THE STEAM-TURBINE EXPANSION LINE ON THE MOLLIER DIAGRAM, AND A SHORT METHOD OF FINDING THE REHEAT FACTOR

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CONTENTS

INTRODUCTION

The so-called "heat diagram," or $\theta\varphi$ diagram, in which the state of a mass of steam is represented by a point on a plane with absolute temperature (θ) and entropy (φ) as rectangular coordinates, has in recent years been much used by writers on technical thermodynamics, and for many purposes it is most instructive, though some caution is needed in interpreting it. But for the quantitative solution of problems in steam-turbine design it is by no means comparable in convenience with the "Mollier diagram" or total heat entropy diagram, in which the representation is on a

plane with the total heat (H) and entropy (φ) as rectangular coordinates. The idea of using a surface with H, φ, and p (pressure) as rectangular coordinates is due to Willard Gibbs, H being the same as his "heat function" χ. The $H\varphi$ diagram may be regarded as the projection of this surface on the $H\varphi$ plane, and it was introduced to the notice of engineers by Prof. Mollier, of Dresden, by whose name it is commonly known.

The first six sections of the following paper serve as an introduction, for those not familiar with the subject, to some of the technically important properties of quantities H and φ and of the Mollier diagram. The remainder of the paper contains a discussion of the form, on this diagram, of the expansion line for wet steam flowing through a multistage turbine of known stage efficiency, and the development of a practical method for use by designers for drawing the expansion line on the $H\varphi$ diagram without the use of the laborious step-by-step method.

1. THE TOTAL-ENERGY EQUATION FOR STEADY FLOW OF A FLUID

Let a fluid of any sort be flowing steadily along a channel C_0PC (Fig. 1). Let A_0 and A be two sections of the channel which we shall call the entrance and exit sections.
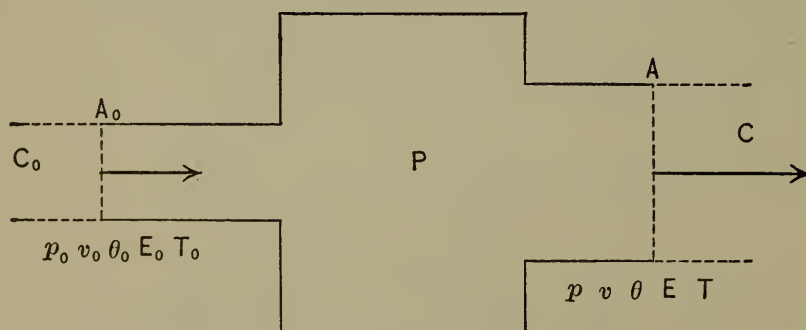
Fig. 1

Let $p_0v_0\theta_0E_0$ be the pressure, specific volume, absolute temperature, and internal energy per unit mass of the fluid as it crosses the entrance section, and let $pv\theta E$ be the corresponding quantities at the exit section. These are to be averages over the section, and the variations from one point to another of the section are to be small.

Let T_o be the kinetic energy per unit mass at A_o, of the axial component [1] of the velocity, and T the corresponding quantity at A. The channel at A_o and A shall be varying in cross section so slowly that the kinetic energy of the radial velocity is negligible. If 1 per cent of the total kinetic energy is a negligible quantity, a total taper of one in four, for a cone, is permissible, so that no severe demands are made on the constancy of cross section at A_o and A.

Subject to the foregoing restriction in the immediate vicinity of A_o and A, the shape of the channel between A_o and A is a matter of complete indifference. There may, if we please, be included, as forming a part of the channel or completely inclosed within it, a motor actuated by the flow of the fluid and delivering work outside the channel, or a pump actuated by the application of power from without and doing work on the fluid. The walls of the channel must be tight so as to prevent leakage of fluid, but they need not be thermally insulating. The fluid may be any liquid, vapor, or gas, but for concreteness we shall usually refer to it as steam.

The first law of thermodynamics, if applied to the passage of one pound of steam from A_o to A, gives us the following statement; the total energy per pound—internal plus kinetic—is increased by the amount of the work done on the steam in crossing the entrance section by the steam behind it, and is decreased by the amount of the work it does against the steam ahead of it in crossing the exit section, by the work given out by the motor, and by any heat which may have been lost by conduction or radiation through the walls of the channel, which may in places coincide with the walls of the motor. We thus have the equation

$$(E + T) - (E_o + T_o) = p_o v_o - pv - W - Q \qquad (1)$$

in which W is the work done outside the channel by the motor, and Q is the heat loss, both measured per pound of steam. The work W includes work done against friction at any bearings which are outside the channel. All the terms in the equation are to be understood as expressed in the same units, e. g., British thermal units (B. t. u.).

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[1] See note at the end of this paper.

Rearranging equation (1), we have

$$T - T_0 + W = (E + pv)_0 - (E + pv) - Q \qquad (2)$$

which is the first fundamental equation of the theory of fluid motors. It is applicable to regularly acting periodic motors as well as to continuous-flow motors such as turbines, if the quantities in the equation are averaged over an integral number of periods or over any very long time. Since no restriction has been imposed upon the sign of either W or Q, the case of continuously or periodically acting pumps is also included, and the equation is, in fact, valid for all cases of steady or—under the above condition as to averaging—periodically varying flow of any sort of fluid.

2. THE TOTAL HEAT OF STEAM

The quantity $(E + pv)$ has a definite value for every state of the fluid in question and has been designated as "heat of formation at constant pressure," "heat contents," and "total heat." We shall adopt the name total heat and write

$$E + pv = H \qquad (3)$$

The quantity to which Regnault gave the name "total heat," and which he measured for dry saturated steam is, for all engineering purposes, sensibly identical with H as just defined. The small outstanding difference is due, first to the fact that the internal energy of water at the ice point is not absolutely though nearly the same at all pressures; and second to the fact that the volume of water at the ice point under any given pressure is not zero though usually quite negligible in comparison with the volume, at the same pressure, of an equal mass of steam dry enough to be suitable for use in a steam motor.

Values of H for water and for dry-saturated steam are given in the steam tables. Their difference is evidently the heat of evaporation, and the value of H for any degree of dryness, x, may be found by linear interpolation between the values for water and for dry steam at the given pressure or temperature. For superheated steam, the value of H is greater than for dry-saturated steam at the same pressure, by the amount of heat needed for the superheating at constant pressure, which depends on the specific heat of superheated steam and is not so well known as might be

desired. Convenient tables for saturated and superheated steam are given in the "Steam Tables and Diagrams" of Marks and Davis [2] from which all the steam data used in this paper have been taken.

We may now write equation (2) in the form

$$(T - T_o) + W + Q = H_o - H \qquad (4)$$

The quantity $(H_o - H)$ will, for short, be called the "heat-drop" of the pound of steam during its passage from A_o to A.

The first two terms of equation (4) represent a quantity of mechanical energy, hence the equation may be read as follows: The mechanical energy produced plus the heat lost to the surroundings, is equal to the heat-drop of the steam. It should be noted that it has not been stipulated that the fluid shall flow without encountering passive resistances such as viscosity. There may be as much internal dissipation of mechanical energy into heat as we please, provided that W represents work actually delivered outside the channel, including work against friction at any *outside* bearings, and that $(T - T_o)$ represents the actual excess of kinetic energy of the axial component of velocity in the exhaust at A over that in the feed at A_o.

3. ADIABATIC FLOW

An adiabatic change of state is defined as one during which the substance in question neither takes in nor gives out heat through its bounding surface. If, therefore, there is no heat leakage through the walls of the channel, the change of state of the steam between A_o and A is adiabatic. We may then set $Q = o$ and equation (4) reduces to

$$(T - T_o) + W = H_o - H \qquad (5)$$

an equation which has several immediate applications to familiar facts. For example, in the case of steady flow, without heat loss, through a simple tube not containing a motor, the work W is zero and equation (5) says that the increase of kinetic energy is equal to the heat-drop. This is the case of flow through a turbine nozzle or through fixed guide blades, if the flow is so rapid or the

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[2] Longmans Green and Co. 1909.

channel so well protected that the heat leakage is negligible. In the case of a properly designed, velocity-compounded, impulse turbine, working with a constant pressure within each stage, the velocity and kinetic energy decrease during flow through the intermediate reversing guide blades. Hence T, at exit from these blades, is less than T_0, at entrance to them, and $H > H_0$. The kinetic energy dissipated in the guide-blade channels thus appears in the steam as "reheat" making H larger than H_0.

The simplest possible case is that of adiabatic throttling or wire-drawing in which there is neither outside work W nor any sensible increase of kinetic energy. The whole first member of equation (5) now vanishes, so that the total heat of the steam remains unchanged during its fall of pressure. Since for steam which remains dry-saturated the value of H decreases with falling pressure, wire-drawing thus tends to superheat dry steam or to dry wet steam, a familiar fact upon which the action of the "throttling calorimeter" is based.

4. DISSIPATIVE FLOW

The passage of heat by conduction or radiation from one point in a body to another involves a waste of availability of the heat thus uselessly let down from a higher to a lower temperature. The internal heating of a body by friction or viscosity involves a waste of the mechanical energy dissipated as work done against the passive resistances of friction or viscosity. In either case, the original state of the body, existing before the waste took place, can not be reestablished except by interference from without; and if we take into account all the bodies involved, the outside bodies used as well as the one concerned in the original process, the initial state of all of them can never be reestablished at all by any means whatever.

Such processes are known, in thermodynamics, as "irreversible" processes. No physical process is entirely free from such elements, and all real changes are therefore irreversible. But if these wasteful elements of the change are relatively so insignificant as to be of negligible importance, the change is sensibly though never exactly "reversible." Evidently any process from which we

desire to get as much work as possible should be freed as far as may be from all causes of waste, i. e., of irreversibility.

The wire-drawing of steam through the ports during admission to the cylinder of a reciprocating engine, especially at the cut-off point and at the beginning of admission with incomplete cushioning, is an irreversible and wasteful action to be avoided as far as may be. The expansion between complete cut-off and the opening of the exhaust valve is, *so far as the steam itself is concerned*, sensibly free from internal irreversible actions and therefore internally reversible, in the thermodynamic sense, though on account of the effect of the cylinder walls it is usually far from adiabatic.

The expansion of steam through a turbine nozzle or through blade channels of decreasing section would, in the ideal case, be free from dissipation, i. e., not retarded by frictional or viscous resistances, and would be in the thermodynamic sense reversible if so rapid that no sensible interchange of heat took place between different parts of the steam. In practice, there are always resistances due to skin friction and eddy currents; some mechanical energy that might otherwise be produced is thus dissipated into heat and the process is irreversible. Since these resistances are wasteful, it is evident that the smaller they are and the more nearly reversible the expansion is, the more closely the gain of kinetic energy $(T - T_0)$ approaches the theoretically possible maximum which is determined, if there is no leakage of heat, solely by the initial state of the steam and the final pressure to which expansion takes place in the space into which the steam jet issues.

During the expansion of steam between any two points in its path through a turbine, the whole mechanical energy produced, or $(T - T_0 + W)$, also approaches its ideal maximum value as all the internal losses due to skin friction and eddy currents in nozzles or blades, windage, and wire-drawing of steam which leaks past blade tips or through bushings, approach zero. The only difference between this case and that of flow through a nozzle is that in the nozzle or in fixed blades the work is necessarily exactly zero, while in the more general case W may have a finite value and $(T - T_0)$ is often negligibly small in comparison with W.

5. ISENTROPIC CHANGES OF STATE

If, during a reversible isothermal expansion at the absolute temperature θ, a body—e. g. a pound of steam—takes in from without a quantity of heat Q, the quantity $\frac{Q}{\theta}$ is known as the "increase of entropy" of the body. Thus, during the evaporation of one pound of water at constant pressure and temperature, ending in its conversion into a pound of dry-saturated steam, the increase of entropy is equal to the latent heat divided by the absolute temperature. This quantity is given in the steam tables as "entropy of evaporation."

If a reversible change of state occurs during which the temperature of the body is not constant, we may cut the process up into a number of small steps; for each of these divide the heat taken in by the average temperature of the body during that step; and finally add all these small quotients. If we then reduce the length and increase the number of the steps indefinitely, the sum approaches a definite limit expressible in the form $\int_A^B \frac{dQ}{\theta}$ where A and B are the initial and final states. The value of this expression is the same for all reversible changes which lead from the state A to the state B, and it is known as the increase of entropy of the body during the change $A\,B$. The elementary case of an isothermal change, considered above, is evidently included in this more general definition of change of entropy.

If, as is usual, we take the temperature of the ice point and the pressure of one atmosphere as our standard conditions, the entropy of a mass of fluid at any other temperature and pressure, referred to this standard state, is the value of $\int \frac{dQ}{\theta}$ from the standard to the actual state along a reversible path; and this value depends only on the end state reached and not on how the body actually reached it, if we assume, as is permissible, that the end state *could* have been reached by a reversible process.

In an adiabatic change of state, dQ is everywhere zero. Hence if an adiabatic process is also reversible, the entropy of the body

in question does not change. Any change of state in which the entropy of the body remains constant is known as an isentropic change.

The ideal adopted for the expansion of steam in the cylinder of a reciprocating engine is that of expansion without wire-drawing or other irreversible internal losses, in a perfectly nonconducting cylinder. Such an expansion would be a reversible and adiabatic, and therefore an isentropic change of state. From this it has come about that the term "adiabatic" is very often loosely used in engineering works with the meaning "isentropic." In reality, however, an adiabatic process need not be isentropic and an isentropic process need not be adiabatic. This becomes so evident in studying steam-turbine theory that there is ground for hope that this confusion of terms may eventually be eliminated from our thermodynamic literature.

If, for example, a pound of steam passes from a state A to a state B, the change of its entropy is definite and depends only on A and B. If the actual process is irreversible because of internal dissipation by which heat is produced within the steam, the increase of entropy is *not* to be found by taking the value of

$$\int_A^B \frac{dQ}{\theta}$$ along the actual irreversible path, but is greater than this.

For heat generated internally has the same effect in changing the temperature or otherwise influencing the state of the steam as an equal amount of heat added from without, so that the final state reached is not the same as if there had been the actual addition of heat from without but no internal generation of heat.

In any actual expansion between two given completely defined states, the final entropy of the expanding substance is always greater than can be accounted for by taking the value of $\int_A^B \frac{dQ}{\theta}$

with Q representing only heat added from without, for even in the best case there is always some internal dissipation. This excess represents heat produced by dissipation of mechanical energy which might, with ideally perfect arrangements, have

been saved and used. It is therefore a measure of the wastefulness of the process of expansion. Irreversibility, wasted availability of energy, dissipation, and needless increase of entropy, are merely different aspects of the same thing. One main reason why entropy, which has been somewhat of a stumbling block in technical thermodynamics, is retained, is that its changes give us the most convenient quantitative expression of the wastefulness or the efficiency of thermodynamic processes from which mechanical work is to be obtained.

The expansion of steam in the cylinder of the reciprocating engine is approximately reversible, so far as the steam itself is concerned; the internal losses due to dissipation inside the steam are small, and poor indicated efficiency is due largely to the fact that the expansion is not adiabatic, the influence of the cylinder walls causing the course of the expansion to be different from that desired. The expansion though nearly reversible is not adiabatic and not isentropic.

The expansion of steam in a turbine may be regarded for most purposes as adiabatic, the external losses of heat from the steam to the casing being small. But the process is subject to a great deal of dissipation by eddy currents, etc., so that though nearly adiabatic it is far from being either reversible or isentropic.

The ideal of the steam turbine is thus the same as that of the reciprocating engine, namely, isentropic expansion. In the one case the ideal is not attained because the changes of state of the steam, though nearly reversible, are far from adiabatic; in the other, because though nearly adiabatic, they are far from reversible.

The assumption that expansion through a steam turbine is adiabatic and fails of being isentropic and ideally efficient only because of internal dissipation losses of various kinds is, of course, only an approximation. For, except possibly in some very unusual cases, there is always some heat lost by conduction and radiation from the turbine, and there is always some longitudinal conduction between the different stages, tending to make separate parts of the expansion not quite adiabatic even though it might be so as a whole. External losses might be reduced or even made negative by jacketing; but while this would improve the efficiency of the turbine, considered by itself, the jacket steam used would

probably much more than offset the gain and cause a considerable increase in the total water rate. It is possible that with small turbines not well protected the external heat losses might be as great as the internal dissipation and so just balance the reheat. In such a case the total result would be the same as if the mechanical energy actually wasted inside the turbine had been used up on bearing friction outside, with no internal heating and no conduction and radiation loss. The expansion might thus be isentropic without being, even approximately, either adiabatic or reversible.

We shall, however, treat the turbine problem as one of adiabatic expansion, having therefore an isentropic expansion as its ideal, and shall treat the external heat losses as negligible and consider the flow as subject to equation (5). Such a simplification would not be permissible in precise physical work, but for the purposes of steam-turbine design, the errors thus introduced are probably always less than those due to the uncertainties in the values of some of the quantities that have to be assumed in the computations—notably, velocity losses in nozzles and blade channels and windage resistance, both of which influence the efficiency, of which an estimate must be made in order to design at all.

6. THE H φ OR MOLLIER DIAGRAM

The total energy equation for adiabatic flow

$$(T - T_o) + W = H_o - H \tag{5}$$

gives us information in terms of the total heat H. We have also seen that the ideal expansion is isentropic, since that corresponds to absence of dissipation and therefore to 100 per cent efficiency in the production of mechanical energy. It is. therefore evident that for the graphical solution of problems in steam flow—turbine design in particular—a chart showing the properties of steam on a plane with the total heat H and entropy φ as rectangular coordinates will be very convenient. On this plane, if H is made ordinate and φ abscissa, an isentropic expansion is represented by a straight line drawn vertically downward from the point representing the initial state; and the heat-drop during the expansion is represented by the length of this line from the initial to the final

position of the state point.　In any adiabatic expansion whatever, the difference of ordinate of the initial and final points gives us, at once, the heat drop and therefore, by equation (5), the mechanical energy developed.

We have now to mention some of the geometrical properties of the $H \varphi$ diagram for steam and may refer to Fig. 2, which gives a qualitative idea of the diagram with the isopiestics, or lines of constant pressure, as well as lines of constant dryness and super-
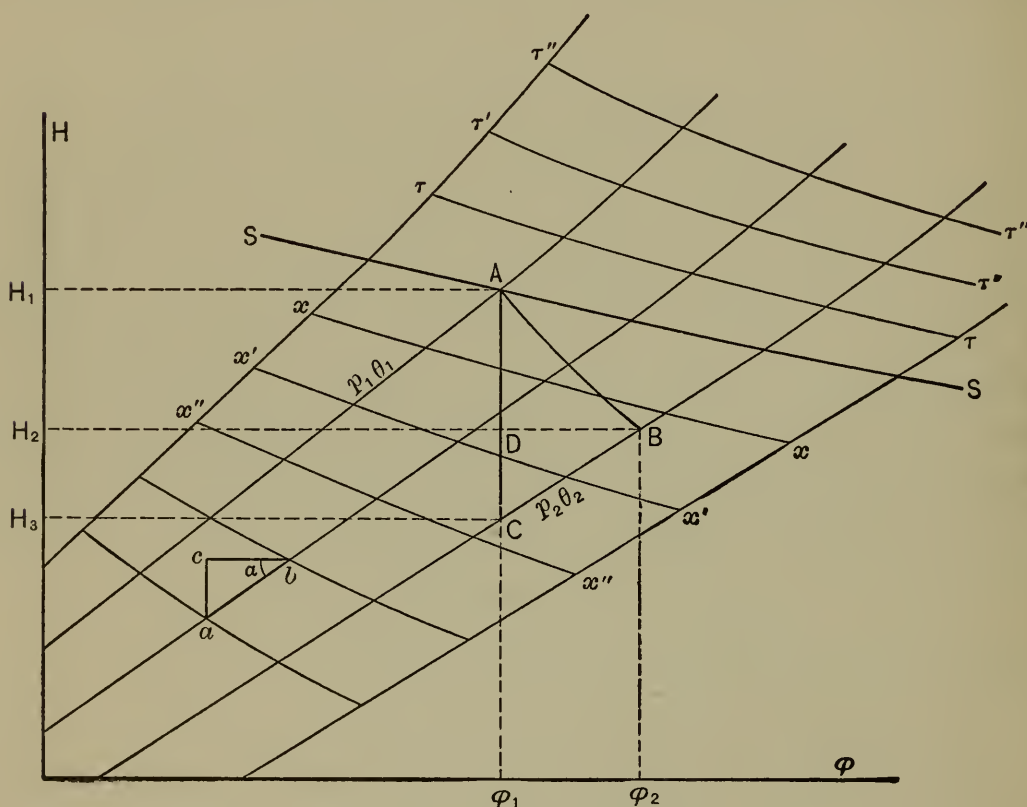
Fig. 2

heat.　An exact plot will not be attempted because it is easily available in the above-mentioned tables of Marks and Davis as well as in other books to which it may be presumed that the reader has access.

Vertical lines are isentropics, and horizontal lines are lines of constant total heat or "throttling lines." A vertical distance read off on the scale of ordinates is a difference of total heat or a heat-drop.　A horizontal distance represents a difference of entropy.

The steam saturation line SS is nearly straight within the range of pressures used in practice. It may easily be plotted by taking simultaneous values of H and φ for dry saturated steam from the steam table. The lines xx etc. are lines of constant dryness; points on one of these lines represent the possible simultaneous values of H and φ for wet steam of the given dryness factor x. They run in the same general direction as the saturation line. The lines $\tau\tau$ etc. running also in this same general direction but above instead of below the saturation line SS, are lines of constant superheat τ.

The lines we are most interested in are those shown sloping upward toward the right, straight for wet steam, i. e., within the saturation field below SS, but curving upward with rising superheat after crossing SS and entering the superheat field. These are the constant-pressure lines or isopiestics. Within the saturation field they are also isothermals, since the temperature of wet steam is fixed by its pressure; but above the saturation line they cease to be isothermals, because superheated steam at a given pressure may have any temperature higher than its saturation temperature.

Starting with one pound of wet steam in the condition of pressure and dryness represented by the point a, let us add to it at constant pressure and temperature a quantity of heat represented by the length ac. The state point moves to b, the entropy increasing by an amount represented on the entropy scale by cb. But since the temperature θ is constant we have, by the definition of change of entropy,

$$\text{length of } cb = \text{length of } ac \div \theta$$

these lengths being measured on the scales of H and φ used in drawing the chart. If these scales are the same, $\theta = ac \div cb = \tan \alpha$. If as is usual the scales are different, we have

$$\theta = k \tan \alpha \qquad\qquad (6)$$

where k is a constant depending on the scales and equal to unity when they are the same.

The slope of the isopiestics for wet steam is therefore proportional to the absolute temperature: the lines have greater slope as the pressure rises and have the familiar fan-shaped arrangement shown in Fig. 2. The reasoning just used is applicable even when

the isopiestics are curved, i. e., for superheated steam, if ac and bc are infinitesimal; so that equation (6) is general if α is understood to be the angle between the φ axis and the tangent to the isopiestic at the given point. There is no sudden change of direction at the saturation line, because there is no discontinuity in the temperature; but with increasing superheat the temperature and therefore, by (6), the slope of the isopiestics increases and they are concave upward, as shown.

Lines of constant dryness drawn for regularly changing values of x, e. g., $x=0.9$, $x=0.8$, $x=0.7$, etc., cut off equal segments on any given isopiestic, and the lengths of the segments on two different isopiestics are proportional to the values of $\frac{l}{\theta}\sqrt{1+\theta^2}$ where l is the latent heat, and $\frac{l}{\theta}$ is the quantity tabulated as "entropy of evaporation."
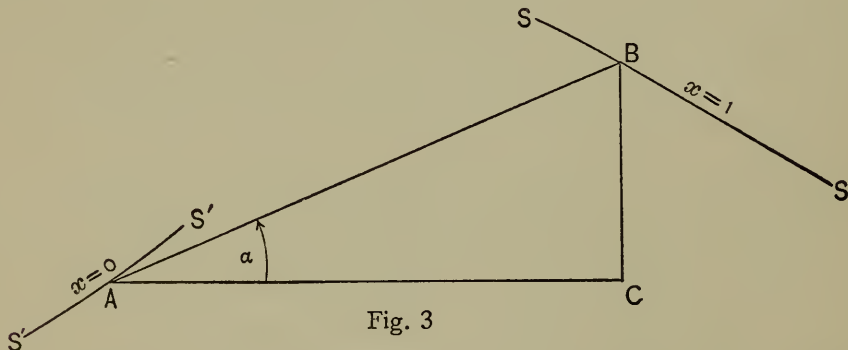
Fig. 3

To prove this we consider Fig. 3, in which SS represents the steam saturation line and $S'S'$ the water line, i. e., the constant dryness line for $x=0$. The whole length of any isopiestic AB between the two saturation lines is given by $\overline{AB}=\sqrt{\overline{AC^2}+\overline{CB^2}}$. But $\overline{CB}=l=$ the heat of evaporation at the given pressure and temperature $p\,\theta$; while if the scale of φ is the same as that of $H,$ $\overline{AC}=\frac{l}{\theta}=$ the entropy of evaporation. We therefore have

$$\overline{AB}=\sqrt{\frac{l^2}{\theta^2}+l^2}=\frac{l}{\theta}\sqrt{1+\theta^2}$$

If the scales are not the same there will be a proportionality factor different from unity.

Any line of constant superheat crosses all the isopiestics at points where their slopes are greater by a constant amount than their slopes at and below the saturation line.

7. REPRESENTATION OF EXPANSION ON THE H φ PLANE—EFFICIENCY

Let us start with one pound of steam at the pressure p_1. If the steam is dry-saturated, its initial state is represented by the point A (Fig. 2), otherwise by the intersection of the same iso-piestic with the appropriate dryness or superheat line. Let p_2 be the final pressure to which expansion takes place. Then C represents the final state reached in isentropic expansion, and $\overline{AC} = H_1 - H_3$ is the ideal maximum heat-drop available for conversion into mechanical energy during adiabatic expansion. The ideal yield may thus be read off at once from the scale at the side of the chart and the ideal water rate computed for a steam motor in which steam expands adiabatically from the initial state A to the final pressure p_2. It is necessary to say "the state A" and not simply "the pressure p_1"; for it is evident that on account of the non-parallelism of the isopiestics, this isentropic heat-drop is greater for dry or superheated than for wet steam. This is qualitatively in accordance with the observed fact that, with given limiting pressures, a steam engine or a turbine has a lower water rate when the initial superheat is raised.

In the actual expansion of steam through a turbine to the final pressure p_2, there is internal dissipation and the working of the machine is not ideally efficient. By reason of this dissipation, part of the mechanical energy which might have been obtained with a perfect machine is either not produced at all or if produced is again immediately dissipated. The whole of this lost mechanical energy appears as "reheat" in the steam, diminishes the actual heat-drop, and increases the entropy just as much as if it had been heat added to the steam from without. The final state of the steam at p_2 will therefore be represented by some point B, to the right of and higher than C. The greater the dissipation the farther B will be from the ideal final state C.

The ideal heat-drop being $H_1 - H_3 = \overline{AC}$, the actual heat-drop is $H_1 - H_2 = \overline{AD}$, and the reheat is $H_2 - H_3 = \overline{CD}$. The efficiency of the process in converting available heat into mechanical energy

is evidently equal to $\overline{AD}/\overline{AC}$. This is not necessarily the same as what would ordinarily be called the efficiency; for, by equation (5), $(H_1 - H_2)$ goes to the production of the kinetic energy $(T - T_0)$ as well as to the production of the outside work W, which is the important result of the process and commonly the only result considered in computing efficiency. But in general the quantity $(T - T_0)$ is negligible in comparison with the useful work W delivered by the steam to the rotor inside the glands. If this is true and if A and B (Fig. 2) represent the state of the steam in the steam chest and in the exhaust chamber, the efficiency of the machine in the usual sense is given by $\epsilon = \overline{AD}/\overline{AC}$. If ϵ were given a priori, D and therefore the final steam condition represented by B would thereby be fixed.

In order to discuss the efficiency of the separate parts of a multistage turbine we have first to define the term stage. In a multistage impulse turbine, of the Rateau type for instance, points at the entrances to the various nozzles and at exit from the last compartment to the exhaust are "similarly situated." In a turbine of the Parsons type, points in the clearance spaces at entrance to the fixed blades, together with a point at exit from the last moving row to the exhaust space, are "similarly situated." We shall define a stage as the part of the turbine between any two such *adjacent* similarly situated points. This agrees with the usual definition of a stage for the impulse turbine, but not with that sometimes adopted for turbines of the Parsons type in which each row of blades whether fixed or moving is regarded as a separate stage. Our definition makes a "stage" consist of a fixed row and the next following moving row. This use of the term seems more rational than that which divides the turbine into two kinds of stages—the moving rows and the fixed rows—in one of which no work at all is done by the steam on the rotor. At all events, it is convenient for the purposes of this paper and will be adopted.

If, then, we consider not a whole turbine but a single stage, we may say that the change of kinetic energy $(T - T_0)$ through any stage is negligible. In an impulse stage T and T_0 are usually separately negligible; in a Parsons stage the *change* of steam speed is so small as to make $(T - T_0)$ negligible except in the first stage, and since turbines of this type always have a great many stages, an error

of this sort regarding a single one is of no importance in regard to the turbine as a whole.   If the kinetic energy term is negligible in comparison with W for each separate stage, it is so for the whole turbine, and if ϵ is the efficiency in the usual sense we have (Fig. 2)

$$\epsilon = \frac{\overline{AD}}{\overline{AC}}$$

When ϵ refers to a single stage it is called the "stage efficiency," for, as we shall see in section 9, a distinctive term is needed.

8. FORM OF THE EXPANSION LINE—GRAPHICAL CONSTRUCTION

It is evident that the expansion of steam in passing through a multistage turbine is not a perfectly regular and continuous process
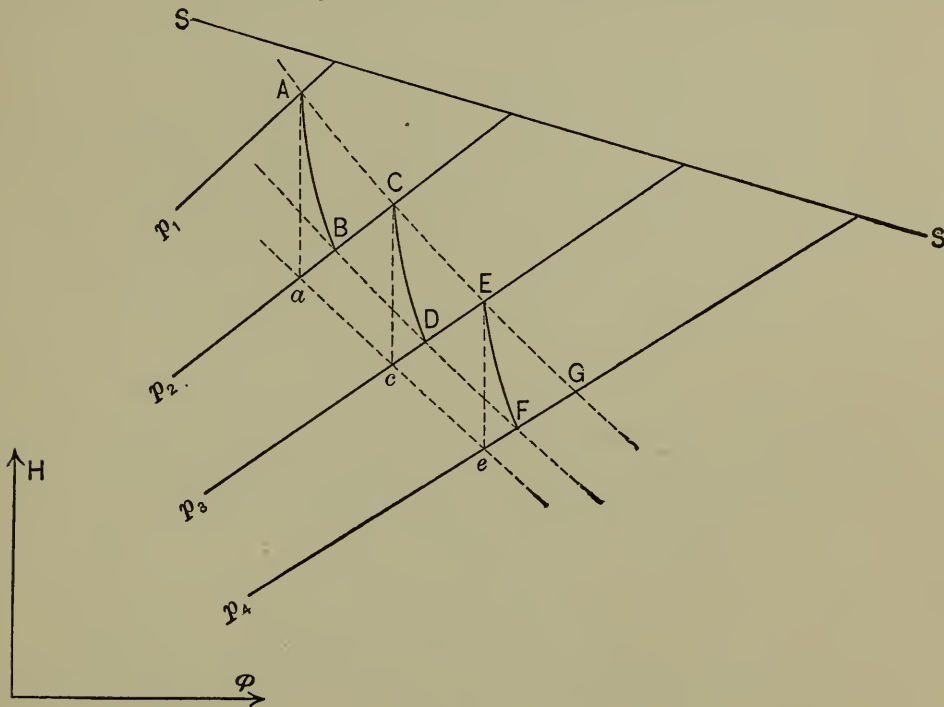
Fig. 4

and that the expansion line can therefore not in reality be a smooth curve like AB in Fig. 2.   Let us consider the simple case of a three-stage impulse turbine working with wet steam between the pressures p_1 and p_4, the intermediate pressures at entrance to the second and third stage nozzles being p_2 and p_3.

In the first-stage nozzles the pressure drops from p_1 to p_2 if the machine is properly designed.   There is some reheat due to dissipation in the nozzles before the pressure has fallen to p_2, but by far

the greater part of the reheat is due to blade losses, windage, leakage, and "carry-over" i. e., the kinetic energy of the steam leaving the last set of moving blades which is in general wasted and not available for driving the issuing steam directly into the next set of nozzles. All these except the nozzle loss occur at the constant pressure p_2. The true expansion through this stage will therefore be represented by a line something like ABC (Fig. 4) and for the second and third stages, if of approximately the same design as the first, the true expansion lines will be of somewhat similar shape, as shown at CDE and EFG. It would evidently be difficult to predict the precise form of the expansion line $ABCDEFG$ in all its small details, but fortunately this is not necessary; for in practice we do not need to know the steam condition exactly for every point but only for a few sets of similarly situated points.

Let us suppose, for example, that we want to know the steam condition at entrance to the three sets of nozzles and in the exhaust; this information is given by the positions of the points A, C, E, G. Now the position of C is fixed if we know the efficiency of the first stage: For if ϵ is this efficiency, we have

$$H_A - H_C \doteq \epsilon_1 (H_A - H_a)$$

in which H_a is the value of H that would be reached if the expansion were isentropic and the final state represented by the point a. Another similar step from C to the next pressure p_3 fixes the point E, and a third the point G.

The points C, E, G can thus be found if the stage efficiencies ϵ_1, ϵ_2, and ϵ_3 are known. Before designing can begin, they must be known, either by experiment on single stages similar to the ones in question, or by computation from a speed diagram; for the stage efficiency is a datum which is fundamental to the design and can not be dispensed with.

If there were more than three stages the same method might be continued, and for any moderate number of stages this would not be a laborious operation; but the practical problem is not so simple, for in general only the terminal pressures are given while the intermediate pressures are not given but are to be determined in accordance with some further condition. It may, for instance, be desired that the work done on the rotor shall be

the same in all the stages. No general solution of the problem can be given in such a case, and a graphical solution by trial and error might with a large number of stages involve a great waste of time which must be avoided if possible.

At this point the problem simplifies itself. Usually we are required to distribute between two limiting pressures a given number of *similar* stages which may be assumed to have the *same stage efficiency* if so designed that there shall be the same heat drop in each. The points A, C, E, G, etc., will then lie on a certain smooth curve of which the form is determined by the constant stage efficiency and the initial state. If this curve can be constructed, we may satisfy the requirement of equal heat drop by distributing the points A, C, E, etc., on this line so as to be at equal vertical distances apart. The points having been thus determined, pressure and quality may be read off from the chart. It will be shown how such a curve may be constructed more easily then by the step-by-step method, if the number of stages is large.

Up to this point we have considered only the similarly situated points A, C, E, G. But let us suppose that what is wanted is not the steam state at entrance to the nozzles but that at exit from them, after the reheat in the nozzles but before the further reheat at the constant lower pressure has occurred. This will evidently be given by the points B, D, F (Fig. 4). If the stages are all alike and the intermediate pressures have been so determined, by distributing A, C, E, G, etc., at equal vertical intervals, that the heat drop is the same in each stage, the steam speed and the reheat will be very nearly the same in each set of nozzles. The reheat after passing the nozzles will then be the same in all the stages and B will be as far below C, measured vertically, as D is below E, F below G, and so on. It is true, this amount can not be determined exactly, but it can be estimated with sufficient accuracy, and furthermore if there are many stages so that the total heat drop in each is itself small, the vertical distance between C and B will be still smaller and no very great error can be made in the position of B. If, therefore, the curve $ACEG$ has been drawn the points BDF, etc., may all be

found very easily if any one can be found.   The same is true of any other set of similarly situated points.

We have referred particularly to the impulse turbine, but in designing a turbine of the Parsons type a knowledge of the form of the curve through any set of similarly situated points is equally useful. In either case it is essential that we should estimate the steam quality at various points in the turbine, for otherwise the cross-sectional areas can not be properly proportioned to pass the required amount of steam at the velocities for which the blading and nozzles have been designed, the desired pressure distribution will not exist, and the whole working of the machine will depart from the intentions of the designer, presumably to its disadvantage.

Whatever the set of similar points chosen, it is evident that the true expansion line will approach a smooth curve drawn through these points more and more closely as the number of stages between the two limiting pressures increases and the heat-drop through each stage diminishes.   We shall speak of this limiting curve, for an infinite number of stages of equal stage efficiencies, as "the expansion line."

9. COMBINED EFFICIENCY OF SIMILAR STAGES IN SERIES—THE REHEAT FACTOR

Let us consider a single stage, of efficiency ϵ, working with either wet or superheated steam between two pressures which differ very little.
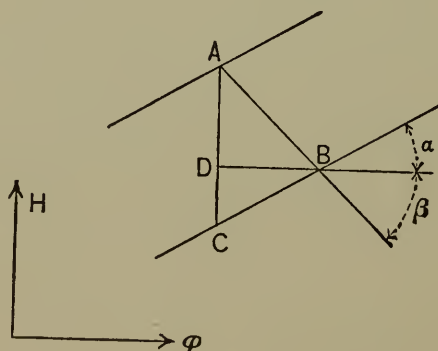

Fig. 5

Let A (Fig. 5) be the initial and B the final state of the steam. To construct the expansion line we may proceed as follows: Measure a short distance AC downward from the initial

point. Divide this at D so that $\overline{AD}/\overline{AC}=\epsilon$, the given stage efficiency; then AB will be an element of the expansion line. Go on in the same manner from B, and continue till the lower pressure limit has been reached. The smaller the steps the more approximate the result if the graphical work is exact.

From the geometry of the figure we have

$$tan\ \beta = \frac{\epsilon}{1-\epsilon} tan\ \alpha \tag{7}$$

or by equation (6)

$$k\ tan\ \beta = \frac{\epsilon}{1-\epsilon} \theta \tag{8}$$

As the pressure falls, the temperature θ also falls; hence β decreases and the expansion line is concave upward.

Let us next consider expansion of wet steam through a number of successive stages, of equal stage efficiency ϵ, between the
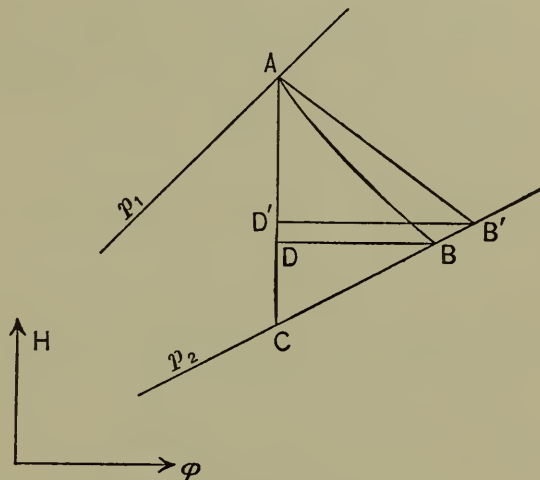
Fig. 6

pressures p_1 and p_2 which differ so that the pressure-ratio p_1/p_2 is a rather large number, 30 for example.

Let A (Fig. 6) be the initial state of the steam and C the state after isentropic expansion to p_2. Let AC be divided at D' so that $\overline{AD'}/\overline{AC}=\epsilon$. Then if the combined efficiency of the whole set of stages were the same as the stage efficiency, the point B' would represent the final state of the steam.

In reality, however, the expansion line starts from A with a sharper slope than the line AB' because the isopiestic p_1 has a

sharper slope than the isopiestic p_2. The expansion line then curves gradually, crossing the intermediate isopiestics at such angles that equation (8) shall be satisfied, and meets the final isopiestic p_2 at such an angle that its tangent is parallel to AB' and at a point B which is evidently somewhat below B'. The actual heat-drop AD is therefore greater than AD' and the combined efficiency, $\epsilon_c = \overline{AD}/\overline{AC}$, is greater than the stage efficiency ϵ in the ratio $\overline{AD}/\overline{AD'}$. This ratio is known as the "reheat-factor" and will be denoted by R. Its value, which in practice is seldom greater than 1.1, depends on the ratio of the initial and final temperatures and on the stage efficiency. It could be found in any particular case by step-by-step construction of the expansion line which would give us the position of B, while that of B' is given by $\overline{AD'}/\overline{AC} = \epsilon$. If, on the contrary, the value of R is known, the point B may be found without this graphical work.

We have now to show by a consideration of the form of the expansion line how the value of R may be determined *a priori*, for wet steam, i. e., for any part of the expansion line which lies within the saturation field.

10. FIRST APPROXIMATION FOR THE REHEAT FACTOR R

The total curvature of the expansion line AB (Fig. 6) is fixed by the stage efficiency and the difference in slope of the limiting isopiestics; but the distance BB' and the value of $(R-1)$ depend on how this curvature is distributed, hence on how the isopiestics are distributed between p_1 and p_2. If they change rapidly in direction in the vicinity of p_1, the expansion line AB will curve rapidly at first, B will be close to B' and $(R-1)$ will be small. In the opposite case $(R-1)$ will be large. We have therefore to consider the distribution of the isopiestics.

It will be found, upon examination of an accurate chart, that the isopiestics if produced do not meet in a point; but that their intersections are so far to the left and their whole divergence within the range of pressures used in practice is so small that only a very slight change would be needed to make them all meet in a single point. We shall first proceed upon the assumption

that they do thus meet in a single point, comparing the results later with those obtained from a more exact assumption.

Let O (Fig. 7) be the point of intersection of the isopiestics; let H and φ be measured from this point, and let them be plotted
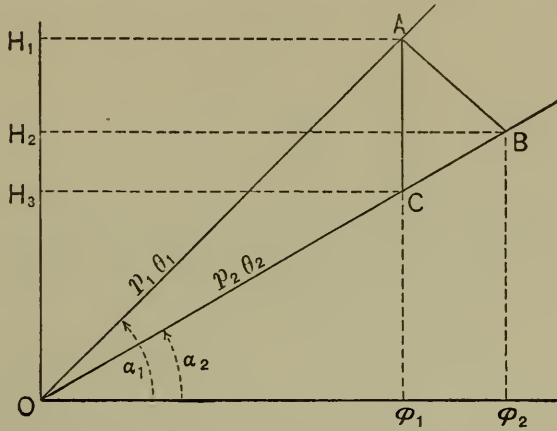
Fig. 7

on equal scales so that $k = 1$ in equation (8). We then have to substitute in equation (7)

$$\tan \beta = -\frac{dH}{d\varphi} \quad ; \quad \tan \alpha = \frac{H}{\varphi}$$

which gives us

$$\frac{dH}{d\varphi} + \frac{\epsilon}{1-\epsilon}\frac{H}{\varphi} = 0$$

or, after integrating between any two points on the expansion line,

$$H_2\varphi_2^{\frac{\epsilon}{1-\epsilon}} = H_1\varphi_1^{\frac{\epsilon}{1-\epsilon}} \tag{9}$$

Noting that by the geometry of the figure

$$\frac{\varphi_1}{\varphi_2} = \frac{H_3}{H_2} \quad \text{and} \quad \frac{H_3}{H_1} = \frac{\tan \alpha_2}{\tan \alpha_1} = \frac{\theta_2}{\theta_1}$$

equation (9) may be reduced to the form

$$\frac{H_1}{H_2} = \left(\frac{\theta_1}{\theta_2}\right)^{\epsilon}$$

and the heat-drop from A to B is therefore given by the equation

$$H_1 - H_2 = H_1\left[1 - \left(\frac{\theta_2}{\theta_1}\right)^{\epsilon}\right] \tag{10}$$

If the combined efficiency of the stages were the same as the stage efficiency, the heat-drop would be $\epsilon(H_1 - H_3) = \epsilon H_1\left(1 - \dfrac{\theta_2}{\theta_1}\right)$ Hence the value of the reheat factor is

$$R = \frac{1 - \left(\dfrac{\theta_2}{\theta_1}\right)^{\epsilon}}{\epsilon\left(1 - \dfrac{\theta_2}{\theta_1}\right)} \qquad\qquad (A)$$

From this equation we obtain, for example, the following values of $(R-1)$, which give an idea of the order of magnitude:

<div align="center">TABLE I</div>

p_1	p_2	$\epsilon = 0.2$	$\epsilon = 0.6.$
300	50	0.072	0.034
50	2	.096	.047
300	2	.168	.080
300	20	.100	.048
20	1	.083	.040
300	1	.186	.088

11. PRACTICAL WORKING METHOD FOR FINDING THE VALUE OF R

While equation (A) is not very complicated, it is inconvenient because it requires our working with absolute saturation temperatures instead of with pressures, and it would not in this form be of any practical value. We therefore proceed to develop a simpler method for obtaining values of R, which shall be sensibly the same as those given by equation (A).

Values of $(R-1)$ were computed by equation (A) for expansion from 277.4 pounds to eleven lower pressures, the lowest being 0.31 pound, with stage efficiencies $\epsilon = 0.2$, $0.3 \ldots 0.7$. When these values of $(R-1)$ were plotted against $\log \theta_2$, they gave sensibly a straight line for each value of ϵ. It was found that all the values could be represented very well by the equation

$$R - 1 = 1.207\,(0.975 - \epsilon)\,(\log_{10}\theta_1 - \log_{10}\theta_2) \qquad\qquad (B)$$

If this equation represented equation (A) exactly, we should evidently have the relation

$$(R_{12} - 1) + (R_{23} - 1) = (R_{13} - 1) \qquad\qquad (C)$$

where R_{12}, R_{23}, and R_{13} are the values of the reheat factor for expansion between θ_1 and θ_2, θ_2 and θ_3, and θ_1 and θ_3, respectively. It will be seen by reference to Table I that this relation is in fact nearly satisfied in the four cases there given.

Taking equation (C) as sufficiently exact, we could then represent equation (B) by a straight line for any given value of ϵ and find the value of $(R-1)$ for expansion with this stage efficiency between any two absolute saturation temperatures, by taking the difference of the ordinates at these two temperatures. If, finally, we plot $(R-1)$ against p as abscissa instead of against the inconvenient $log\ \theta$, we shall have reduced our method for finding R to practical shape. A single curve is applicable only to a single value of ϵ, but we may either work to other values of ϵ by means of equation (B), or plot a number of curves for different values of ϵ and interpolate when making the readings of $(R-1)$.

In Table II are values of R computed by equation (B) for expansion from 350 pounds to 18 lower pressures, with the stage efficiencies $\epsilon=0.1$, 0.2, .. etc. .. 0.8

TABLE II

p_2	$\epsilon=0.1$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
300	1.0075	1.0066	1.0058	1.0049	1.0041	1.0032	1.0023	1.0015
250	1.0161	1.0143	1.0124	1.0106	1.0088	1.0069	1.0051	1.0032
200	1.0265	1.0235	1.0205	1.0174	1.0144	1.0114	1.0083	1.0053
150	1.0394	1.0349	1.0304	1.0259	1.0214	1.0169	1.0124	1.0079
100	1.0569	1.0504	1.0439	1.0374	1.0309	1.0244	1.0179	1.0114
70	1.0717	1.0635	1.0553	1.0471	1.0389	1.0307	1.0225	1.0143
40	1.0936	1.0829	1.0722	1.0615	1.0508	1.0401	1.0294	1.0187
30	1.1047	1.0927	1.0808	1.0688	1.0568	1.0449	1.0329	1.0209
20	1.1191	1.1055	1.0919	1.0783	1.0646	1.0510	1.0374	1.0238
15	1.1292	1.1144	1.0997	1.0849	1.0702	1.0554	1.0406	1.0258
10	1.1429	1.1265	1.1102	1.0939	1.0775	1.0612	1.0449	1.0285
7	1.1545	1.1369	1.1192	1.1016	1.0839	1.0663	1.0486	1.0309
5	1.1651	1.1462	1.1274	1.1085	1.0896	1.0708	1.0519	1.0330
3	1.1808	1.1601	1.1395	1.1188	1.0981	1.0775	1.0568	1.0361
2	1.1926	1.1706	1.1486	1.1266	1.1045	1.0825	1.0605	1.0385
1.5	1.2102	1.1782	1.1552	1.1322	1.1092	1.0862	1.0632	1.0402
1.0	1.2120	1.1878	1.1636	1.1394	1.1151	1.0909	1.0667	1.0424
0.5	1.2309	1.2045	1.1781	1.1417	1.1253	1.0989	1.0725	1.0460

From these figures any one who desires can plot the set of curves for himself. The curve shown on Plate I was plotted from the values for $\epsilon = 0.1$; for larger values of ϵ the curves would differ from this only in a linear reduction of the ordinates. Having this curve for $\epsilon = 0.1$ we may then find the value of $(R-1)$ by the following practical

RULE: *Read from the curve the values of $(R-1)$ at the two pressures between which the expansion takes place; their difference is the desired value of $(R-1)$ for $\epsilon = 0.1$. For any other value of ϵ, multiply by the factor* $\dfrac{0.975 - \epsilon}{0.875}$.

Many numerical tests have shown that values of R found by this rule agree with those found from equation (A) much more closely than any stage efficiency is ever known a priori, the discrepancy being seldom over 0.1 per cent.

The rule may therefore be regarded as a satisfactory substitute for the use of equation (A); but it remains to be shown that equation (A) itself, obtained from an admittedly only approximate assumption regarding the distribution of the isopiestics, gives sufficiently correct values of the reheat factor. This question has now to be taken up.

12. DISTRIBUTION OF THE ISOPIESTICS ALONG AN ISENTROPIC

Let φ_0 be the value of the entropy at some fixed isentropic line. Let Z be used to denote the value of H at a point on this line. In section 10 it was assumed that the isopiestics all met in a single point. If H and φ were both measured from that point we should then have $Z/\varphi_0 = \theta$. If H and φ were measured from some other. point, we should have in general

$$\theta = a + bZ \qquad (11)$$

where θ is the absolute saturation temperature of any isopiestic and Z is the value of H at the point where this isopiestic crosses the isentropic $\varphi = \varphi_0$. Equation (11) therefore represents the assumption from which equation (A) was deduced. Readings on the chart show that it is by no means exactly fulfilled. By readings of Z from the $H\varphi$ chart of Marks and Davis at $\varphi_0 = 1.384,\ 1.544,$

and 1.704, it was found that in each case the values could be represented by an expression of the form

$$\log \theta = A + BZ \qquad (12)$$

nearly though possibly not quite as closely as the readings could be made. The values of A and B varied with the value of φ_0. A number of readings were made between the isopiestic for $p = 1$ pound and the saturation line. This empirical equation for the change of H during isentropic expansion is one which I do not remember to have seen given and which might prove useful in other work than the present.

Since readings from the chart can not be made with any great accuracy, recourse was now had to interpolation in the tables. Values of H (i. e., of Z) were computed at $\varphi_0 = 1.384$ for 21 pressures from 0.5 pound to 513 pounds. Values of Z for the same temperatures were then computed by the equation

$$Z = 1645 \, log_{10}\theta - 3750.2 \qquad (13)$$

The greatest difference between Z obtained from the table and Z calculated by equation (13) was 0.7 B. t. u. and the average difference was only 0.3 B. t. u. The differences showed a systematic run, but it was wave-like and not progressive, the value passing through zero in the vicinity of $550°$, $700°$, and $900°$ absolute F.

Equation (13) is therefore a quite exact empirical representation of the facts, as given in the tables, over the whole range of pressures used in present steam practice. At any other isentropic where the entropy has the value φ, equation (13) leads evidently to the equation

$$Z = 1645 \, log_{10} \theta - 3750.2 + \theta(\varphi - 1.384) \qquad (14)$$

which must diverge from the tabulated values by precisely the same amounts as equation (13). For security this was checked through by interpolation in the tables for $\varphi = 1.684$, from the isopiestic $p = 0.505$ lb/in² up to the saturation line.

Since equations (13) and (14) are quite exact, it is evident that equation (12), which is equivalent to (13), must be less exact when φ_0 differs much from the value 1.384, though it is an approximation nearly or quite sufficiently close for use with the chart alone.

13. SECOND APPROXIMATION FOR R

Since equation (14) or its special case (13) is a much closer representation of the facts then equation (11) from which (A) was deduced, it follows that an expression deduced from (13) or (14) will give more nearly correct values of the reheat factor than are given by equation (A). We now proceed to deduce such an expression.
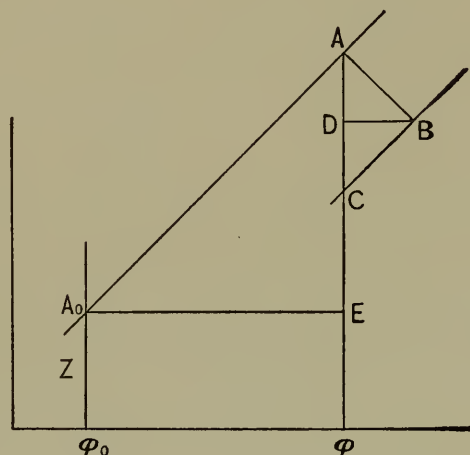

Fig. 8

Let AB (Fig. 8) be an infinitesimal element of the expansion line. Then, if ϵ is the stage efficiency, we have

$$\frac{\overline{CD}}{\overline{DA}} = \frac{1-\epsilon}{\epsilon} \qquad (15)$$

Now we have

$$\frac{\overline{CD}}{\overline{DB}} = \left(\frac{\partial H}{\varphi}\right)_p = \theta$$

Also

$$\frac{\overline{DA}}{\overline{DB}} = \frac{-dH}{d\varphi}$$

where the symbol d refers to change along the expansion line. Substituting these values in (15) gives us

$$\theta \frac{d\varphi}{dH} = -\frac{1-\epsilon}{\epsilon} \qquad (16)$$

as an equation which must be satisfied at all points of the expansion line and in all cases, regardless of the distribution of the isopiestics.[3]

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[3] The validity of this general equation is not limited to the case of wet steam.

Let A_0 be the intersection of the isopiestic through A with the isentropic $\varphi = \varphi_0$, and let Z be the ordinate of this point. Then we have

$$\frac{\overline{EA}}{\overline{A_0E}} = \frac{\overline{CD}}{\overline{DB}} = \frac{H-Z}{\varphi-\varphi_0} = \theta$$

Whence

$$\varphi - \varphi_0 = \frac{H-Z}{\theta}$$

Differentiating by H and multiplying by θ gives us

$$\theta\frac{d\phi}{dH} = 1 - \frac{dZ}{dH} - (H-Z)\frac{1}{\theta}\frac{d\theta}{dH} \tag{17}$$

another general relation, which must hold for all points within the saturation field and for motion of the state point in any direction whatever.

By comparison of equations (16) and (17) we get

$$(H-Z)\frac{d\log\theta}{dH} + \frac{dZ}{dH} - \frac{1}{\epsilon} = 0 \tag{18}$$

If we now let $\phi_0 = 1.384$, we may, as shown in section 12, set

$$\log\theta = A + BZ \tag{19}$$

and equation (18) may be reduced to the form

$$\frac{dH}{dZ} = \epsilon\left[1 + B(H-Z)\right] \tag{20}$$

a differential equation for the expansion line in terms of H and Z.

This equation is satisfied by setting

$$H = Me^{\epsilon BZ} + Z + \frac{1-\epsilon}{\epsilon B} \tag{21}$$

in which M is the arbitrary constant, to be adjusted so that the curve shall pass through the given initial position of the state point.

From equation (19) we get $BZ = \log\theta - A$, whence

$$e^{\epsilon BZ} = \frac{\theta^\epsilon}{e^{\epsilon A}}$$

and equation (21) may be written

$$H = \frac{M\theta^\epsilon}{e^{\epsilon A}} + Z + \frac{1-\epsilon}{\epsilon B} \tag{22}$$

Let θ_1 H_1 Z_1 refer to the initial state, the starting point of the expansion line, and let

$$\frac{M\theta_1^\epsilon}{e^{\epsilon A}} = H_1 - Z_1 - \frac{1-\epsilon}{\epsilon B} = L \tag{23}$$

Then equation (22) takes the form

$$H = L\left(\frac{\theta}{\theta_1}\right)^\epsilon + Z + \frac{1-\epsilon}{\epsilon B} \tag{24}$$

and the heat-drop along the expansion line between two points at the absolute temperatures θ_1 and θ_2 is given by

$$H_1 - H_2 = Z_1 - Z_2 + L\left[1 - \left(\frac{\theta_2}{\theta_1}\right)^\epsilon\right] \tag{25}$$

For isentropic expansion $\epsilon = 1$. Hence by setting $\epsilon = 1$ in equations (22), (23), and (24) we have, along the isentropic through the starting point of the expansion line,

$$H = N\frac{\theta}{\theta_1} + Z \tag{26}$$

where

$$N = H_1 - Z_1 \tag{27}$$

and the isentropic heat-drop from the initial point at $\theta = \theta_1$ to the final pressure where $\theta = \theta_2$ is given by

$$H_1 - H_3 = Z_1 - Z_2 + N\left(1 - \frac{\theta_2}{\theta_1}\right) \tag{28}$$

The reheat factor therefore has the value

$$R = \frac{Z_1 - Z_2 + L\left[1 - \left(\frac{\theta_2}{\theta_1}\right)^\epsilon\right]}{\epsilon\left[Z_1 - Z_2 + N\left(1 - \frac{\theta_2}{\theta_1}\right)\right]} \tag{D}$$

in which

$$N = H_1 - Z_1 \tag{D$_1$}$$

$$L = N - \frac{1-\epsilon}{\epsilon B} \tag{D$_2$}$$

Beside necessitating our working with absolute temperatures, the determination of R by equation (D) requires our finding the values of Z_1 and Z_2 either graphically or by computation from equation (13). As a means for the practical computation of R for use in designing, equation (D) is evidently so inconvenient as to be entirely worthless; but it has a value as confirmatory of the

results obtained by the practical rule given in section 11; for it has been shown that that rule gives sensibly the same results, within the range of present steam-turbine practice, as equation (A), and furthermore that equation (D), based on a much more accurate assumption than that which underlies the deduction of (A), must give more accurate values than (A) and therefore than the rule. If, therefore, equation (D) gives sensibly the *same* values as the rule, the values obtained by the rule may be relied upon as sensibly correct.

14. TESTS OF THE RULE FOR FINDING THE VALUE OF R

The tests consisted in computing R by equation (D) and comparing the results with values obtained by the rule given in in section 11.

The values of Z read from the $H \varphi$ chart at $\varphi = 1.384$ were plotted against $log_{10} \theta$ and a straight line drawn through the points; the scales were 100 B. t. u. $= 75$ mm and, for $log_{10} \theta$, 1.5 mm $= 0.01$. The values of Z_1 and Z_2 needed in the computations were read from this plot. The slope of this line when reduced to terms of naperian logarithms for use in equation (D_2) gave $1/B = 723$. The readings of H_1 for use in equation (D_1) were made from the $H \varphi$ chart. More exact results might have been had by obtaining all these values by computation instead of graphically, but the precision of the graphical method is sufficient for our purpose. It will not do, however, to compute H_1 from the table, but take Z_1 and Z_2 from a plot or from an equation derived from readings on the chart, for considerable errors are thereby introduced. The chart, while self-consistent and quite accurate enough for all ordinary purposes, is not an exact representation of the tables, owing doubtless to difficulties in printing. This, however, need not occasion any lack of confidence in the chart, for the test of its accuracy is a severe one, as will be found upon making a few computations by means of equations (D), (D_1), and (D_2).

A number of test computations covering the part of the saturation field, which is of practical importance, were made by the foregoing method. Values of R for the same cases were computed by the rule of section 11 and in many instances also by equation (A). On the whole the values obtained by the rule agreed with

those obtained by the use of equation (D) somewhat more closely than did values from equation (A). A summary of the results of these tests is given in the following table:

TABLE III

Value of $\epsilon \times$ [R by equ. (22) $- R$ by rule] expressed as percentage of $(H_1 - H_3)$

$\dfrac{p_1}{p_2}$	p_1 lbs/in² abs	x_1	$\epsilon=0.1$	$\epsilon=0.2$	$\epsilon=0.4$	$\epsilon=0.6$	$\epsilon=0.7$
12.7	277.4	0.981		+0.03	−0.02	−0.02	
45.5	277.4	.981		+ .04	− .09	− .10	
185.0	277.4	.981		− .03	− .20	− .22	
44.0	66.2	1.00		− .06	− .09	− .10	
33.7	21.9	.99		+ .04	− .04	+ .05	
33.7	21.9	.88		+ .04	− .06	+ .07	
33.7	21.9	.77		+ .06	+ .07	− .08	
20.0	20.0	.99	+0.04		− .01		+0.03
20.0	20.0	.84	+ .04		+ .04		− .02
20.0	20.0	.79	+ .04		− .02		− .01
20.0	120.0	.99	+ .07	+ .05	+ .03		+ .04
20.0	120.0	.84	+ .08	+ .04	+ .03		+ .04
300.0	300.0	1.00		− .01	− .19	− .25	
110.0	110.0	1.00		− .06	− .13	− .10	
35.0	35.0	1.00		+ .07	+ .01	− .01	
8.0	8.0	1.00		+ .03	+ .01	+ .02	

In the first three columns of the table are given the pressure ratio, the initial pressure, and the initial dryness factor; and it will be seen that the tests are sufficiently varied.

The object in using the reheat factor is to determine the position of a point B with an error in vertical position which shall be negligible in comparison with the isentropic heat drop $(H_1 - H_3)$. This difference of position, in percentage of $(H_1 - H_3)$, is found by multiplying the difference of the two values of R—found by the two methods—by the value of ϵ, since R is always nearly unity. Accordingly, the values in the table may be regarded as the changes of height of the point B, in per cent of $(H_1 - H_3)$, caused by

changing the method of computing R. The positive sign means that the rule gives B a higher position than it should have according to equation (D).

On account of errors in reading H, Z, and $R-1$ from the curves, differences of less than 0.1 have little or no significance unless p_1/p_2 is very large. It will be seen that the discrepancy never exceeds one-fourth of 1 per cent and attains the value 0.1 per cent only when the pressure ratio is large.

Since one-fourth of 1 per cent is, for the designer, a negligible quantity, we may conclude that since one of the methods certainly gives a much more accurate value than the other, the error involved in either method is negligible and the value of R is accurate enough when found in either way. It follows that the values of R obtained by the working rule given in section 11 may be relied upon *for wet steam* as being sufficiently accurate for use in designing.

15. USES OF THE REHEAT FACTOR

Having thus a practical method of finding the reheat factor, we are able to plot the expansion line. If p_1 and p_2 are the terminal pressures, the position of the final point may be found by the equation

$$H_1 - H_2 = R\epsilon \; (H_1 - H_3) \tag{29}$$

We next take, instead of p_2, the pressure corresponding to some isopiestic about halfway (on the diagram, not numerically) between the two terminal isopiestics. Using the rule to find R for expansion from the initial to this intermediate pressure, we compute the position of a point on the expansion line where it intersects this intermediate isopiestic. Having now three points of the expansion line, we may sketch it in as a circular arc. It will then be evident whether or not it is worth while to go on to determine still more points. Unless the expansion ratio p_1/p_2 is large, even a single intermediate point will usually be needless and, after the final point B has been found, a straight line AB will be close enough to the true expansion line, even in the middle. This, however, is a matter for the discretion of the individual designer, who knows what reliance he may place on the value of the stage efficiency on which his work is based.

In designing a turbine for new conditions, it may be worth while to make a number of preliminary sketch designs which need not be worked out in detail further than to compute the probable influence on the water rate of variations in the arrangement of the stages. In such work, what is needed is not the form of the expansion line but merely the actual heat drop $(H_1 - H_2)$, which may be expected in a given set of similar stages. If the expansion ratio is large, neglecting the reheat factor may introduce an error of 5 per cent or more, which is undesirable and may be avoided by a single computation of the reheat factor for use in equation (29).

16. GENERAL REMARKS

The foregoing methods are valid only when the isopiestics are straight lines; they therefore fail for superheated steam. The curvature of the isopiestics in the superheat field depends on the specific heat of superheated steam at constant pressure. Even if this were exactly known and expressible in simple mathematical form, the development of a general differential equation for the expansion line would be a difficult matter and the task of integrating it would very possibly present insuperable mathematical difficulties. But without attempting this general solution a few pertinent remarks may be made.

(a) Consideration of the $H\varphi$ chart shows that an isentropic expansion between two given isopiestics involves a greater change in temperature for superheated than for wet steam, as may also be seen from the pv chart. A dissipative expansion line starting at a point in the superheat field will therefore fall more sharply at first than if it started at the same pressure within the saturation field and the reheat factor for a given pair of terminal pressures and given stage efficiency will be greater than for wet steam between the same pressures. Hence, if we use the value of R computed for these pressures for wet steam, we shall underestimate the actual heat-drop and combined efficiency and be on the safe side as regards the economy expected.

(b) Up to 100° F superheat the curvature of the isopiestics is so small that the error can at most not be important if we simply

use the value of R for wet steam between the same pressure limits.

(c) The value of the quantity $(R-1)$ decreases with the pressure ratio p_1/p_2 and decreases as the stage efficiency ϵ increases. Examination of the $H\varphi$ chart in connection with Table II shows that, unless the efficiency is very poor, with superheats up to 150° or 200° F, which are unusual as yet, the expansion ratio, and therefore $(R-1)$, can not be large for expansion entirely within the superheat field, so that $(R-1)$ can not be subject to a large error. With poor efficiency the reheat might be enough to keep the steam dry a long way down in the turbine, but such a case is not of great commercial interest and does not demand close designing.

(d) A high initial superheat has not as yet been much used in the Parsons type of turbine, so that the superheat field is of interest mainly to the designer of turbines in which the first few stages, at least, are of the impulse type, not requiring fine radial clearances. In such a turbine, the first one or two stages will usually have rather large pressure ratios and will get rid of the superheat, so that the remaining stages are working with wet steam. It is not a serious matter to treat one or two stages separately, by determining the state-points graphically, and it is usually necessary to do so for at least one stage. For the first stage is usually not quite like the later stages and has not the same stage efficiency, so that an expansion line drawn for constant stage efficiency could not fit the facts in any event.

(e) The whole idea of the reheat factor and the need of drawing the expansion line arises from the fact that the combined efficiency of a number of similar stages in series is greater than the efficiency of each stage separately. The method developed for finding the reheat factor was based on the assumption that the heat-drop in each stage was small, for it consisted in developing and integrating a *differential* equation for the expansion line. For a few stages with very high steam velocities, the results would not be exact, but the whole matter is also, in that case, of no importance; for the graphical construction for a few stages is a simple operation and involves no great waste of time.

(*f*) The notion of a reheat factor, as the term has been used in this paper, is not applicable to a single stage. Let p_1 and p_2 be the limiting pressures of a given single stage, and A (Fig. 9) the initial state.
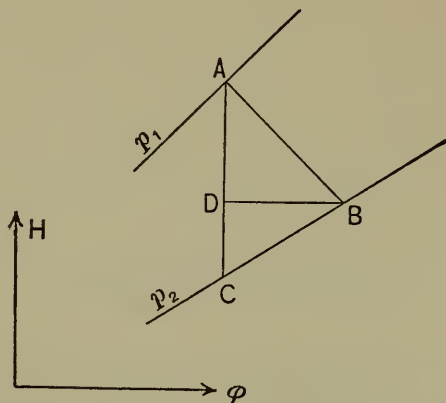

Fig. 9

Then if B is the final state

$$\frac{\overline{AD}}{\overline{AC}} = \frac{H_1 - H_2}{H_1 - H_3}$$

is the ratio of the mechanical energy produced to the maximum possible in adiabatic expansion. If, as may be assumed in an impulse stage, the difference of kinetic energy between similarly situated points is negligible, $\overline{AD}/\overline{AC} = \epsilon$ is the efficiency of the stage. This is true regardless of whether the isopiestics are straight or curved, i. e., whether the steam is wet or superheated, so long as B is at p_2, AC is vertical and DB is horizontal. If the net efficiency of the stage is given beforehand, B is thus fixed by the initial state A and the final pressure p_2 and there is no question of a reheat factor.

But the efficiency ϵ may have been obtained in either of two ways. If it was obtained by experiments on a stage similar to the one in question and working under similar conditions, there is nothing more to be said. But if it was computed from a velocity diagram drawn with proper allowances for velocity losses in nozzles and blades, with subsequent correction for windage and leakage, the case may be a trifle different in theory though hardly in practice.

Let A and therefore C be given, and let D be so placed as to make $\overline{AD}/\overline{AC} = \epsilon$, where ϵ is the computed efficiency. It now becomes a question of when the various elements of the total dissipation occur and of the temperature of the steam when the various elements of the reheat are added to it. In an impulse stage, in general, only a small fraction of the dissipation occurs in the nozzles, hence the reheat is almost all added to the steam after the lowest pressure and temperature have been reached. The actual expansion line is therefore nearly coincident with the broken line ACB, which would be the expansion line if *all* the dissipation and reheating took place at p_2. Hence, if ϵ were computed by some method which treated the reheat as *all* added at the low pressure, while in reality a little of it is added at a higher pressure and temperature in the nozzles, the efficiency thus computed would be a trifle too small and should be multiplied by a factor analogous to the reheat factor which, in effect, is introduced to allow for the fact that in a series of stages the reheat is distributed instead of being concentrated at the end of the last stage. The improvement in accuracy due to the use of such a factor would, however, be altogether illusory until our experimental knowledge is sufficient to permit of a far more accurate *a priori* computation of the stage efficiency than is at present possible.

To apply a reheat factor computed for the case of continuous expansion through an infinite number of stages, to expansion between the same pressure limits through a single stage, is altogether erroneous.

(*g*) In conclusion we may say that, so far as present practice is concerned, the fact the method given in this paper for finding the value of the reheat factor is applicable only to wet steam, is not an important restriction on its usefulness. In the designing of turbines with many similar stages, its use may save a great deal of time that would otherwise have to be spent over the drawing board.

NOTE (TO SEC. 1).—LIMITATIONS OF THE THEORY

In the total energy equation, the terms T_0 and T are taken as including only kinetic energy of the axial component of the velocity of the fluid. This is done because the only kinetic energy

which can be utilized mechanically is that due to the general motion of the fluid as a whole. Kinetic energy which is not utilizable, such as that of eddy currents, is for our purposes not to be counted as "mechanical" energy. It must ultimately be dissipated into heat; and motions which are already so unordered as to be incapable of mechanical utilization may be considered as already, for our purposes, completely dissipated. It is of no importance to the practical theory of steam flow whether they have already become completely unordered in the molecular sense or not.

A question then arises as to what is meant by the "state" of a mass of fluid when it is the seat of eddy currents which are still on a relatively large scale though already beyond our power to utilize their kinetic energy directly. In going on to complete dissipation, they "produce heat," which is merely our convenient way of saying that the dissipation tends to raise the temperature of the fluid. We are thus led to ask precisely what is meant by the temperature of the fluid at a given point in its course, and a very little consideration at once makes it clear that the "state" of a mass of fluid can not be precisely defined at all, unless it is a state of quiescence. Turbulent states are not capable of precise description, and the terms pressure and temperature have no *precise* meaning when referring to a fluid in a condition of turbulent motion nor even when referring to a volume element of a fluid moving as a whole with a rapid acceleration.

But though the *precise* treatment of the vastly complicated process of steam flow through a turbine is quite beyond our powers, the only important question is whether our theory is nearly enough correct for practical purposes—whether it represents the facts sufficiently well to be useful in predicting what will happen practically in a future case. This question may without hesitation be answered affirmatively. We neglect certain recognized errors in the theory and while we can not say, a priori, just how large these errors are, we find a posteriori that they are negligible if we use the theory with good judgment and do not throw common sense to the winds. If one attempted to measure the temperature of the steam issuing from a de Laval nozzle by putting the bulb of a mercurial thermometer directly in the jet, and then—assuming

that the thermometer could endure such treatment—compared the readings with the theory, one might find very large discrepancies.   But no sensible person would expect anything else.

Every physical theory is built upon a simplified ideal picture of the main outlines of the known facts.   The simplifications, when recognized, constitute the assumptions or hypotheses stated as a basis for the theory.   The degree of simplification permissible depends on the accuracy expected in using the theory for prediction and is limited by the accuracy attainable in the experimental verification of the results of prediction.   In these respects the theory of the steam turbine does not differ at all from other physical theories; only it happens that the accuracy required of the theory is low on account of experimental difficulties, and that the things which the theory neglects in making its ideal picture of the facts are so obvious that anyone can see them.

WASHINGTON, March 2, 1911.

REHEAT FACTOR

PRESSURE LBS./IN² ABSOLUTE

SCALE ABOVE

SCALE BELOW

Plate I.