THE RESOLVING POWER OF OBJECTIVES.

By P. G. Nutting.

The mathematical theory of the resolving power of lenses, developed long ago, gives a relation between the radius of the lens r, the wave length λ of the light used in illumination, and the least angular separation φ between the objects to be clearly separated in the image.

$$\varphi = a\frac{\lambda}{r}$$

The value of the constant a depends upon the quality of the definition required in the image.

This constant a has never been determined experimentally, although it is in its very nature an experimental constant. Theory tells us that when the image of one of the objects to be resolved lies in the first dark ring surrounding the other, $a = 0.61$; but it does not tell us how good a definition this means in practice nor to what quality of image other values of the constant correspond.

The work here described was undertaken to determine the constant a experimentally and to establish a relation between different specified qualities of definition in an image and different values of a. Incidentally we wished to find out how near to this theoretical resolving power different objectives could be operated and to study the differences between lack of resolution and residual coma and spherical aberration.

The prime requisite for such work is, of course, an object of homogeneous texture in one dimension, illuminated by monochromatic light. Such an object was found in one half of an uncemented half-tone screen. This consists of parallel rulings on glass, the rulings being etched and filled with opaque material and accurately equal in width to the spaces between them. The rulings run 50 to 200 to the inch, the finest being suitable for small objectives and laboratory distances.

In this work a screen whose spacing was 0.127 mm was placed over the second slit of a monochromatic illuminator lighted by a Nernst lamp. The image was viewed both with high-power objectives and low-power microscopes, care being taken to use only those of greater aperture ratio than the objective being tested; otherwise the effects observed might have been due to lack of resolving power in the observer's eye or in the ocular used.

This arrangement proved to be very sensitive. When near the limit of resolution, varying the distance from objective to screen by as little as 1 per cent, or shifting the wave length of the illuminating light by 15 $\mu\mu$, produced a very perceptible variation in the sharpness of the image. An auxiliary iris stop at the objective proved very convenient in testing for sensibility.

As the limit of resolution is approached, the bright lines become hazy at the edges and then broaden, filling in the dark lines chiefly *from the sides* apparently. Spherical aberration, on the other hand, appears to fill in the dark lines, not from the sides, but from the bottom, leaving the edges more or less sharp. Coma produces a shading off similar to lack of resolution but easily distinguishable from it by its dissymmetry.

Telescope objectives we find fulfill this theoretical resolving power even at full aperture. This is true even of small reading telescopes with objectives of the old cemented doublet type. On the other hand, none of the four photographic objectives tested, all of the best modern types, gave this theoretical resolving power except when stopped down to $F/10$ or smaller. This defect appears to be due to a slight residual spherical aberration that is sacrificed to the oblique corrections.

The chief advantages of this method lie in its sensibility and its concreteness. One disadvantage is the difficulty in obtaining precise numerical results. This is due, not to lack of sensibility, but lack of any fixed criteria of quality. The most easily determined fixed point is when all structure just leaves the image. This is for a value of the constant a of 0.48 or 0.50 with an uncertainty of not over 0.02. At $a=0.60$, outlines are well established in the image, but so rounded that it is barely possible to tell the nature of the object. With $a=0.70$, outlines are firm, and the image bears a close resemblance to the object but lacks sharpness

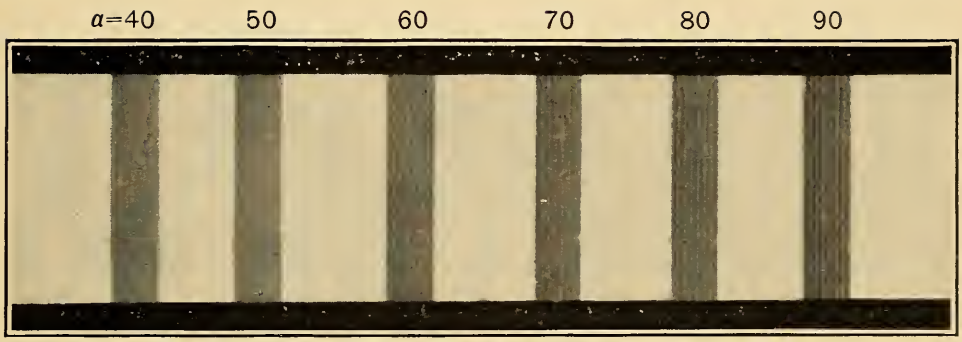

Figure showing variation in resolution in an image photographed near the limit of resolving power of an objective. Enlarged 2½ times. Object 200 lines to the inch.

of definition. At $a=0.80$ the image is just perceptibly deficient in definition, while at $a=0.90$ it shows no defects. The uncertainty in a setting on the beginning of lack of definition is about 0.05, considerably larger than for a setting on disappearance of all structure.

The appearance of the image in various stages of resolution is shown in the cut. The image was photographed at about three diameters with a "Planar" lens of 20-mm focal length used as photographic ocular. Exposures were made at $a=0.40$ (very diffuse), $a=0.50$, 0.60, 0.70, 0.80, and 0.90. The changes are just easily perceptible from step to step. A variation in resolution from side to side of each image, due to varying wave length, may also be noted. The range of wave length in question was about 12 $\mu\mu$.

When a galvanometer is used with telescope and scale, the galvanometer mirror is in effect a stop half way between the object (scale) and the objective lens of the telescope. In this case the resolving power of the telescope is double what it would be were a stop the size of the mirror placed at the objective. The telescope objective need not be more than four times the diameter of the galvanometer mirror to obtain the best results with proper eyepieces. Experimental tests of resolving power were made with stop placed in front of objective at a distance of ½, ¾, and ⅞ of the distance from the objective to object, and the theory verified with nearly the same precision as with full aperture.

In the resolving-power formula, $\varphi=a\lambda/r$, φ is the ratio of size to distance of the test object and hence applies to the image as well. Writing then $\varphi=\delta/F$ where δ is the size of the smallest resolvable image and F the equivalent focal length of the objective

$$\varphi=\frac{\delta}{F}=a\frac{\lambda}{r}$$

hence

$$2a\,A=\frac{\delta}{r}$$

or the smallest resolvable detail in the image, measured in wave lengths, is equal to the aperture ratio $A\left(=\frac{F}{2r}\right)$ of the objective. This is a useful rule in designing or selecting objectives. $2a=1$ represents the extreme limit of resolution, $2a=2$ corresponds to a sharply defined image.

When an image is to be viewed by an ocular, δ and λ are necessarily the same for the ocular as for the objective, hence if the ocular has an aperture ratio A equal to or greater than that of the objective, then the resolving power of the system will equal that of the objective alone. It can not be greater, for the light cone is of the same angle for the ocular as for the objective.

CONCLUSIONS.

The resolving power of objectives may be determined experimentally with simple but sensitive apparatus.

The resolving power constant varies from 0.50 for an image showing no detail whatever to 0.90 for an image just free from defects. The theoretical value 0.61 corresponds to a very diffuse but recognizable image.

Every lens fulfills its computed resolving power, but this is masked by residual aberration except in the case of telescope and other objectives that are carefully freed from third-order axial aberrations. The camera objectives examined show best resolution at about $F/10$.

In any lens system to be used at full power, no lens should have a smaller aperture ratio than the objective.

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