard this evidence as roughly conformatory of that presented in section 3, which shows that when the medium is homogeneous the exponent of ρ in equation (7) can not differ much from unity.

6. *Remarks on the Comparison of Dissimilar Wheels.*—The considerations already set forth having shown that an equation of the form (7) describes the behavior of wheels of any one shape, it remains for us to find out if possible how the coefficient N depends on the shape of the wheel and its casing.

Since N depends on shape and not on absolute size, any correct expression for N must contain as variables only ratios of lengths. An equation for P which can not be reduced to the form (3), or practically to the form (7), with the N's satisfying the above condition, is not a rational equation and can not have any general validity, even approximate, though it may be satisfactory as an empirical working formula over limited ranges of the variables.

In attacking the problem of finding a satisfactory expression for N we are met at the outset by the obvious fact that the shape of a turbine wheel and its casing, even if confined to general conformity with commercial practice, may vary in a great many ways. Thus N must be regarded as a function of a large number of variables which are, at least within certain limits, all independent. But while a complete solution of the problem of predicting the value of N from geometrical measurements is thus out of the question, we may nevertheless make some progress if it is found that in practice one or a very few variables are of so much importance that the effects of changing the others are small or negligible.

It is evident, both a priori and from experiment, that two very important geometrical elements to be considered are the blade length and the closeness with which the casing surrounds the blades, both measured in terms of the disk diameter. Other important elements are the roughness of the disk; the width, pitch, and angles of the blades; the number of rows of blades,

and whether they are shrouded or not. Of these most evidently important data, the blade length ratio, $\frac{l}{D}$, and the number of rows of blades are often the only ones given in published accounts of experiments, the others either not being given at all or having been varied so unsystematically that they can not be used. To start with, we shall of necessity confine our attention to wheels with one row of blades, together with their limiting form, bladeless disks. As regards pitch, width, and angles of the blades, we can only say that in practice different single-row steam-turbine wheels are usually somewhere near similar in respect to these points, as they are also, though perhaps less nearly, in respect to roughness and profile of the disk.

Stodola's experiments [1] showed that the amount of clearance between wheel and casing has a large influence on the windage resistance, and he made a few measurements relating to this piont. But since his results are not numerous and we have no other satisfactory data on the effect of altering the casing which incloses a given wheel, we are reduced to the expedient of eliminating the effect of the casing by removing it altogether; in other words, we must, at least in starting, use only data obtained from wheels running in the open without any casing. We may then hope to get an approximate expression for N in terms of the blade-length ratio, the hope being founded on the expectation that this will prove to be much more important in specifying the shape of the wheel itself than all the other variable elements combined, so long as these others remain within the limits which obtain in practice.

We then have available for study, Odell's results on disks and Stodola's results on one disk and five wheels. Tentatively, we shall also use Holzwarth's results on inclosed wheels in steam, the assumption being that the casings were similar. Since the nature of these data does not warrant the use of any refinements in analyzing them, we shall accept the equation

$$P = N\rho n^3 D^5 \tag{15}$$

as a sufficiently approximate description of the facts for wheels of any given shape.

7. *Expressions for N in terms of Roughness and Blade Length.*—If we assume that the differences of resistance of different single-row wheels of the same disk diameter, when driven at the same speed in the same medium, are expressible in terms of the roughness of the disk and the blade length l, the coefficient N must have the form

$$N = A + f\left(\frac{l}{D}\right) \tag{16}$$

in which f is some function which vanishes with its argument, and A is the constant limiting value of N when the blades are shortened indefinitely. For simple flat wheels the "disk coefficient" A will depend mainly on the superficial roughness of the metal of the disk; but it will also, in general, depend on the profile of the wheel and the nature of the shrouding over the ends of the blades. For brevity we may include all these factors in the single term "roughness" since we have no data which would enable us to separate them.

If, further, we admit that equation (7) is correct in having only a single term in the second member, it follows that any general equation for P which contains no other elements of shape than roughness and blade length must necessarily have the form

$$P = \rho^{1-\delta} n^{3-\delta} D^{5-2\delta} \mu^{\delta} \left[A + f\left(\frac{l}{D}\right) \right] \tag{17}$$

or approximately

$$P = \rho n^3 D^5 \left[A + f\left(\frac{l}{D}\right) \right] \tag{18}$$

The three equations which the writer has seen given for computing the power dissipated in windage, do not satisfy this requirement and so are not general, i. e., they can not safely be used for extrapolation to values of the variables outside the limits of the experiments from which the equations were deduced. These three equations are the following:

Lasche is quoted by Stodola [1] as having deduced, from experiments on wheels with 1, 2, 3, and 4 rows of blades, equations which may, for comparison with (18) be written in the form

$$P = \rho n^3 D^5 \left[B\left(\frac{l}{D}\right) D^{-3} \right] \tag{19}$$

This equation does not profess to be valid except within rather narrow limits, hence there is no occasion for criticising the fact that it leads to an absurdity for bladeless disks or that it violates the dimensional requirements.

Jude,[9] after an examination of the experimental results of Odell, Lewicki, Stodola, and Holzwarth, gives an equation which may be written

$$P = \rho n^3 D^5 \left[A + B \left(\frac{l}{D} \right)^{\frac{3}{2}} D^{-2} n^{-\frac{3}{2}} \right] \tag{20}$$

The second or blade term of this equation violates the dimensional conditions completely.

Stodola [1] gives an equation which may be written

$$P = \rho n^3 D^5 \left[A + B \left(\frac{l}{D} \right)^{\frac{3}{2}} D^{\frac{1}{2}} \right] \tag{21}$$

This, too, violates the dimensional requirements but to a less extent than equation (20); it could therefore be used over a wider range of diameters before involving excessive errors. Equation (20) can apply only over a limited range of speed as well as of diameter, while equation (21) involves time correctly to the same degree of approximation as equation (15).

It has seemed to the writer worth while to attempt to represent the same data as were used by Stodola and Jude, or such of them as it appeared might legitimately be used, by an equation which should be free from the defects just noticed. The very simple form

$$P = \rho n^3 D^5 \left[A + B \left(\frac{l}{D} \right)^{x} \right] \tag{22}$$

was therefore tested, to see if it was possible to find satisfactory values of the constants A, B, and x. The disk coefficient, A, must evidently depend on the roughness of the disk and we can not expect all disks to give the same value. The "blade coefficient," B, should doubtless involve the width, pitch, angles, profile, and form of shrouding of the blades, but it will, tentatively, be treated as a constant, as will the exponent x. The question is whether such an equation can be made to describe the observed facts with reasonable accuracy and completeness.

8. *The Form of the Coefficient N for Stodola's Unenclosed Wheels.*—We may first examine Stodola's data on his five wheels running in the open air. He gives his observed value of the power absorbed by each wheel, only for one speed, namely the highest used with that wheel. The data which concern us here are collected in Table 6.

TABLE 6

Stodola's Observations on Unenclosed Wheels in Air

I	II	III	IV	V	VI	VII	VIII
	$\frac{l}{D}$	Disk diameter D (ins)	Maximum speed n (r. p. m.)	Power absorbed P (hp.)	$10^{16}N$	P (calc)	P (obs) $-P$ (calc) in per cent
D	0.0311	34.84	1600	2.306	1.569	2.315	− 0.4
C	0.0364	26.50	2200	1.778	1.786	1.736	+ 2.4
A	0.0396	19.88	2200	0.536	2.319	0.446	+18.
E	0.0476	45.47	980	2.895	2.264	3.000	− 3.5
B	0.1190	19.84	2100	1.850	9.306	1.833	+ 0.9

N is defined by $P=N\rho n^3D^5$ and the density of the air is assumed to have been $\rho=0.0699$ lb/ft^3

$$P\text{ (calc)}=10^{-16}\rho n^3D^5\left[1+593\left(\frac{l}{D}\right)^2\right]$$

If the observed values of P. are expressible by equation (22), i. e., by setting $N=A+B\left(\frac{l}{D}\right)^x$, we have

$$\log\,(N-A)=\log B+x\log\frac{l}{D} \tag{23}$$

and the values of A, B, and x may be found by trial. We assume a value of A, and plot $\log\,(N-A)$ against $\log\frac{l}{D}$. If we can find a value of A such that the points all lie on a straight line within the observational errors, equation (22) is satisfied, the slope of the line is the value of x, and the position of the line gives the value of B.

By this method it was found that the observations on four of the five wheels are satisfactorily represented by writing equation (22) in the particular form

$$P=10^{-16}\rho n^3D^5\left[1+593\left(\frac{l}{D}\right)^2\right] \tag{24}$$

and that no other values of A, B, and x are sensibly better. Values computed by equation (24) are given in column VII of Table 6 and the agreement of the results with the observed values in column V is shown by column VIII to be good except for wheel A. The value for this wheel can evidently not be brought into accord with those observed for the others by any simple expression for N as a function of $\frac{l}{D}$. It is possible that the discrepancy may be due to an experimental error, but this seems very unlikely. Unless it is merely a typographical error, it probably arises from some geometrical peculiarity of this wheel which is not evident from the description given. The writer has assumed that the blades of all the wheels were shrouded, because that is the usual construction; but if the blades of wheel A were open-ended, the relatively greater windage would be easily understood.

However, there is no advantage to be gained from splitting hairs in analysing so small a number of data by means of confessedly only roughly approximate assumptions, and we may be content to say that equation (24) represents the results of Stodola's experiments on single-row wheels of ordinary forms running in the open air, quite as well as could be demanded of any equation deduced from so inadequate data.

9. *Deduction of an Expression for N from Holzwarth's Data.*— Holzwarth's results and the manner in which they were treated for finding the relation of P to n, have been described in section 3. The values found for $\beta = 3 - \delta$ are given in Table 7 as they were read from the straight lines drawn on the plot of log K against log n and they are uncertain by from 0.1 to 0.2, because the lines were drawn merely by inspection.

TABLE 7

Values of K from Holzwarth's Diagrams

Blade length (inches)	$D=20$ inches	$D=30$ inches	$D=40$ inches	$D=50$ inches
0.5	3.61	3.39	2.51	3.24
1.0	3.62	*3.00*	2.51	3.30
1.5	2.28	*3.00*	2.58	2.58
2.0	*2.83*	*2.98*	*2.90*	*2.89*
2.5	2.69	3.39	*3.09*	*3.14*

From the great variations in β it appears either that the values of Holzwarth's coefficient K read from his diagrams are erroneous or that the wheels, even for the same blade-length ratio, were far from geometrically similar. It would be a waste of time to attempt to reconcile all the readings of K with an equation of the form (22) in which $\beta=3$, and I have therefore arbitrarily selected the eight series which gave values of β (*italicized* in Table 7) between 2.8 and 3.2, i. e. values which we can not be sure are different from 3.0, and I shall confine my analysis to these series.

For each of the selected eight series, the mean value of $\frac{10^{16}K}{n^3D^5}$ was found by averaging over all the values of K for the higher speeds, omitting a few points where the readings from the diagrams were obviously liable to exceptionally large errors. We thus get eight separate values.

In the units we have been using we have, approximately,

$$P=0.54\,\rho\,K \tag{25}$$

Since Holzwarth's diagrams are drawn for a constant value of the density we may treat K as we have previously treated P, which is proportional to it when ρ is constant. If we proceed in this way, we find that by selecting an appropriate disk coefficient A' for each diameter and plotting $\log\left(\frac{10^{16}K}{n^3D^5}-A'\right)$ against $\log\frac{l}{D}$, seven of the eight points may be made to fall very close to a single straight line of which the slope is 2. The exception is the one rather doubtful point for the 20-inch diameter. We thus find that those of Holzwarth's results which are comparable with Stodola's in giving $\beta=3$, approximately, may be represented by an equation very like (24) which describes Stodola's results. The exponent of $\left(\frac{l}{D}\right)$ is the same and the disk and blade coefficients are not very different, though somewhat smaller for Holzwarth's wheels, in accordance with the fact that the wheels were inclosed instead of run in the open.

If we write

$$P=\rho n^3D^5\left[A+B\left(\frac{l}{D}\right)^2\right] \tag{26}$$

we have the values of A and B given in Table 8.

TABLE 8

Values of the Coefficients of Equation (26)

	Disk diameter D (inches)	Disk coefficient $A \times 10^{16}$	Blade coefficient $B \times 10^{16}$
Stodola's observations on wheels B, C, D, E, in open air	19.8–45.5	1.0	593
Holzwarth's observations. Wheels inclosed in steam	20	(0.54)	(243)
	30	0.86	414
	40	0.32	414
	50	0.22	414
Stodola's bladeless disk run in open air	21.1	0.63	
Odell's disk D in open air	47.1	0.5	
Jude; from observations by Lewicki, Odell, Stodola, and Holzwarth, for wheels run in open air		0.44	

The difference between the coefficients deduced from Stodola's and from Holzwarth's observations would probably have been greater if Holzwarth's values had all been obtained from experiments in air. His results were obtained "from occasional experiments carried on for some years—partly in air, partly in steam of different density." Since he says nothing to the contrary, the steam was doubtless saturated and probably by no means dry, so that we may infer from Lewicki's observations, described in section 4, that the value of N or of the coefficients A and B would have been smaller if the experiments had all been made in air or perfectly dry steam.

On account of the varying density in Holzwarth's experiments, some sort of reduction to a standard density must have been made in order to get the values of K for drawing the diagrams. Holzwarth sets $P \propto \rho$ and we must assume that he used this relation in the reduction. But if in equation (7) the value of β is very different from 3, the exponent of ρ must differ by the same amount from unity and a reduction of this sort is not permissible. Although no detailed criticism is possible, it is evident that we have here an additional ground for omitting from consideration those series of Holzwarth's results which do not conform approximately to the equation $P = const \times n^3$.

The degree of approximation to which equation (26) with the coefficients given in Table 8 reproduces the readings from Holz-

warth's diagrams, may be seen from Table 9 in which are given the values K(obs) read from the diagrams, and the values K(calc) computed from equations equivalent to (26) with the coefficients in Table 8. The readings may be in error by one unit at the low values and several units at the higher.

TABLE 9

Comparison of Observed and Computed Values of Holzwarth's Coefficient K

Speed in r. p. m. n=			1000	1250	1500	1750	2000	2500	3000	3500
D (in.)	l (in.)									
20	2.0	obs	5	6	8.5	10	14.5	24.5	45.5	(120)
		calc	.....	.....	5.9	9.4	14.1	27.5	47.5	75.5
30	1.0	obs	10.5	13.5	20	31.5	45	94	160	293
		calc	6	11.6	20.1	31.9	47.7	93.1	161	256
	1.5	obs	12.5	19	28.5	44	62.5	131	238	.....
		calc	8.5	16.7	28.8	45.8	68.3	133	231	.....
	2.0	obs	13	20	40	70	109.5	205	344	.....
		calc	12.2	23.8	41	65.2	97.3	190	328	.....
40	2.0	obs	26	46	85	150	224	375	527	.....
		calc	25.7	50.3	86.9	138	206	402	695	.....
	2.5	obs	34	63	123	211	334	560	.....	.....
		calc	36.8	71.8	124	197	294	574	.....	.....
50	2.0	obs	50	86.5	161	288	.....	.....	.....	.....
		calc	50.8	99.2	172	272	.....	.....	.....	.....
	2.5	obs	65	118	236	446	.....	.....	.....	.....
		calc	72.3	141	244	388	.....	.....	.....	.....

For the smaller wheels the observed resistance at the lower speeds is considerably larger than the calculated, and the law of resistance is evidently not the same as at the high speeds. At 750 r. p. m. the observed K is larger than the calculated in every instance. But if we omit the low speeds, where agreement is not to be expected, and the highest speeds, where slight errors in the drawing of the diagrams cause large errors in the values of K read from them, the agreement of observed and calculated values is fair—certainly as good as would be demanded of any formula by one who has studied the diagrams.

10. Further Values of the Disk Coefficient.—We have now exhausted the few published data which are available for investigating the dependence of the coefficient N of equations (7), (8), (9), etc., on the blade-length ratio, and there remain for consideration only experiments on bladeless disks which will give values of the disk coefficient for comparison with the values obtained from experiments on wheels with blades.

Stodola's boiler-plate disk of 21.1-inch diameter absorbed 0.147 hp when running 2000 r. p. m. in air of density $\rho = 0.07$ lb. per cu. ft^3. From these values we find $A = 0.63 \times 10^{-16}$. The only remaining experiments on a rigid disk, which gave $\beta = 3$ nearly, are those of Odell on his largest disk. From data already given in section 4 we find for this disk $A = 0.5 \times 10^{-16}$ approximately. Both these values are included in Table 8. The difference between the values of A deduced from Stodola's observations on wheels and on the disk need not cause any surprise, for we do not know anything about the relative roughness in the two cases. The shrouding over the blades and the difference of profile between wheels and disk may account for the difference, which appears to be genuine.

One more value of A is included in Table 8, namely, $A = 0.44 \times 10^{-16}$, deduced by Jude from a general examination of all the experiments already mentioned in this paper. It is applicable according to its author to flattish disks in open air.

11. The Influence of Axial Clearance.—For information on this point we have only Stodola's comparative runs of three of his wheels in the open air and in casings. The results are given in Table 10, some of the data being repeated from Table 6.

TABLE 10

Stodola's Experiments on Varying Clearance

I	II	III	IV	V	VI
	$\frac{l}{D}$	D inch	Axial clearance $\div D$	$10^{16}N$	Ratio
			∞	2.32	
A	0.0396	19.88	0.0079	1.28	0.55
				2.26	
E	0.0476	45.47	0.0056	0.57	0.25
			∞	9.31	
B	0.1190	19.84	0.0079	3.53	0.38

In column V of the table are given the values of N in the equation $P = N\rho n^3 D^5$ for the axial clearances given in column IV, the clearance ∞ meaning that the wheel was run in the open air. In column VI there is given for each wheel the ratio in which the dissipation was reduced by the presence of the casing.

It appears from this table that the effect of a close casing is much greater on the blades than on the disk. By comparing wheels A and B we find that the presence of a casing, which left at the blade edges an axial clearance of about 0.008 times the disk diameter, reduced the resistance considerably more in the case of B, which had blades three times as long as those of A, the ratio of reduction being 0.38 for the long blades as against 0.55 for the short ones. If we compare wheels A and E, which did not differ very widely as to blade-length ratio, we find that reducing the clearance from 0.0079 D to 0.0056 D, i. e., to two-thirds, decreased the resistance, expressed as a fraction of the open-air resistance, by about one-half, or from 0.55 to 0.25. We have also already found in section 4, when comparing wheel B with Lewicki's wheel of about the same blade-length ratio, that increasing the axial clearance by about one-half, or from 0.008 D to 0.012 D, increased the resistance some 35 per cent.

These few isolated data are evidently not a sufficient ground for any general quantitative statement about the effect of clearance on windage, but they may be valuable as a basis for guesswork in cases which happen to be nearly similar to the ones mentioned.

In this connection we may also note the results obtained by Stodola[1] in comparing wheels run in the normal or forward direction with the same wheels run backward. In open air the resistance backward was in one example as much as 5.4 times the resistance forward, though in other examples the ratio was not so large. But inclosing the blades reduced the difference very much, and the longer the blades and the greater the part of the resistance due to the blades the greater is this effect of the casing, so that with very small clearances the resistance when the wheel is run backward is very little greater than when it is run forward. With wheel B for which $\frac{l}{D} = 0.119$, when the axial clearance was 0.008 D, the resistance backward was only 13 per cent more than the resistance forward.

12. *Remarks on Further Experimental Results.*—In the foregoing discussion the symbol P has everywhere denoted the power required to drive a wheel against the resistance of the otherwise stagnant medium surrounding it, and the experiments noted have referred to this state of affairs. But the conditions of ordinary practice are different, and it remains a question whether, in designing a steam turbine, a windage correction based on even completely satisfactory data of the kind considered could be considered reliable. The only answer that can be made to this question is that we do not know; and the best, because the only, thing we can do at present is to compute windage corrections for designing purposes as if the turbine were to be driven independently from without, acting merely as a brake, a condition which occurs only in the case of marine turbines with reversing stages or with cruising stages which are by-passed at full power.

Another pertinent question is: How much ought the computed windage loss to be reduced when a part or all of the blades are working in the ordinary manner, with steam from the nozzles passing through them? The experiments of Lasche, quoted by Stodola,[1] and of Jasinsky [10] are not sufficiently comprehensive to tell us more than that the windage decreases as the admission arc increases and fewer blades are idle. With rectangular nozzles and a continuous steam belt, it will probably be not far wrong to multiply the blade term of the computed resistance by the fraction of the whole circumference which is not occupied by open nozzles, i. e. by the fraction of the whole number of blades which is idle at each instant.

The experiments we have considered having referred only to disks or single-row wheels, one further question remains: How does the windage, i. e. the value of N in our equations, depend on the number of rows of blades? Here again we have only the most meager information. Experiments by Lasche, quoted by Stodola,[1] gave for wheels with from 1 to 4 rows but otherwise, presumably, similar, resistances which stood in the following relation:

Number of rows of blades	1	2	3	4
Relative resistance	1	1.2	1.6	2.4

But the experiments were on wheels which were not closely encased so that they do not give us much practical assistance. The increase of resistance with number of rows of blades would probably be very much less rapid if the wheels were run in casings with fine clearances.

13. *Dynamically Similar Wheels.*—The one fact which emerges clearly from the foregoing discussion is that the task of providing data for the development of a satisfactory general formula for computing the windage losses of high-speed steam turbines, has hardly been begun. Suggestions as to future lines of experiment are therefore in place here.

Up to peripheral speeds of at least one-half the speed of sound, or about 700 feet per second for steam, we are justified in treating the medium as incompressible and the resistance phenomena may be described by an equation of the general form (4). If we introduce the "kinematic viscosity" $\nu=\frac{\mu}{\rho}$, the equation may be written

$$P=\rho n^3 D^5 \varphi\left(\frac{\nu}{nD^2}\right) \tag{27}$$

φ being a function of which the form, though unknown, is fixed by the shape of the wheel and casing.

Let us compare two geometrically similar wheels of diameters D and D_0 running in media of densities ρ and ρ_0 and kinematic viscosities ν and ν_0 at speeds of rotation which stand in the ratio

$$\frac{n}{n_0}=\frac{\nu}{\nu_0}\left(\frac{D_0}{D}\right)^2 \tag{28}$$

Speeds thus related are "corresponding speeds."

At corresponding speeds

$$\frac{\nu}{nD^2}=\frac{\nu_0}{n_0 D_0^2}$$

and since the *form* of φ depends only on the shape, which is the same for the two wheels, we have

$$\varphi\left(\frac{\nu}{nD^2}\right)=\varphi\left(\frac{\nu_0}{n_0 D_0^2}\right)$$

no matter what the shape of the wheels and the form of the function φ may be. The ratio of the dissipation by the two wheels at corresponding speeds is therefore, by equation (27)

$$\frac{P}{P_o}=\frac{\rho n^3 D^5}{\rho_o n_o^3 D_o^5}$$

or by equation (28) which defines corresponding speeds,

$$\frac{P}{P_o}=\frac{\rho}{\rho_o}\left(\frac{\nu}{\nu_o}\right)^3\frac{D_o}{D} \tag{29}$$

Any two geometrically similar wheels running at corresponding speeds constitute a pair of "dynamically similar" systems, and the power dissipated by either may be determined from an experiment on the other by means of equation (29), if the diameters of the wheels are measured and the densities and kinematic viscosities of the media are known.

If the experiments are all made in the same medium so that $\rho=\rho_o$ and $\nu=\nu_o$, corresponding speeds are inversely as the squares of the diameters and the powers dissipated at corresponding speeds are inversely as the diameters.

If T and T_o are the torques required to drive the wheels at corresponding speeds, since $P\propto nT$ we have by (28) and (29)

$$\frac{T}{T_o}=\frac{\rho}{\rho_o}\left(\frac{\nu}{\nu_o}\right)^2\frac{D}{D_o} \tag{30}$$

At corresponding speeds in a given medium the torque is directly as the diameter.

The shearing stress in the metal of the shaft, so long as the shaft is not sensibly distorted, is proportional to the torque divided by the cube of the shaft diameter, or with similar wheels to $\frac{T}{D^3}$. Hence the ratio of the shearing stresses, S and S_o, in the shafts of two dynamically similar wheels is

$$\frac{S}{S_o}=\frac{\rho}{\rho_o}\left(\frac{\nu}{\nu_o}\right)^2\left(\frac{D_o}{D}\right)^2 \tag{31}$$

At corresponding speeds in a given medium the shearing stresses are inversely as the squares of the diameters.

The centrifugal stresses C and C_0 in the metal of the two wheels, if they are of the same density, will stand approximately in the ratio.

$$\frac{C}{C_0}=\left(\frac{\nu}{\nu_0}\right)^2\left(\frac{D_0}{D}\right)^2 \tag{32}$$

At corresponding speeds in a given medium, the centrifugal stresses are inversely proportional to the squares of the diameters if the wheels are of the same density.

14. The Use of Model Wheels in Studying Windage.—To avoid the difficulties of working with large wheels, it may be desirable to utilize the results of model experiments as is done in determining ship resistance.

If we limit ourselves to tne use of a single medium, the practicable range of scale reduction is not great, unless the full-sized original runs very slowly, and this case is not interesting because we know that at low speeds the windage losses are too insignificant to demand much attention. For dynamical similarity in a given medium, a quarter-scale model must, by equation (28) run at 16 times as many revolutions per minute as the full-sized original; and by equation (32), the centrifugal stresses in the wheel will then be 16 times as great in the model as in the original; an increase which would usually not be permissible. Furthermore, the peripheral speed of the model will be 4 times that of the original and may approach the acoustic speed so closely as to invalidate our fundamental assumption that the medium behaves sensibly as if it were incompressible. There is evidently not much information to be got from small scale models unless they can be run in a medium of much less kinematic viscosity than steam, so that the speed of the model may be reduced, in accordance with equation (28).

Water is such a medium. Using values from Landolt and Börnstein's tables, and comparing water with air which is known to act like dry steam, we have at 20° C and 1 atmosphere pressure.

Air	Water
$\rho=0.001205$	$\rho=0.9982$
$\mu=0.0001898$	$\mu=0.01012$
$\nu=0.1575$	$\nu=0.01014$

Let it be desired to find the power dissipated by a wheel of diameter D_0 running n_0 revolutions per minute in air, by means of experiments on a model of diameter $D=\frac{D_0}{r}$ running in water, the temperature being 20° C in each case and the air being at normal pressure. We then have to substitute in equations (6) to (10)

$$\frac{\rho}{\rho_0}=828 \qquad \frac{\nu}{\nu_0}=0.0644$$

the uncertainty of $\frac{\nu}{\nu_0}$ being about $\pm$ 3 per cent.

For any given scale ratio r, the ratio of corresponding speeds, n for the model and n_0 for the original will be, by equation (28),

$$\frac{n}{n_0}=0.0644r^2\pm 3 \text{ per cent} \tag{33}$$

The ratio of the powers dissipated at corresponding speeds will be, by equation (29),

$$\frac{P}{P_0}=0.222\ r\pm 6 \text{ per cent} \tag{34}$$

from which P_0 can be found if P has been measured in an experiment on the model.

We must next consider whether the stresses in the wheel and shaft of the model will rise too high when we make a convenient reduction of diameter. Taking first the centrifugal stresses in the wheel, we have by substitution in equation (32)

$$\frac{C}{C_0}=0.00415\ r^2 \tag{35}$$

Setting $C=C_0$ and solving for r, we find that the stress of any point in the model will not exceed that at the corresponding point in the original until $r>15.7$. As a 10 to 1 reduction will usually be ample, there will be no difficulty regarding the bursting strength of the model if it is made of the same material as the full-sized wheel.

For the ratio of the shearing stresses in the shaft, equation (31) gives us

$$\frac{S}{S_0} = 3.43\ r^2 \tag{36}$$

A scale reduction $r = 10$ would thus give us 343 times as great a stress in the shaft of the model as in the original shaft. This, at first sight, looks quite impracticable. But in fact turbine shafts are generally made much stronger, for the sake of stiffness, than is required by torsional strength; and, furthermore, the torque due to windage when the wheel is driven light is only a few per cent of the torque of the wheel running at full load. Hence the ratio 343 would probably not always be excessive. However, doubling the diameter of the shaft of the model would have hardly any effect on its windage resistance, and by such a small sacrifice of exact geometrical similarity the use of a model in water might always be made practicable, so far as the strength of the shaft is concerned.

To make the matter more concrete we may carry out the computations for a few typical cases of wheels run in air in comparison with models of 12 inches diameter run in water. We have the values given in Table 11.

TABLE 11

Full-sized diameter D_0 (inches)	Wheel speed n_0 (r. p. m.)	Diameter ratio for 12-inch model $r=\frac{D_0}{D}$	Speed of model n (r. p. m.)	Power ratio $\frac{P}{P_0}$	Stress ratio in shafts $\frac{S}{S_0}$
36	3600	3	2090	0.67	31
60	1800	5	2900	1.11	86
84	900	7	2840	1.55	168
120	450	10	2900	2.22	343

It therefore appears that the investigation of such cases as are commonly met with, by means of 12-inch models run in water, would be quite practicable so far as the points already treated are concerned, and only one further point remains to be considered. This is the question of cavitation.

If cavitation occurs in the water the similarity of the model and the original will cease to exist, and equation (29) will not give correct results. To avoid cavitation, the water surrounding the model must be under pressure and the casing must be constructed accordingly. At atmospheric pressure and high speeds cavitation will certainly occur. If the pressure on the water is increased the cavitation will decrease and the torque at a fixed speed will change until the pressure has been increased so much that cavitation has been eliminated. By observing the variation of torque or power with the pressure at any given speed we have thus a means for finding what pressure is needed in order to eliminate cavitation and make sure that the model in water is comparable with its original in air or steam. It remains to be seen whether the pressures required would be impracticably high, but it appears that the method of model experiments is worth trying.

15. *Summary.*—I. The power P required to drive a turbine wheel of diameter D at n revolutions per unit time against the resistance of a homogeneous medium of density ρ and viscosity μ, when the peripheral speed does not exceed one-half that of sound, may be represented by an equation of the general form

$$P = \rho n^3 D^5 \varphi\left(\frac{\mu}{\rho n D^2}\right) \tag{I}$$

in which the form of the unknown function φ depends solely on the shape of the wheel and its casing.

II. Throughout the practical range of the experimental data, equation (I) has the simpler form

$$P = N \rho^{1-\delta} n^{3-\delta} D^{5-2\delta} \mu^{\delta} \tag{II}$$

the abstract numerical constant N having a value which is determined solely by the shape of the wheel and casing. All the reliable data which we have agree with equation (II) or with direct deductions from it.

III. At low speeds the value of δ is nearly unity and the resistance is directly proportional to the viscosity of the medium. At

the speeds at which stationary turbines are usually run, δ is a small quantity and we have approximately

$$P = N\rho n^3 D^5 \tag{III}$$

A closer approximation is obtained by setting $\delta = 0.1$ whence

$$P = N\rho^{0.9} n^{2.9} D^{4.8} \mu^{0.1}$$

IV. For wheels of ordinary shapes, running either in the open or in casings with fairly large clearances, the "shape coefficient" N may be expressed approximately by the equation

$$N = A + B\left(\frac{l}{D}\right)^2 \tag{IV}$$

in which l/D is the ratio of the blade length to the disk diameter. The disk coefficient A increases with roughness of the disk; the blade coefficient B decreases when the clearance round the blades is decreased, but no more definite statements are warranted.

V. (*a*) For designing purposes we may first compute the windage loss by the following equation deduced from Stodola's results:

$$P = 10^{-16}\rho n^3 D^5\left[1 + 590\left(\frac{l}{D}\right)^2\right] \tag{V}$$

In this equation

P = the horsepower dissipated
ρ = the density of the medium, in pounds per cubic foot
n = the speed, in revolutions per minute
D = the diameter, in inches, at the root of the blades
l = the length of the blades, in inches

The equation is applicable to wheels of ordinary shapes with one row of shrouded blades, running in the open or in casings which leave large clearances, in a homogeneous medium such as air or dry steam.

(*b*) Reducing the clearances, especially round the blades, reduces the windage. In some cases the amount of this reduction may be estimated from Table 10 but no general quantitative statement is possible. The reduction affects mainly the blade term.

(*c*) Open-ended blades have more resistance than shrouded blades, to which equation (V) refers, but there are no data to

show how much. When the radial clearance over the blade tips is small, the presence or absence of shrouding will have little effect.

(*d*) A wheel run backward experiences a greater resistance than the same wheel run forward, i. e., in the normal direction. When running in the open the difference may be considerable, but if the clearances round the blades are small, the resistance backward is not much greater than the resistance forward.

(*e*) Each additional row of blades increases the resistance. When run in the open, a four-row wheel may have two and one-half times the resistance of a one-row wheel, but there are no adequate data published. With short blades and fine clearances the effect of increasing the number of rows will be much less than that indicated.

(*f*) A wheel run in wet steam experiences more resistance than in dry steam or air of the same density. In Lewicki's experiments the increase was 30 per cent; we have no other information on this point.

(*g*) Equation (V) gives the power absorbed when the wheel is driven from without. If the wheel is working in the usual manner we may reduce the blade term in the ratio of the number of idle blades to the whole number.

(*h*) The values given by equation (V) may be too large, especially for smooth wheels with short blades; for the disk coefficient used, viz.: $A = 10^{-16}$, is larger than the values deduced from other experiments.

VI. The necessary indefiniteness of many of the statements made under V is due to lack of experimental data. No formula based only on our present knowledge can be trusted to give accurate results, and no formula which has not the general form (II) should be used for extrapolation beyond the limits of the experiments from which it was deduced. Equation (V), if used as indicated, probably gives safe maximum values.

VII. The method of model experiments is applicable to the study of the windage losses of steam turbines and might prove very useful by decreasing some of the difficulties encountered in making systematic experiments on a large number of full-sized wheels.

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