WATTMETER METHODS OF MEASURING POWER EXPENDED UPON CONDENSERS AND CIRCUITS OF LOW POWER FACTOR.

By Edward B. Rosa.

The power factor of an alternating current flowing into and out of a good condenser, or a cable on open circuit, is so small as to make its measurement by a wattmeter somewhat difficult. Measurements made by the resonance method[a] and by the calorimetric method[b] gave power factors for some paraffined paper condensers of less than half of 1 per cent. These methods are, however, not adapted to general laboratory work. A simple wattmeter method of measuring power involves two corrections. First, for the power expended upon the fixed coil of the wattmeter, which is measured along with that of the condenser, and, second, for the change of phase of the potential current due to the combined inductance and capacity of the potential circuit. The correction (which is chiefly due to the inductance of the coil of the instrument which carries the potential current) is made small by using a large resistance in the potential circuit. Incandescent lamp filaments have extremely small inductance and capacity, and may be employed in series with one another, as the high resistance of the potential circuit; or wire coils may be employed if wound so as to avoid both inductance and capacity. The potential current, being very small, requires a relatively delicate suspension in order to give a satisfactory deflection. I have found difficulty in securing sufficient stability and sensitiveness at once, and have hence been led to the use of a series of null methods for measuring the power factor of a condenser or cable current, thereby avoiding the measurement of a deflection, and dispensing with the requirement of so great stability. These methods were devised and employed in a long series of measurements in 1898, but the work was interrupted before its completion, and circumstances have since prevented me from resuming the work.

[a] Rosa and Smith: Phys. Rev., Jan., 1899.
[b] Rosa and Smith: Phys. Rev., Feb., 1899.

The most obvious null method consists in using a variable inductance in the potential circuit. The difference of phase between the two currents being nearly 90°, if the potential current is slightly retarded by an added inductance the phase difference can be made 90° and the deflection reduced to zero. Knowing the value of the added inductance, the frequency of the current, and the resistance, the change of phase, and hence the power factor, can be readily computed.

USING AN AUXILIARY COIL ON THE WATTMETER.

There are, however, several other methods of getting a difference of phase of 90° and securing a zero deflection, any one of which can be employed for this purpose, according to the instruments one has available. These all depend upon the use of an auxiliary coil of fine wire wound over the fixed coil of coarse wire, having about the same number of turns as the fixed coil and made exactly equivalent to the fixed coil magnetically. This equivalence is shown by causing the same current to flow through the main coil and the auxiliary coil in opposite directions. The resulting magnetic field is then zero at the position of the suspended coil if the latter is not deflected when the current flowing through the main and auxiliary coils in opposite direction passes through the suspended coil also. Suppose, then, the condenser current i_1 passes through the fixed coil, the potential current i_2 passes through the suspended coil, and a small current i_3 (in phase with i_2) passes through the auxiliary coil. Then if K is the constant of the instrument, the deflection d_1 of the suspended coil due to current i_1 in the main coil and i_2 in the suspended coil will be

$$d_1 = Ki_1i_2 \cos \phi_1$$

where ϕ_1 is the angular difference of phase between i_1 and i_2.

The current i_3 through the auxiliary coil would by itself (supposing there is no current in the main coil) produce a deflection d_2 such that

$$d_2 = K i_2 i_3.$$

If the current i_3 flows in such a direction as to make the deflection d_2 opposite to d_1, then when the currents i_1 and i_3 flow simultaneously, the deflection is the difference between d_1 and d_2; and when this is made zero by adjusting i_3 we have

$$i_2 i_3 = i_1 i_2 \cos \phi_1$$

$$\therefore \cos \phi_1 = \frac{i_3}{i_1}$$

Another way of expressing this result is to say that the current i_3 (which is in phase with i_2, but *reversed by the connections*, and therefore differs in phase by 180° from i_2) is added to i_1, and the vector sum is thereby made to differ by 90° in phase from i_2. The ratio of i_3 to i_1 then gives the angle through which i_1 has been turned, to make it differ by 90° from i_2. Or, again, the magnetic field of i_3, added to the magnetic field of i_1, gives a resultant magnetic field which differs by 90° in phase from the potential current i_2. Hence, knowing i_3, we can find the power factor cos ϕ_1. The corrections for the resistance of the fixed coil and the inductance of the suspended coil being applied, we have cos ϕ, the true power factor.

The auxiliary or compensation current i_3 may be secured by several different devices, as follows:

(1) From the terminals of a noninductive resistance r_3 in the main circuit a shunt circuit is carried to the auxiliary coil, having a second condenser in series with it. Thus the current i_3 is proportional to the main current i_1, to the resistance r_3, to the capacity of the secondary condenser C_3, and to the frequency of the current. That is

$$\dot{i}_3 = pC_3 i_1 r_3 \text{ and } \frac{\dot{i}_3}{i_1} = \cos \phi_1 = pC_3 r_3$$

Thus, knowing the capacity of the secondary condenser C_3, the frequency of the current ($p=2\pi n$), and the variable resistance r_3, we readily compute the power factor cos ϕ_1, which, corrected as before, gives cos ϕ.

(2) A portion of the potential current i_2 is shunted off through the auxiliary coil, and the value of i_3 becomes known if i_2 and the resistances of the divided circuit are known.

(3) By transforming down from the high e. m. f. E impressed upon the condenser and shunt circuit, a small e. m. f. e_3 is obtained, differing in phase by 180° almost exactly. A current $i_3 = \frac{e_3}{R_3}$ is thus obtained, which differs in phase from the shunt current i_2 by 180°, and this is used as a compensation current. The resistance R_3 is varied until the deflection is zero, and e_3 is measured by an alternating current voltmeter; i_3 thus becomes known, and from it cos ϕ_1 as before.

When the capacity of the condenser is small, and therefore the main current i_1 is small, it may be better to pass i_1 through the suspended coil and the potential current through the fixed coil. This gives rise to two other arrangements of the circuits. But the principle is the same in every case, the compensation current i_3 being determined when the deflection of the wattmeter is zero, and the phase angle then found by a very simple calculation.

The measurements which I made by these various methods on the same condensers showed closely agreeing results, although the apparatus employed did not permit of measurements of the highest precision. I proceed to give a description of these methods, and a few examples of their use in measuring the power absorbed in some paraffined-paper condensers.

METHOD 1.—SIMPLE DEFLECTION METHOD.

The connections of the wattmeter to the source of alternating current and the condenser are shown in fig. 1, and the phase diagram is shown in fig. 2. F is the fixed coil and M the movable coil of the wattmeter, i_1 the main condenser current, and i_2 the potential current

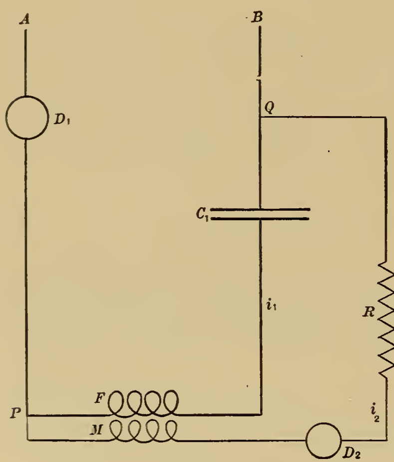

FIG. 1.—Connections for Method 1.

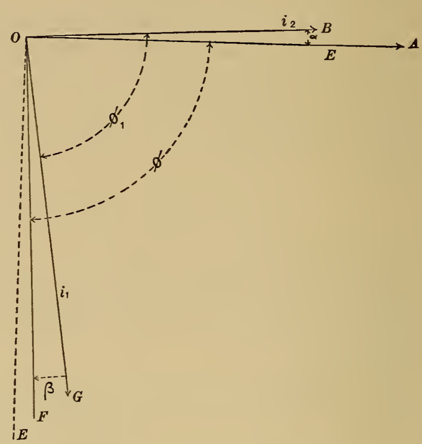

FIG. 2.—Phase diagram for Method 1.

passing through the large noninductive resistance R. An alternating potential of several hundred volts (up to 1,500 in my experiments) was applied to A B. D_1 is the electrodynamometer or alternating current ammeter for measuring the current i_1+i_2, and D_2 measures i_2 alone. Inasmuch as these two currents differ in phase by nearly 90°, and i_2 is relatively small, the reading of D_1 is practically unaffected by the current i_2. The noninductive resistance is several thousand ohms, and if it is known accurately, and also the voltage E, the instrument D_2 may be omitted and i_2 calculated from E and R. The two coils F and M are joined together at the instrument in order that they may have the same potential and so avoid electrostatic attraction between them, a serious source of error when relatively high potentials are used, if the coils are not joined together.

$O\ A$ (fig. 2) represents the phase of the electromotive force E. The potential current lags behind it by an angle α, due to the inductance of the moving coil and of the resistance R. The high resistance itself often possesses a greater capacity than inductance, and hence the combined inductance of the circuit (L_2) may be less than that of the moving coil of the wattmeter alone. It may even be negative, and therefore the potential current may be ahead in phase of E. In my experiments the inductance and capacity of the resistance R were both very small, and L_2 was taken as that due to the known inductance of the moving coil of the wattmeter. The line $O\ G$ represents the condenser current i_1, differing in phase from i_2 by the angle ϕ_1. $O\ F$ would have been the phase of this current if the fixed coil of the wattmeter had no resistance. The angle β is the difference of phase $G\ O\ F$, due to this resistance, and $\tan\ \beta = r \div \frac{1}{pC} = pCr$, where $p=2\pi n$, n being the frequency of the alternating current, C the capacity of the condenser, and r the resistance of the wattmeter coil F and the connections; that is, the resistance between P and Q when the condenser is short circuited. We then have the following expressions, d being the deflection of the wattmeter, K the constant of the latter, ϕ_1 the measured difference of phase $B\ O\ G$ and ϕ the corrected difference $A\ O\ F$, due to the condenser.

$$d = Ki_1i_2 \cos\ \phi_1, \text{ or } \cos\ \phi_1 = \frac{d}{Ki_1i_2}$$

$$\tan\ \alpha = \frac{pL_2}{R},\ \tan\ \beta = pCr.$$

$$\phi = \phi_1 - \alpha + \beta$$

As stated above, ϕ_1 is the measured phase difference, and α and β are respectively the small angular corrections due to the inductance of the potential circuit and the resistance of the fixed coils of the wattmeter

Table I.—RESULTS BY METHOD 1.

[n=120 cycles; E=1,400 volts.]

Condenser employed.	i_1	i_2	K	d	$\cos\ \phi_1$	Power factor, uncorrected.
	Amperes.	*Amperes.*				*Per cent.*
Condenser No. 4..	1.653	0.1470	23300	26.0	0.00459	0.459
Condenser No. 5..	3.494	0.1666	23100	76.0	0.00565	0.565
Condenser No. 6..	3.494	0.1707	23100	75.0	0.00545	0.545

and its connections. Cos ϕ is then the power factor of the condenser. Table I gives some measurements made on three condensers, the second and third having a capacity about double the first.

The corrections α and β are here very slight, so that ϕ is sensibly equal to ϕ_1. The condensers are Stanley paraffined paper condensers, and No. 4 has a sensibly smaller energy loss than the others.

METHOD 2.—INDUCTANCE IN SHUNT CIRCUIT.

Method 2 is similar to the first, except that a variable inductance L_3, fig. 3, is inserted in the shunt circuit and the current i_2 is made to

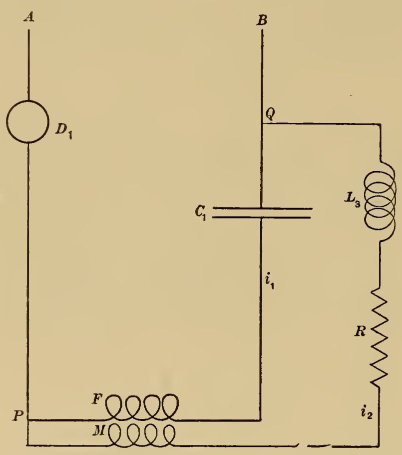

FIG. 3.—Connections for Method 2.

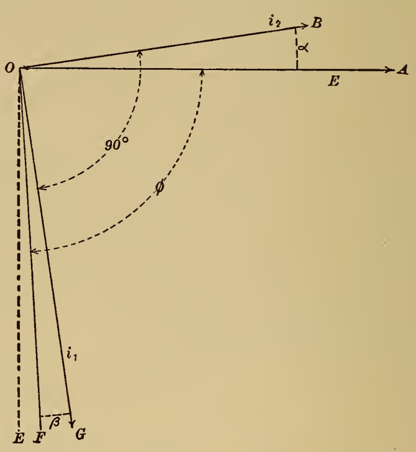

FIG. 4.—Phase diagram for Method 2.

lag by a larger angle than before, so that ϕ_1 is now 90°, and the deflection is zero. If L_2 is the same as before and L_3 is the added inductance,

$$\tan \alpha = \frac{p(L_2+L_3)}{R}$$

$$\tan \beta = pCr$$

$$\phi = 90° - \alpha + \beta$$

This is a very convenient method if a variable known inductance of suitable magnitude is at hand, and it gives excellent results. The frequency of the current must be known more accurately than before, for α is now a much more important quantity than in method 1, since the value of cos ϕ_1 depends mainly on α. But d, K, i_1, and i_2, the quantities measured in method 1, need not be measured at all in this method.

Table II.—RESULTS BY METHOD 2.

[n=120, E=1,260 volts.]

Condenser employed.	$p=2\pi n$	L_2+L_3	R	cos ϕ_1	Power factor, uncorrected.
		Henry.	*Ohms.*		*Per cent.*
Condenser No. 4	754	0.0530	8,548	0.00468	0.468
Condenser No. 5	754	0.0627	8,548	0.00553	0.553

These results do not agree exactly with those by the first method. The difference is partly due to errors of the experiment, and partly to a change in the temperature of the condensers and of the wave form of the electromotive force employed. For want of a suitable variable inductance, very little was done with this method.

METHOD 3. AUXILIARY CONDENSER SHUNT ON THE MAIN CIRCUIT.

In this and the following methods the auxiliary compensation coil A is employed. As already explained, this coil is of fine wire and is

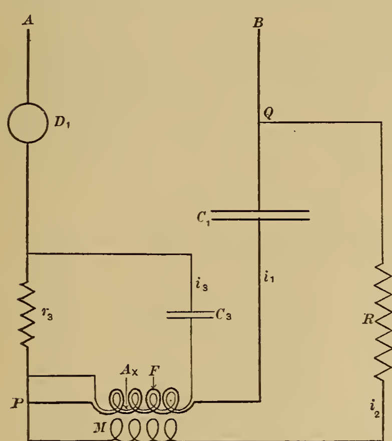

FIG. 5.—Connections for Method 3.

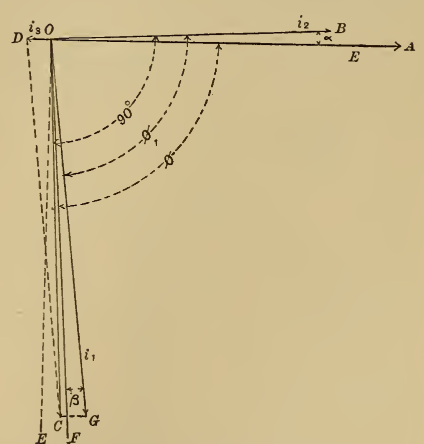

FIG. 6.—Phase diagram for Method 3.

wound on the outside of the main coil of the wattmeter, but thoroughly insulated from the latter. It is magnetically equivalent to the main coil, so that a current flowing in series through the two in opposite directions produces no magnetic field at the position of the moving coil.

A small variable resistance r_3, fig. 5, is inserted in series with the main current i_1, and an auxiliary condenser C_3 is placed in series with the compensation coil on the terminals of this resistance. The compensation current i_3 is equal to $pC_3\, r_3\, i_1$, and is nearly 90° in advance in phase of i_1; that brings it practically opposite in phase to i_2, as appears in fig. 6. We may compound i_1 and i_3 just as though they were in parallel in the same wire. The resultant is OC, and when i_3 has such a value that there is no deflection of the movable coil, CO is exactly 90° ahead of OB. Then, since $BOG=\phi_1$,

$$\cos\,\phi_1=\frac{\dot{i}_3}{\dot{i}_1}=pC_3r_3$$

Thus, it is unnecessary to measure E, i_1, i_2, or R, but only C_3 and r_3 in addition to the frequency of the current. To apply the corrections α and β to obtain the value of ϕ, it is of course necessary to determine C, r, and R approximately, as in method 1. The results obtained with five paraffined paper condensers are given in Table III. Nos. 2, 3, and 4 have a capacity of about 1.7 microfarads and 5 and 6 have a capacity of about 3.2 microfarads. These condensers were intended for 500 volt circuits. In these experiments, however, they were subjected to voltages between 910 and 1,386, most of the readings being taken at 1,260. This is a very convenient method.

Table III.—RESULTS BY METHOD 3.

Condenser employed.	E	$\dot{i}_1$	$p=2\pi n$	C_3	r_3	$\cos\phi_1=pC_3r_3$
	Volts.	*Amperes.*		*Farads.*	*Ohms.*	
Condenser No. 2....	1, 260	1. 673	745	1.70×10^{-6}	3. 46	0. 00438
Condenser No. 2....	1, 260	1. 686	745	1.70×10^{-6}	3. 48	. 00440
Condenser No. 3....	1, 260	1. 633	745	1.70×10^{-6}	3. 48	. 00440
Condenser No. 3....	1, 260	1. 643	745	1.70×10^{-6}	3. 48	. 00440
Condenser No. 4....	1, 386	1. 756	745	1.70×10^{-6}	3. 50	. 00443
Condenser No. 4....	1, 260	1. 673	745	1.70×10^{-6}	3. 52	. 00445
Condenser No. 2+3.	1, 260	3. 305	745	1.70×10^{-6}	3. 62	. 00458
Condenser No. 3+4.	1, 260	3. 286	745	1.70×10^{-6}	3. 66	. 00463
Condenser No. 5....	910	2. 329	745	1.70×10^{-6}	4. 35	. 00551
Condenser No. 5....	910	2. 319	745	1.70×10^{-6}	4. 36	. 00552
Condenser No. 5....	1, 260	3. 176	745	1.70×10^{-6}	4. 26	. 00540
Condenser No. 6....	910	2. 277	745	1.70×10^{-6}	4. 20	. 00531
Condenser No. 6....	1, 260	3. 146	745	1.70×10^{-6}	4. 12	. 00521
Condenser No. 5+6.	910	4. 810	745	1.70×10^{-6}	4. 50	. 00570

EFFECT OF THE SELF AND MUTUAL INDUCTANCE OF THE FIXED AND AUXILIARY COILS.

The self-inductance of the fixed coils of the wattmeter employed in these measurements is 0.075 millihenry, of the auxiliary coil 0.111 millihenry, and the mutual inductance of the two is 0.072 millihenry. At a frequency of 120, $p = 754$, and the reactance of the auxiliary coil is 0.083 ohm. This is wholly negligible in its effect upon the magnitude or phase of the compensation current $\dot{i}_3$. The self-inductance of the fixed coil has no appreciable effect upon the magnitude or phase of the main current $\dot{i}_1$. The mutual inductance between the coils, however, may have an appreciable effect upon the magnitude of the compensation current i_3 in the auxiliary coil, provided the electromotive force e_3 on the compensation circuit is small. Thus, the back e. m. f. in this circuit, due to the current $\dot{i}_1$ in the fixed coil, is pMi_1, and this is equal to 0.16 volt when $\dot{i}_1 = 3$ amperes and $p = 754$. In methods 4 and 5, e_3 was usually 45 to 70 volts, and hence the correction γ due to mutual inductance is only one or two units in the last decimal place of the values given for cos ϕ_1 in Tables IV and V. In method 3, however, e_3 was smaller and the correction is 5 to 6 in the last decimal place of the values of cos ϕ_1 in Table III.

The expression for cos ϕ_1 in which the mutual inductance of the fixed and auxiliary coils is taken into account, is

$$\text{For Method 3, } \cos\phi_1 = \frac{\dot{i}_3}{\dot{i}_1} = pC(r_3 - pM)$$

$$\text{For Method 4, } \cos\phi_1 = \frac{\dot{i}_3}{\dot{i}_1} = \frac{\left(\frac{\dot{i}_1}{\dot{i}_2} r_3 - pM\right)}{R_3 + r_3}$$

$$\text{For Method 5, } \cos\phi_1 = \frac{\dot{i}_3}{\dot{i}_1} = \frac{\left(\frac{e_3}{\dot{i}_1} - pM\right)}{R_3}$$

There is also a slight correction δ to be made in method 3, for the potential current $\dot{i}_2$, which also passes through r_3; that is, the resultant of $\dot{i}_1$ and $\dot{i}_2$ passes through r_3. This makes a slightly larger current and shifts the phase a little. The shifting of the phase, however, does no appreciable harm. The correction γ can be made insignificant by increasing r_3 and using an auxiliary condenser of correspondingly smaller capacity, while the correction δ is reduced to an insignificant quantity by having the potential current $\dot{i}_2$ sufficiently small in comparison with $\dot{i}_1$. In fact, it is easy under most circumstances to eliminate the three corrections β, γ, δ by properly proportioning the coils, leaving only a small correction α to be applied, due to the resistance of the fixed coils and connections.

METHOD 4.—SHUNT ON POTENTIAL CIRCUIT.

In this case, fig. 7, the compensation current $\dot{i}_3$ is shunted off from $\dot{i}_2$, the resistance r_3 being in the potential circuit through the moving

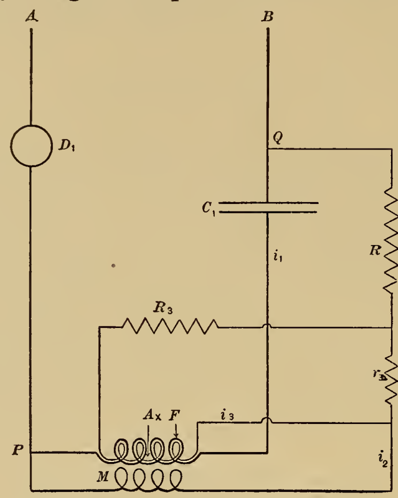

FIG. 7.—Connections for Method 4.

coil instead of the main circuit as in method 3. The compensation current is thus in phase with $\dot{i}_2$, but by reversing the terminals of the compensation coil it has the same effect as though it were opposite in phase. Thus, fig. 6 represents this method also.

Therefore, since

$$\dot{i}_3=\dot{i}_2\frac{r_3}{R_3+r_3},\quad \cos\phi_1=\frac{\dot{i}_3}{\dot{i}_1}=\frac{\dot{i}_2}{\dot{i}_1}\frac{r_3}{R_3+r_3}$$

Table IV.—RESULTS BY METHOD 4.

[E=1,260 volts.]

Condenser employed.	i_1	i_2	r_3	R_3	$\cos\phi_1$	Power factor uncorrected.
	Amperes.	*Ampere.*	*Ohms.*			*Per cent.*
Condenser No. 5	3.016	0.1396	507	3800	0.00544	0.544
Condenser No. 5	3.037	0.1729	507	4830	0.00540	0.540
Condenser No. 6	2.962	0.1735	507	5100	0.00529	0.529

METHOD 5. USING A TRANSFORMER FOR THE COMPENSATION CURRENT.

In this case the high potential winding of a transformer is joined across the terminals A B of the circuit, fig. 8, and the low potential

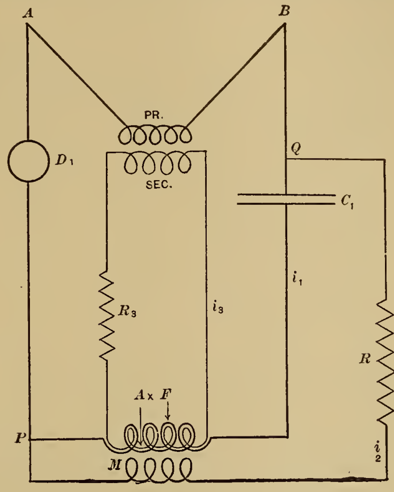

FIG 8.—Connections for Method 5.

coil supplies the current $\dot{i}_3$ to the compensation coil through a resistance R_3. This current is opposite in phase to $\dot{i}_2$, as in methods 3 and 4.

$$\dot{i}_3=\frac{e_3}{R_3}$$

$$\cos\,\phi_1=\frac{\dot{i}_3}{\dot{i}_1}=\frac{e_3}{\dot{i}_1R_3}$$

The small electromotive force e_3 may be determined by the ratio of transformation, or by direct measurement. It is very convenient to put a voltmeter on the secondary of the transformer and measure e_3 directly, and then get E by multiplying by the ratio of transformation. Some of the results obtained by this method are given in Table V.

The transformer used here is the potential transformer which was employed to get the voltage on the main condenser circuit. In these experiments it had a ratio of about 14, and hence, for 90 volts on the secondary there was 1,260 on the primary. The secondary consisted of two equal coils, and the compensation coil was joined to the terminals of one of them. The slight current used (about a hundredth of an ampere) did not alter the ratio of transformation of the transformer.

METHOD 6. SHUNT ON MAIN CIRCUIT.

This method corresponds to method 4, but the condenser is placed in the moving coil circuit, fig. 9, and a portion of i_1 is shunted off into

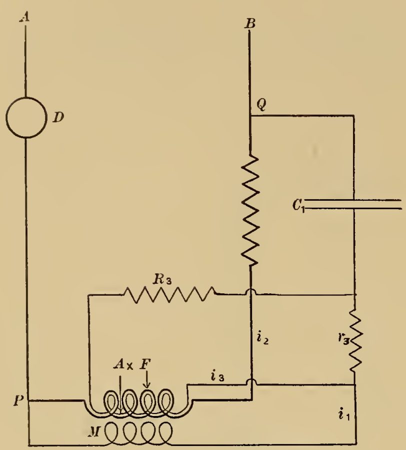

FIG. 9.—Connections for Method 6.

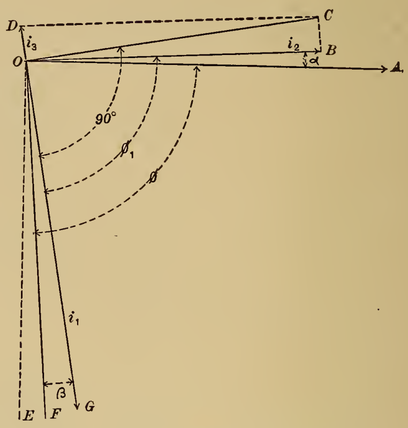

FIG. 10.—Phase diagram for Method 6.

the compensation coil. Thus, i_3 is opposite in phase to i_1 and is combined with i_2 so that the resultant of i_2 and i_3, $O\ C$, is 90° different in phase from i_1, fig. 10.

Thus:
$$i_3 = i_1 \frac{r_3}{R_3 + r_3}$$

This method is especially adapted to small condensers, where the current i_1, is very small and i_2 can be increased above its usual value.

Table V.—RESULTS BY METHOD V.

[E=1,260 volts=voltage on the condensers. e=45 volts=voltage on the compensation circuit.]

Condensers employed.	R_3	$i_3 = \frac{e}{R_3}$	i_1	$\cos \phi_1 = \frac{i_3}{i_1}$
FEBRUARY 11.		*Ampere.*	*Amperes.*	
Condenser No. 2	6000	0.00750	1.633	0.00459
Condenser No. 3	6250	0.00720	1.614	0.00446
Condenser No. 4	6000	0.00750	1.633	0.00459
Condenser No. 2+3	2950	0.01525	3.206	0.00476
Condenser No. 3+4	2965	0.01517	3.181	0.00477
Condenser No. 5	2560	0.01758	3.146	0.00559
Condenser No. 6	2730	0.01648	3.089	0.00533
Condenser No. 5+6	1310	0.03435	6.330	0.00542

Table V.—RESULTS BY METHOD V—Continued.

Condensers employed.	R_3	$i_3=\frac{e}{R_3}$	i_1	$\cos\phi_1=\frac{i_3}{i_1}$
FEBRUARY 12.		*Amperes.*	*Amperes.*	
Condenser No. 2	6290	0.00715	1.604	0.00445
Condenser No. 3	6530	0.00689	1.573	0.00438
Condenser No. 4	6290	0.00715	1.594	0.00448
Condenser No. 2+3	3080	0.01461	3.146	0.00464
Condenser No. 3+4	3050	0.01475	3.105	0.00475
Condenser No. 5	2650	0.01698	3.022	0.00561
Condenser No. 6	2790	0.01613	2.978	0.00541
Condenser No. 5	2680	0.01679	3.042	0.00552

METHOD 7. AUXILIARY CONDENSER ON TRANSFORMER.

This corresponds to method 5, where the transformer is used as a source of i_3, but as in method 6, the condenser under test is placed in the moving coil circuit. In order to bring the compensation current 90° out of phase with i_2, with which it is combined, an auxiliary con-

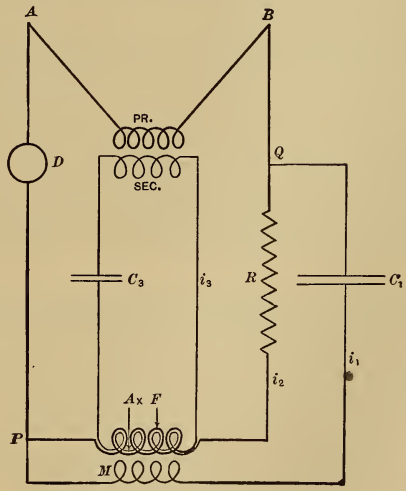

FIG. 11.—Connections for method 7.

denser C_3 is placed in the i_3 circuit instead of a resistance, fig. 11. The phases are shown in fig. 10. Thus:

$$i_3=pC_3e_3 \qquad i_2=\frac{E}{R}$$

$$\cos\phi_1=\frac{i_3}{i_2}=pC_3e_3\frac{R}{E}=\frac{pC_3R}{n}$$

where n is the ratio of transformation of the potential transformer.

Or, the formula may be written $\cos\phi_1 = \frac{pC_3e_3}{i_2}$, and e_3 determined directly by a voltmeter and i_2 by an ammeter or electrodynamometer.

Table VI.—Summary of Quantities to be Measured in the Various Methods.

Method 1	d	K	i_1	i_2							
Method 2	.....	......	....	....	p	L	R				
Method 3	......	......	....	....	p	......	......	r_3	C_3		
Method 4	......	......	i_1	i_2	....	......	......	r_3	......	R_3	
Method 5	......	......	i_1	....	....	......	......	....	......	R_3	e_3
Method 6	......	......	i_1	i_2	....	......	......	r_3	......	R_3	
Method 7	......	......	....	i_2	p	......	......	....	C_3	......	e_3

The quantities in Table VI are as follows:

d is the deflection of the electrodynamometer in scale divisions.

K is the constant of the electrodynamometer.

i_1 is the main current, through the condenser under test.

i_2 is the potential current, through the large resistance R.

p is 2π times the frequency.

L is the variable inductance added to the potential circuit.

R is the large potential resistance, as free as possible from inductance and capacity.

r_3 is the small resistance shunted by the auxiliary circuit.

C_3 is the auxiliary condenser, of constant value.

R_3 is the resistance in series with the auxiliary coil.

e_3 is the relatively small electromotive force supplying the auxiliary current.

Thus it appears that each method requires the determination of three or four quantities, and a choice of method will be determined in part by what instrumental facilities are available for the work. All the methods are capable of giving good results, but the null methods are more satisfactory than method 1, unless one has a wattmeter which is both sensitive and stable, and which has a nearly uniform field, so that the constant K does not vary too rapidly. All the methods require the two correction terms α and β to be applied to derive the true power factor $\cos\phi$ from the measured power factor $\cos\phi_1$. These two corrections are of opposite sign, as already explained, and should be small. In the experiments here described the resistance of the fixed coils of the wattmeter and the leads through which the main current flowed—that is, from P to Q with the condenser short circuited (figs. 1, 3, 5, etc.)—was 0.08 ohm. When the current is 1.6 amperes, $i^2{}_1r$ is 0.20 watt, and this requires a correction of about 0.0001 in the power factor. The correction for inductance in the moving-coil is of the same order of magnitude and hence the results of the measurements cited above as examples are not very different from the true power factor $\cos\phi$. At the time these measurements were made I did not have facilities for measuring the various quantities involved in these several methods with sufficient accuracy to make a crucial test of the methods. The results, however, show that the various methods agree substantially, and there is no reason to doubt the entire reliability of any of them. The values given above for the power factor are somewhat larger than those found for the same condensers by the calorimetric method,[a] but in those experiments the high electro-motive force impressed upon the condensers was obtained by resonance from a lower electro-motive force and the harmonics were therefore largely suppressed. In all the work described in this article the high electro-motive force was obtained by transforming up, and hence the harmonics were retained and magnified by the condensers. Since the power factor is higher for higher frequencies, it is higher for the harmonics and therefore for the distorted wave. This is one of the subjects to be investigated when this work is resumed.

By taking careful note of the temperature of the condensers, and determining accurately the frequency of the current and the exact values of the α and β corrections, we would obtain not only a crucial test of the various methods, but also data as to how the energy losses vary with the frequency, temperature, and wave form. I hope soon to repeat these measurements with improved apparatus, and hope to obtain results of sufficient precision to give valuable information of this character.

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[a] Rosa and Smith: Phys. Rev., Jan., 1899.