A COMPARISON OF THE VARIOUS METHODS OF DETERMINING THE RATIO OF THE ELECTROMAGNETIC TO THE ELECTROSTATIC UNIT OF ELECTRICITY.

By E. B. Rosa and N. E. Dorsey.

In the masterly report to the British Association in 1863 "On the Elementary Relations Between Electrical Measurements," prepared by Prof. J. Clerk Maxwell and Mr. Fleming Jenkin,[1] occur the following paragraphs concerning the experimental determination of the ratio, **v**, between electromagnetic and electrostatic measures of quantity:

"In order to determine the value of **v**, it is necessary and sufficient that we should obtain a common electrostatic and electromagnetic measure of some one quantity, current, resistance, electromotive force, or capacity. There are thus five known methods by which the value can be obtained:

"First. By a common measure of *quantity*. Let a condenser of known capacity, s, be prepared. Let it be charged to a given potential P. Then the quantity in the condenser will be sP in electrostatic measure. The charge can next be measured by discharge through a galvanometer in electromagnetic measure. The ratio between the two numbers will give the ratio **v**. The only difficulty in this method consists in the measurement of the potential P, entailing the measurement of an absolute force between two electrified bodies. The method was proposed and adopted by Weber.

"Second. By a comparison of the measure of *electromotive force*. The electromotive force produced by a battery, in electrostatic measure, can be directly weighed. Its electromotive force, in electro-

[1] Constituting Appendix C of the Report of the Committee on Standards of Electrical Resistance, B. A. Report, pp. 130–163; 1863.

magnetic measure, can be obtained from the current it produces in a given resistance. The ratio of the two numbers will give the value of **v**.

"Third. By a common measure of *resistance.* We know how to measure resistances in electromagnetic and in electrostatic measure. The ratio between these measures is equal to $\mathbf{v}^2$. The measure of resistance in electrostatic measure is not as yet susceptible of great accuracy.

"Fourth. By a comparison of *currents.* The electromagnetic value of a current produced by a continuous succession of discharges from a condenser of capacity *s* can be measured. The electrostatic value of the current will be known if the potential to which the condenser is charged be known. The ratio of the two numbers is equal to **v**.

"Fifth. By a common measure of *capacity.* The two measurements can be effected by the methods given. The ratio between the two measurements will give $\mathbf{v}^2$. This method would probably yield very accurate results."

During the nearly forty-four years that have elapsed since the above was presented to the British Association, many determinations of the ratio of the electrical units have been made. We present in this number of the Bulletin the results of our work by the last of the methods given by Maxwell and Jenkin, in which we have striven to carry the precision of the determination much further than has been done before. It seems a proper time to review the various methods that have been employed, or that might be employed, in order to show why we have chosen these particular methods and also to see whether in any future work other methods could be used with as good promise of obtaining an accurate value of **v**.

In the first stage of the work of finding **v**, extending up to about 1886, no value was found that could be depended on to nearer than 1 per cent, and most of the values were several per cent in error. In the second stage, during which a number of determinations were made between 1886 and 1898, the best results could be depended on to within one or two tenths of 1 per cent. We have now entered the third stage in the development of the work, in which results must be reliable to within one or two hundredths of 1 per cent in

order to be satisfactory in fixing the value of v. The question now is: How many of the various methods that have been employed or that might be employed would probably yield results to this order of accuracy if they were carried out under the most favorable circumstances and with the facilities available in a well-equipped, modern laboratory? The velocity v is so fundamental, and a knowledge of its value is so important, not only because of its theoretical identification with the velocity of light, but also because its numerical value is required in many electrical measurements and calculations, that it should be determined with all possible accuracy.

THE QUANTITY METHOD.

A determination by this method was carried out by Weber and Kohlrausch in 1857 and by Rowland in 1879. Practically every other method involves the use of a resistance, the value of which in absolute measure must be known. Hence this method, which does not require a knowledge of resistance, appealed to the early investigators, when resistances were not well known. The method is, however, complicated and difficult. In the first place, in common with all methods except the second (the electromotive-force method), it requires a condenser, the value of which is known in electrostatic measure, C_s. This is the easiest condition to meet, as a condenser can be built and measured so as to give its electrostatic value with high precision. The electromotive force to which it is charged must then be measured by an absolute electrometer. This is easy to do with a precision of a few tenths of 1 per cent, may perhaps be done to a tenth of 1 per cent, but is extremely difficult to do much better than this. The guard ring form has serious intrinsic difficulties, requiring as it does that the plane of the disk coincide closely with that of the ring. The force to be measured is very small unless the distance between the two plates is reduced unduly, and to get the precision of adjustment necessary while keeping the attracted plate free to move, it being at the same time in unstable equilibrium, is a matter of great difficulty. It is possible that a cylindrical electrometer can be designed that will avoid this trouble, but the latter has other difficulties of its own.

The electrostatic quantity, Q_s, becomes known when C_s and E_s are determined. This quantity of electricity is then measured elec-

tromagnetically by passing it through a ballistic galvanometer. To get Q_m with sufficient accuracy requires not only very accurate readings of the galvanometer deflections, but the determination of the constant of the galvanometer to a high order of accuracy. This can not be done satisfactorily by getting the strength of the earth's field in the usual way and calculating G from the dimensions of the galvanometer. The earth's magnetic field must be eliminated, as was done by Rowland, by the use of an absolute electro-dynamometer. This makes the experiment very difficult and complicated and renders the high precision we have specified practically impossible.

THE ELECTROMOTIVE-FORCE METHOD.

In this method no condenser need be employed. A certain constant electromotive force of considerable magnitude is measured by means of an absolute electrometer in electrostatic units E_s, and then it is obtained in electromagnetic measure by measuring the current I_m, flowing under this same electromotive force through a known resistance R_m.

$$E_m = I_m R_m, \text{ and then } \mathrm{v} = \frac{E_m}{E_s}$$

If we assume that the value of the international ohm in absolute measure is known, then there are two measurements to be made in this method of determining v, namely, E_s by an absolute electrometer and I_m by an absolute electrodynamometer. We have spoken above of the difficulty of measuring E_s with precision. The measurement of I_m is not so difficult, although by no means simple. It is doubtful whether this method of finding v can compete in accuracy with the capacity method discussed below, and we believe that the most useful application of this method may be to *reverse the process* and, taking the value of v as given by other methods, find E_m ($=E_s$ v) by measuring E_s by the absolute electrometer. This will give a check on the value of the standard cell as determined by means of absolute electrodynamometers. Inasmuch as v is now probably as well or better known than the standard cell, this would be a valuable contribution to our knowledge of the value of the standard cell, provided the absolute electrometer measurements are sufficiently accurate. If they are not, they would be worth nothing in fixing the value of v.

Maxwell made the first determination (1868) by this method, combining the two instruments, the absolute electrometer and the absolute electrodynamometer, in one, so as to make it a null method. The apparatus was complicated and the experimental difficulties serious. As a pioneer experiment it was a success, although the result was 3 per cent too small. Hurmuzescu in 1896 published a determination of v by the method of Maxwell; that is, combining the electrometer and electrodynamometer in one instrument. The former was a double cylindrical electrometer, and suspended inside a horizontal solenoid in the axis of rotation of the electrometer was the moving coil of the electrodynamometer. The instrument was constructed, and a long series of observations was made. The results were about one-tenth per cent too large. This seems to be one of the best determinations that has been made.

In the series of experiments carried out by Thomson and King (1869), McKichan (1874), Shida (1880), Thomson, Ayrton and Perry (1888), the electromotive force was measured by a Thomson absolute electrometer and the current by an absolute electrodynamometer or a tangent galvanometer. None of these determinations were very exact, the values being several per cent too small in every case.

The very careful determination by Pellat (1891) was made, using a Thomson electrometer and a Pellat absolute current balance. Full particulars of the work were not published, but the author claims no better accuracy than 1 in 500 for the electrometer. The result was nearly one-half per cent too high.

Perot and Fabry (1898) used an absolute electrometer of parallel plates, consisting of glass disks worked to optical surfaces and lightly silvered, the small distance apart being measured by the method of interference. The current through a standard resistance was measured in terms of the electrochemical equivalent of silver. The result is affected, as Abraham points out, by the uncertainty in the electro-chemical equivalent of silver, and the approximations used in calculating the constant of the electrometer. The result happens to be, however, only very slightly (0.1 per cent approximately) too high.

Some at least of the above experiments have been carried out by the most skilled observers working with excellent facilities. It seems doubtful whether very great improvement could be made by a repetition of the work.

THE RESISTANCE METHOD.

Assuming as in the previous method that the value of the international ohm is known in absolute measure, the problem in this method is to find the value of some particular resistance in ohms and in electrostatic measure. Since the ratio $\frac{R_m}{R_s} = \mathbf{v}^2$ the uncertainty in the absolute value of the ohm enters by only half its value in **v**. In this respect this method is superior to the preceding. If the resistance be a well-constructed coil of manganin wire, or a series of such coils, so mounted as to permit accurate comparisons with standard resistances, the determination, with the required precision, of their values in international ohms is of course not a difficult matter. Practically the whole problem lies in measuring the resistance in electrostatic units.

If a condenser of capacity C_s, charged to a potential V_1, discharge itself through a resistance R_s, the current at any moment is

$$i_s = \frac{V_s}{R_s} = -C_s \frac{dV_s}{dt}$$

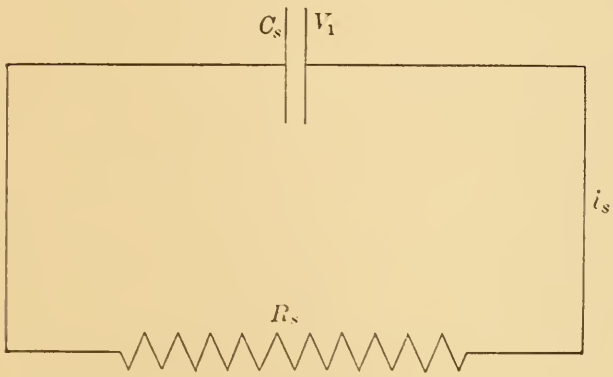

Fig. 1.

$$\therefore\ C_s \frac{dV_s}{dt} + \frac{V_s}{R_s} = 0 \qquad (1)$$

$$\int_{V_1}^{V_2} \frac{dV_s}{V_s} = -\int_{t_1}^{t_2} \frac{dt}{C_s R_s}$$

$$\log \frac{V_1}{V_2} = \frac{t_2 - t_1}{C_s R_s}$$

$$R_s = \frac{t_2 - t_1}{C_s \log \frac{V_1}{V_2}} = \frac{t}{C_s \log \frac{V_1}{V_2}} \qquad (2)$$

Thus, if a condenser of electrostatic capacity C_s discharges itself in t seconds through a resistance R_s from an initial potential V_1, to a

lower potential V_2, the value of this resistance in electrostatic measure is given by equation (2). Substituting $R_s = \frac{R_m}{v^2}$, we have

$$v^2 = \frac{C_s R_m \log \frac{V_1}{V_2}}{t}$$

Suppose that $V_2 = \frac{V_1}{2}$, that is, that the final potential is exactly one-half the initial potential, then

$$v^2 = \frac{C_s R_m \log_e 2}{t}$$

To get v, therefore, we must have in addition to the value of the resistance in absolute electromagnetic measure two quantities: (1) The capacity in electrostatic units of the condenser; (2) the time t in which the condenser discharges to one-half its initial potential.

In order to find the approximate magnitude of the quantities C, R, and t, we may use equation (3), assuming v known. The condenser must be a very well-constructed air condenser; suppose its capacity is 0.1 microfarad, which is rather large for an air condenser, but readily attainable. Let $t = 0.1$ sec. and take $v = 3 \times 10^{10}$. Substituting these values in equation (3) we find, since $\frac{C_s}{v^2} = C_m$

$$R_m = \frac{t}{C_m \log_e 2} = \frac{0.1}{0.1 \times 10^{-6} \times .693} = 1{,}443{,}000 \text{ ohms.}$$

This is a reasonable resistance, and hence we see that the experiment is perfectly practicable if we can find a way of measuring t with the required accuracy, and getting the value of the condenser in electrostatic measure with sufficient accuracy.

The electrostatic capacity can of course be determined only by comparing the condenser with another condenser, the value of which can be computed from its dimensions. Such a condenser will have a value about 1,000 times less than the condenser proposed to be used, that is, about 0.0001 mf. The difficulties of making directly an accurate comparison of two condensers of so different values are very great. It would probably be better to measure each absolutely by means

of the Maxwell bridge, and so obtain their ratio. If one were to attempt to work directly with the small air condenser whose capacity is determined from dimensions, and discard altogether the large air condenser of 0.1 mf, he would be obliged to use very great resistances, 1,000 megohms or more, or very short intervals of time. In either case the difficulty of getting accurate results would be increased. Supposing, however, that the electrostatic value of the large air condenser has been satisfactorily measured, let us see how to get the

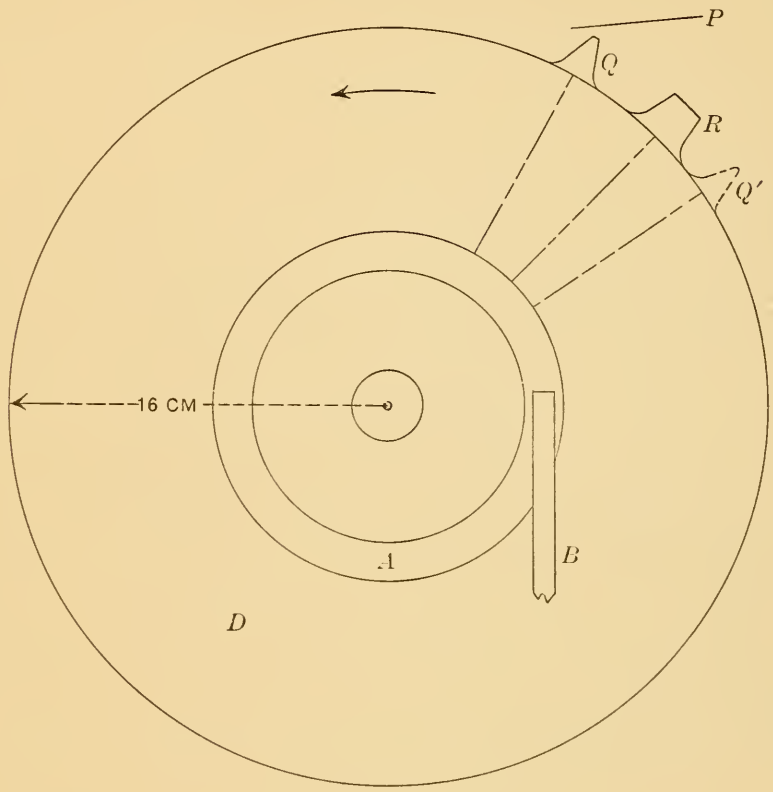

Fig. 2.

time t, during which it is discharged to one-half its initial potential, through the large resistance R.

Let D (Fig. 2) be a disk of hard rubber 32 cm in diameter mounted on a shaft and driven by an electric motor at a uniform speed of 600 revolutions per minute, or any other desired speed. At Q is a boss of phosphor-bronze which makes contact with the brush P for an instant only. R is a second boss making a longer contact. By contact rings and brushes one on each side of the disk, Q is joined to the middle of the charging battery and R to the end (Fig. 3). The

brush P is joined to the terminal B of the condenser. The condenser is then charged as P makes contact over R, and discharges itself through the high resistance during nearly one whole revolution of the disk, occupying 0.1 second, until the brush touches R again, when it is recharged. If as P touches Q the potential of the terminal B of the condenser is at a different potential from the middle point of the battery which is joined to Q, the galvanometer (or electrometer) will give a deflection. By adjusting the speed or the resistance or the potential of Q or R, the galvanometer deflection could be made zero. The simplest way, of course, would be either to keep the ratio of the potentials exactly 1 to 2, by means of a potentiometer and adjustable rheostat, and the resistance constant, and then adjust the speed of the contact disk so that the deflection is zero on the average for several minutes, recording the speed on a chronograph; or else, hold the speed steady by an auxiliary bridge and adjust the resistance to give zero deflection. In practice both methods should be used.

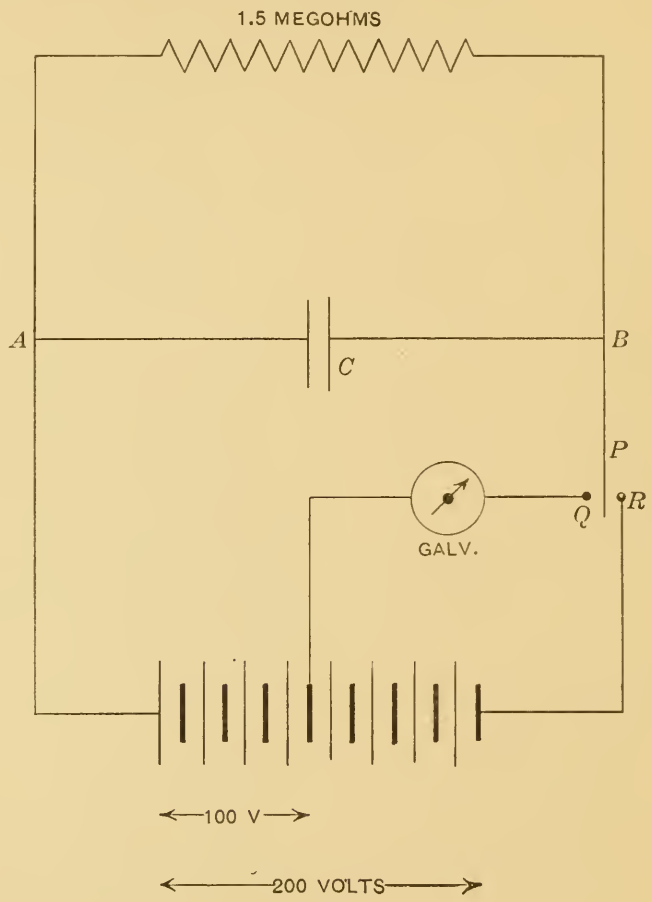

Fig. 3.

In order to determine the exact time between the breaking of contact at R and the middle of the contact at Q one could measure the angle by which Q must be turned back (to the position Q′) in order to show a very small fall in potential, say one-half per cent, and calculate the time from this measured fall in potential between the breaking of R and the contact at Q′. This would give, know-

ing the angle QQ′ exactly, the effective angle between the break of R and the middle of Q. This effective angle θ would depend on the adjustment of the brushes. Perhaps two exactly similar studs, QQ′, offset from R, and two brushes would do better. One brush would charge the condenser and one would make contacts on Q and Q′. A switch outside would put Q and Q′ in service alternately. Q would show a drop of 100 volts; Q′ of only about 1 volt. Then, without stopping the commutator or touching the brushes, the two observations would give a very exact value of the effective angle, θ, and from this and the speed, t would become known. This null method of working, with a large capacity (which would make the capacity of the lead wires, commutator, and brushes of little consequence) and a fairly high voltage which would reduce the disturbance due to electrostatic charges would probably give very accurate results. Possibly a revolving brush, P, with a stationary ring carrying Q and R would be better. Then Q could be moved back through a known angle, a, and Q′ would be represented by Q in its new position, now, of course, joined to a different point of the battery, 1 volt (say) from the end.

It would seem that this experiment could be carried out with success, and give an accurate value of R_s and so of **v**. As already stated, however, it is a serious undertaking to obtain C_s, the electrostatic capacity of the large condenser. If we do it by getting electromagnetic capacities of two condensers and taking their ratio, we have $\mathbf{v}^2$ in the ratio of C_s to C_m for the one absolute condenser. Hence all the work in getting R_s is *really unnecessary, except as affording by a roundabout way a value of* **v** *more easily obtained by the method of capacities.* By constructing an absolute air condenser having a capacity 100 times as great as assumed above (i. e. 0.01 mf) the experiment of getting R_s might be carried out on the absolute condenser itself. This would greatly simplify the work, but it would be a difficult matter, if not impossible altogether, to construct such a condenser so that its capacity could be certainly measured to 1 part in 10,000 by measurements of its dimensions alone. It would probably be a multiple-plate condenser, and the connecting wires would offer a serious difficulty in getting the capacity by calculation. This method was attempted by Klemenčič at Vienna in 1886, but without much success.

THE CURRENT METHOD NOT AVAILABLE.

There is no way of measuring directly the value of a steady current in electrostatic units. The method suggested by Maxwell, of charging and discharging a condenser rapidly, sending the charges or discharges through a galvanometer and measuring the mean value of such an interrupted current, as though it were a continuous current, requires a condenser of known value and is really included in the method of capacities. Hence we may better regard the current method as unavailable and include the plan proposed by Maxwell under the method of capacities.

THE METHOD OF CAPACITIES.

This is, by all means, the best of the four methods available. Methods 1 and 3 (the quantity method and the resistance method) both involve the use of capacities of known values in electrostatic measure. Hence they also might have been included under this method. The method of electromotive forces is therefore the only one that does not involve the use of a capacity, and that method is not capable of yielding results at all comparable in accuracy with the method of capacities. Hence we see that a precise knowledge of the velocity **v** must come through the use of capacities.

There are several different ways of obtaining **v** by this general method. In every one we must obtain the electrostatic capacity C_s by calculation from the dimensions of the condenser. The difference comes in the various ways of getting C_m, but in all we require to know the value of a resistance in absolute measure. We shall specify seven different methods of finding C_m, the third and fourth of which are the simplest and most direct and capable of the highest accuracy.

(*a*) **By ballistic galvanometer.**—In this method the condenser of known electrostatic capacity C_s is charged by a certain emf. E and then discharged through a ballistic galvanometer, precisely as in the quantity method. The difference between the two cases is this: In the first or quantity method the emf. E_s must be measured by an absolute electrometer, giving $Q_s = C_s E_s$, and the ballistic galvanometer must be calibrated by a current measured by an absolute electrodynamometer or tangent galvanometer; the resist-

ances are not assumed to be known in absolute measure. In this method the same battery is used in charging the condenser and in calibrating the galvanometer, so that it is $\frac{Q_m}{E_m}$ that is measured by the galvanometer, and that is C_m directly, which is of course the quantity sought. In thus eliminating E the value of a resistance is introduced, so that while avoiding the absolute electrometer and electrodynamometer, it is necessary to know resistance in absolute measure.

This method was employed by Ayrton and Perry in Japan in 1879 and by Hockin in the same year. While simple and direct, this method can not compare in accuracy with the null methods (*c*) and (*d*) below.

(*b*) **By steady deflection of a galvanometer,** using a fork or rotating commutator to charge and discharge the condenser periodically. The charges or the discharges may be allowed to pass through the galvanometer, and the steady deflection produced is then compared with that due to a steady current through a known resistance from the same battery. The number of charges and discharges per second must be known as well as the resistance employed in getting the steady current deflection.

This method was employed by Stoletow at Moscow in 1881 and by Klemenčič at Vienna in the same year. Stoletow used a revolving commutator to effect the successive charges and discharges of the condenser. Klemenčič used a tuning fork for this purpose. This was an important improvement over sending a single charge or discharge through the galvanometer.

(*c*) **By the Maxwell bridge, getting C_m in terms of R and t.**—This is the best-known capacity method, and has yielded excellent results. The condenser and commutator are placed in one arm of the bridge, and the bridge balanced by varying either a resistance or the speed of the tuning fork or commutator. Being a null method, the bridge may be made very sensitive by using high voltages and high frequencies of charging. As this and the next method (*d*) have been discussed at length in the foregoing paper we say very very little about either at this time.

This method was first employed for finding **v** by J. J. Thomson at Cambridge in 1883. It has since been employed by Himstedt at

Darmstadt (1887), by Rosa at Baltimore (1888), by Thomson and Searle at Cambridge (1890), and by the present writers at Washington 1904–1907.

(*d*) **By the differential galvanometer.**—This is a modification of (*b*), the interrupted current from the condenser and the steady current from the battery passing simultaneously through the two coils of a differential galvanometer, and so making it a null method. A greatly increased sensibility is thus secured and errors due to measuring deflections eliminated. This method possesses the same possibilities of high accuracy as (*c*), and is subject to practically the same limitations.

The method was first employed by Klemenčič at Vienna in 1884. It has since been used by Himstedt at Freiburg (1886) and Darmstadt (1888), by Abraham at Paris (1892), and by the present writers at Washington (1905–1907).

(*e*) **By oscillatory discharges of a condenser.**—If a condenser of capacity C_m be placed in series with inductance L_m, the period of its oscillatory discharge is given by the expression

$$t = 2\pi\sqrt{C_m L_m}$$

when the resistance is small. By determining t, one obtains a measure of the product $C_m L_m$, and by then measuring L_m the capacity of the condenser in electromagnetic measure becomes known. From C_s and C_m, v becomes known as in the other capacity methods. There are thus three quantities to measure—t, C_s, and L_m.

This method was employed by Colley at Kasan (1886), by Webster at Clark University (1898), and by Lodge and Glazebrook (1899).

The time of oscillation may be determined in various ways. Colley used a galvanometer of very high frequency, Webster employed an electrometer, Lodge and Glazebrook photographed by a revolving mirror the spark produced in the discharge.

Although the accurate determination of the period t is not without difficulty, the chief sources of error probably lie in getting C_s and L_m. In order to obtain t accurately, the oscillations must not be very rapid. This requires a relatively large condenser and a large inductance coil. If the condenser consists of a large number of parallel plates very close together (Lodge and Glazebrook), it is

impossible to get their capacity with precision by measurement and calculation. If the capacity be determined by comparison with a standard condenser of suitable design for accurate calculation of the capacity, the experimental comparison of the two involves greater errors than the absolute measurement of C_m for the small condenser. Similarly, the inductance must be larger than can be accurately calculated from its dimensions, and its electrostatic capacity offers a troublesome factor to obtain except by experiment. We believe that one-tenth of 1 per cent is about the limit of accuracy possible to obtain at present by this method, which is a much higher accuracy than has yet been obtained by the method.

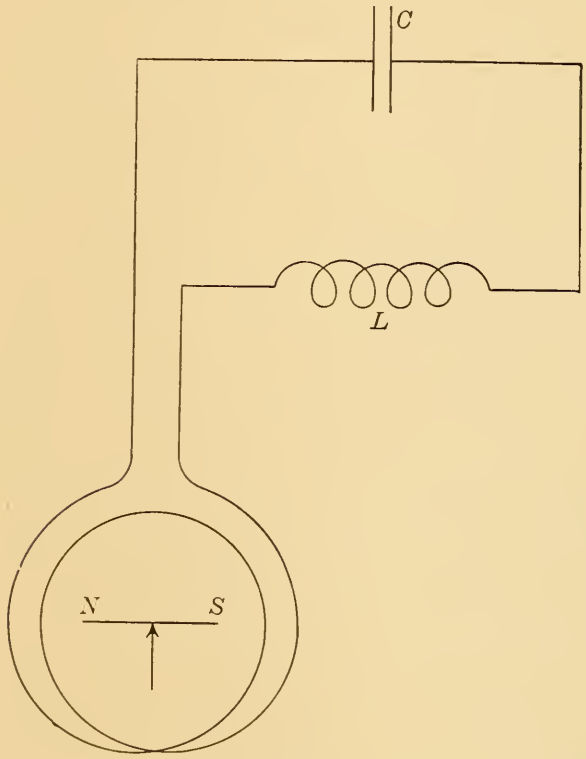

Fig. 4.

(*f*) **By rotating a magnet within a suspended coil.**—This method, which has been suggested by Gray,[2] has never been carried out. A magnet is to be rotated rapidly within a coil, the latter being suspended in a vertical plane by a bifilar so as to turn freely about its axis. The induced current deflects the coil through an angle θ, the value of which depends on the value of the capacity and inductance in series with the coil, as well as upon the coil itself, the suspension, the magnet, and the speed with which it is rotated. The maximum deflection is given by such a speed that $t = 2\pi\sqrt{CL}$, where t is the time of one revolution of the magnet. This requires the product CL to be relatively large—that is, one would have to employ a large capacity and a large inductance. By using three different angular velocities—n_1, n_2, n_3,—and measuring three dif-

[2] Absolute measurements, etc.

ferent deflections—D_1, D_2, D_3—the value of $C_m L_m$ is obtained from the following equation:

$$C^2L^2 = \frac{1}{n_1^2 n_2^2 n_3^2} \cdot \frac{\frac{n_1^3}{D_1}(n_2^2 - n_3^2) + \frac{n_2^3}{D_2}(n_3^2 - n_1^2) + \frac{n_3^3}{D_3}(n_1^2 - n_2^2)}{\frac{n_1}{D_1}(n_2^2 - n_3^2) + \frac{n_2}{D_2}(n_3^2 - n_1^2) + \frac{n_3}{D_3}(n_1^2 - n_2^2)}$$

This equation results from the three equations given by the three sets of observations, after eliminating the resistance and the constants of the coil.

It is obvious that this experiment, while interesting theoretically, can not afford a means of determining v with precision. The three speeds and the three deflections would have to be determined with extraordinary precision to give an accurate value of $C_m L_m$. When that is determined it still remains to find C_s and L_m as we obtain C_m, by dividing $C_m L_m$ by L_m, and then get v^2 from C_s and C_m. But since C_m and L_m are both necessarily large, we can not calculate them directly from their dimensions, but must compare them with absolute condensers and coils that have been so determined or measure L_m in terms of resistance. Thus the method is very elaborate and offers many sources of error.

An idea of the magnitude of C and L can readily be obtained from the condition that for maximum deflection $t = 2\pi\sqrt{CL}$; $\frac{2\pi}{t} = n =$ angular velocity, t being the time of one revolution of the magnet. Suppose $n = 100$, corresponding to about 1,000 revolutions per minute. Then

$$n^2 = 10{,}000 = \frac{1}{CL}$$

Let $\quad C = 10$ microfarads $= \frac{1}{10^5}$ farads

$L = 10$ henrys

Then $CL = \frac{1}{10^4} = \frac{1}{n^2}$

Evidently a capacity of 10 microfarads and an inductance of 10 henrys are far too large to obtain directly from their dimensions. Both can be measured in terms of ohms, but to get C_s requires, as in the preceding method, a comparison with an absolute condenser.

(*g*) **By comparing a capacity and a resistance by use of an alternating current.**—This method was employed by Miss Maltby in 1897, as follows:

An alternating current flows through ABD, the air condenser C being in series with a noninductive resistance W. The applied electromotive force is generated by the rotation of a magnet within a

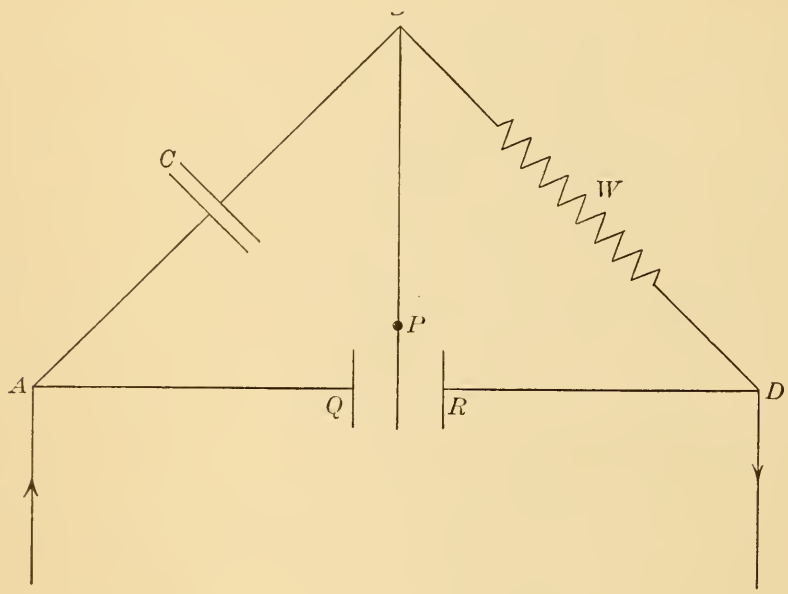

Fig. 5.

coil, and is nearly a sine wave. The electrometer PQR is used to indicate by its balance that the effective emf. on AB is equal to that on BD. In this case

$$W=\frac{1}{2\pi nC}$$

$$\text{or } T=\frac{1}{2n}=\pi CW$$

T being the time of one-half cycle of the alternating emf. Knowing T from the speed of the rotating magnet and W, the capacity C becomes known in electromagnetic measure. It is to be observed that the measured capacity C_m of the condenser will be too large if correction is not made for the capacity of the electrometer.

This simple and interesting method is evidently not suitable for precision measurements, unless the wave form is determined very accurately and allowance made for any harmonics found.

There are other alternating current methods of obtaining capacity in terms of resistance, those of Wien using a bridge and tuned galvanometer being especially favorable to accuracy as the effect of harmonics is eliminated by the tuned galvanometer, and no error is produced by a wave form varying from a sine curve. The chief difficulty in any case is to obtain the electrostatic capacity of the condenser if the latter is large, or the electromagnetic capacity with sufficient accuracy if the condenser is small, the difficulty common to most of the condenser methods.

(*h*) **By comparing C_m with L_m by means of a bridge.**—We can measure the electromagnetic capacity of a condenser in terms of an inductance by means of the bridge methods of Maxwell, Anderson, Wien, or Rosa, using alternating currents. Having thus obtained C_m, we get v as soon as C_s is determined. This appears a simple and direct method of getting the ratio of the units. The difficulty lies chiefly in getting the electrostatic capacity C_s of a condenser large enough to use in the bridge methods for finding C_m. It is the same difficulty noticed already in other methods. There is no way of directly measuring a small capacity (such as that of an accurate absolute condenser) in terms of an inductance with sufficient precision for this work. Hence it is necessary to get the value of a large air condenser by comparing it with a small absolute air condenser, and then compare the former with the known inductance. This of course can be done with considerable precision, but it is less direct and far less likely to give a result of high precision than methods (*c*) and (*d*) above described, and it involves precisely the same kind of errors that are involved in those methods, with others in addition. We do not mean to say that the last three capacity methods are without value. What we wish to emphasize is that not one of them permits as direct a determination of the capacity of an absolute condenser in electromagnetic measure as do methods (*c*) and (*d*), and that while they all possess serious disadvantages as compared with (*c*) and (*d*), none of them possesses any single compensating advantage.

Our conclusions are as follows:

1. Of the four general methods available, the quantity method is most complicated and least accurate.

2. The electromotive force method is better than the first, but can perhaps be of more service in giving us a more accurate knowledge of the absolute value of the standard cell, taking **v** as known, inasmuch as **v** is as well or better known than the value of the standard cell.

3. The resistance method is roundabout, and could not be expected to give as accurate results as the more direct of the capacity methods.

4. Of the seven capacity methods that have been used or suggested, the first two (*a*) and (*b*) have been superseded by the next two (*c*) and (*d*), which are the best of all; the last three are complicated and incapable of high accuracy.

5. With the use of several different forms of absolute condensers, constructed as perfectly as possible, and measured by the capacity methods (*c*) and (*d*), we believe that thoroughly reliable results can be obtained without further use of any of the less direct and more difficult methods.

WASHINGTON, June 1, 1907.