MEASUREMENT OF THE COLOR TEMPERATURE OF THE MORE EFFICIENT ARTIFICIAL LIGHT SOURCES BY THE METHOD OF ROTATORY DISPERSION[1]

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ABSTRACT

This paper describes and discusses a new method for measuring color temperatures between 3000 and 4000° absolute centigrade, and presents some data on the color temperature of the gas-filled tungsten lamp and the carbon arc.

The method is briefly this: Light from a source of known spectral distribution is modified by passage through a quartz plate between nicol prisms. The nicols and the quartz plate constitute in effect a blue filter of adjustable spectral transmission. The spectral transmission is adjusted by rotating one of the nicols. The source and the thickness of quartz are so chosen that the energy emerging from the second nicol has (for all positions to be considered) the spectral distribution characteristic of a complete radiator ("black body," Planckian formula). The light from the source whose color temperature is to be determined is color matched by rotating one of the nicol prisms. The temperature is obtained from this rotation by a method described in the paper.

The subject matter includes:

1. A thorough discussion of the reliability of the standard source.

2. Data upon the precision and accuracy of color temperature measurements at about 2850° K. It is shown that the probable error of a single observation is about ±6°. Individual observers' means (20 observations) differ from the mean of four observers by about 5°.

3. Data upon the color temperature of the gas-filled tungsten lamp as a function of efficiency up to nearly the melting point of tungsten. The filament failed at a temperature of about 3644° K, the efficiency being about 39 l. p. w.

4. Data upon the color temperature of the crater of the carbon arc indicate a color temperature of about 3780° K for solid carbons and 3420° K for cored carbons.

CONTENTS

[1] This paper was first presented at the Rochester Meeting of the Optical Society of America, Oct. 25, 1921. The author is indebted to Dr. K. S. Gibson, E. P. T. Tyndall, and H. J. McNicholas for their assistance in obtaining the data on precision and accuracy shown in Tables 1 and 2, and to Dr. M. Katherine Frehafer and Dr. Gibson for much assistance in computing.

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I. INTRODUCTION

The color temperatures of a number of sources of comparatively low or medium efficiency have been published [2] some years ago. Forsythe has recently communicated results on various lamps including gas-filled tungsten lamps at several efficiencies up to 27.3 lumens per watt.[3] So far as the author knows there are no data extant for higher temperatures than those given by Forsythe and no data duplicating the higher temperatures given by him. Also, so far as we know, no attempt has heretofore been made to determine the color temperature of the carbon arc by direct observation.

The author has previously described an apparatus which may be readily adapted to the measurement of very high color temperatures [4] by the rotatory dispersion method.

The purposes of the present paper are:

1. To illustrate the practical applicability of the rotatory dispersion method to the measurement of color temperatures between 3000 and 4000° K.

2. To present some data on the precision and accuracy of measurements of color temperature at about 2850° K.

3. To present an independent confirmation of Forsythe's data on the color temperature of gas-filled lamps.

4. To present new data on the color temperature of the gas-filled lamp (Mazda C) up to efficiencies of about 39 lumens per watt, which corresponds very nearly to the melting point of tungsten and the consequent failure of the filament.

5. To present some data on the color temperature of the crater of the carbon arc.

II. DEFINITION OF COLOR TEMPERATURE

In this paper, color temperature is understood to mean the temperature at which a hypothetical Planckian radiator ("black body") would emit light competent to evoke a color of the same quality (hue and saturation) as the light from the lamp under test.

The value 14 350 micron-degrees is assumed for the Planckian constant c_2 throughout this paper.[5]

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[2] Hyde and Forsythe, J. Franklin Inst., 183, pp. 353–354; 1917. E. F. Kingsbury, J. Franklin Inst., 183, pp. 781–782; 1917.

[3] Meeting of Am. Phys. Soc., Washington, April, 1921; Phys. Rev. (2) 18, p. 147; August, 1921.

[4] J. Op. Soc. Am., 5, pp. 178–183; March, 1921. Cf. also Phys. Rev. (2), 10, pp. 208–212; 1917, particularly the closing paragraph.

[5] Coblentz, B. S. Sci. Papers, No. 248; p. 470; 1916. Forsythe, J. Op. Soc. Am., 4, p. 332; 1920.

III. THE PRECISION AND ACCURACY OF MEASUREMENTS OF COLOR TEMPERATURE

Before proceeding further it is pertinent to introduce some data on the precision and accuracy of temperature measurements of lamps by the method of color matching in general, and quite aside from the particular features of the method to be described in this paper.

These data were obtained under the following conditions:

1. Type of photometric field: Circular and divided along a diameter (Martens photometer).

2. Angular size of whole field: 6°.

3. Absolute temperature: 2850° K.

4. Method: The observer adjusts lamp voltage to color match while an assistant records the voltages thus set. The differences between single settings and averages are computed and these residuals translated into temperature by means of the known relation between voltage and temperature.

Data on precision are shown in Table 1.

TABLE 1.—Precision of Color Matching Lamps at About 2850° K

[Circular photometric field divided on a diameter. Angular diameter of whole field about 6° (Martens Photometer). Observer sets voltage on test lamp to color match comparison standard. Assistant records voltages. Observed deviations in volts have been reduced to corresponding deviations in temperature. Data from four gas-filled 500-watt lamps, June 29-30, 1921.]

Observer	Average deviations (degrees centigrade) from means of 10 observations for set No. —								Average of average deviations	Probable error of 1 observation	Probable error of mean of 10
	1	2	3	4	5	6	7	8			
									°C	°C	°C
IGP	3.6	9.8	10.8	5.9	6.7	7.5	5.3	6.9	7.1	±6.3	±2.0
KSG	6.7	6.9	5.5	4.5	8.6	4.8	5.8	6.9	6.2	±5.5	±1.7
EPTT	11.0	5.3	6.5	4.3	4.2	7.1	8.8	6.0	6.6	±5.9	±1.8
HJM	4.2	5.0	5.7	7.9	3.4	5.1	4.2	6.1	5.2	±4.6	±1.3
Average									6.3	5.6	1.7

Data on the agreement among the final results of determinations by different observers on the same lamps are shown in Table 2. The systematic differences between observers shown in this table are probably due to the fact that for each observer a constant setting of the comparison lamp was used.

TABLE 2.—Departure of Individual Observer's Means (20 Observations) from Mean of 4 Observers

[Substitution method. Circular photometric field divided on a diameter. Angular diameter of whole field about 6° (Martens Photometer). Data from four gas-filled 500-watt lamps, June 29–30, 1921.]

Observer	Deviations (degrees centigrade) for lamp No. —				Average
	3254	3255	3256	3257	
IGP	−3.2	−4.9	−1.7	−7.0	−4.2
KSG	+9.4	+7.4	+6.7	+10.2	+8.4
EPTT	−9.2	−4.1	−6.9	−5.6	−6.4
HJM	+3.0	+1.8	+1.9	+2.5	+2.3
Average without regard to sign					5.3

IV. STANDARD SOURCE

1. DESCRIPTION OF LAMP

The fundamental reference standard on which the temperature scale in this paper is based is embodied in a particular 500-watt gas-filled concentrated-filament tungsten stereopticon lamp, designated as B. S. Lamp No. 1717, operated at 118.0 volts. The efficiency of this lamp as found by the photometric section, Bureau of Standards, was—

On April 3, 1917, at 118.0 v, 4.06 a, 15.6 l. p. w.

On June 17–18, 1921, at 118.0 v, 4.05 a, 15.75 l. p. w.

2. STANDARDIZATION BY SPECTRAL DISTRIBUTION

The spectral distribution of energy from this standard lamp as determined radiometrically by Dr. W. W. Coblentz, of the Bureau of Standards, in April, 1917, is shown by the circles in Figure 1. The continuous curves in the same figure show the theoretical spectral distribution of energy from a Planckian radiator at 2820 and 2850° K. It may be inferred from this figure that the color temperature of this lamp is approximately 2840 to 2850° K, but from mere inspection of the figure this conclusion is subject to considerable uncertainty.[6] A more precise value has been derived from the same data by the following procedure:

[6] In previous papers (J. Op. Soc. Am., 5, pp. 178–183, March, 1921, and B. S. Sci. Papers, No. 417, 17, pp. 231–265, 1921), the color temperature 2830° K was inferred from these same data. This value was merely a rough approximation as inferred from plotting the data on a small scale and is not accurate enough for the present purpose. The revised value given in the present paper results from a more careful examination of the data and a more precise and reliable method of reducing it.

1. The wave length of the center of gravity of a spectral distribution of light is defined as

$$\lambda_c = \frac{\int V \cdot E \cdot \lambda d\lambda}{\int V \cdot E d\lambda}$$

where λ = wave length;

E = energy per unit wave length for wave length, λ;

V = visibility of radiant energy for wave length, λ.

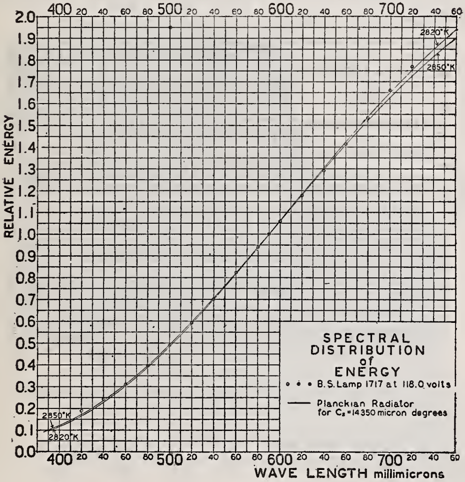

FIG. 1.—*Spectral distribution of energy, B. S. Lamp No. 1717 and Planckian radiator at 2820° and 2850° K*

(The graphic significance of this definition may be explained by reference to Fig. 2. λ is plotted as abscissa. VE is plotted as ordinate. The different curves represent spectral distributions of light from a Planckian radiator at different temperatures. For any temperature, λ_c is the λ-coordinate of the center of gravity of a thin template of uniform density bounded by the λ-axis and the distribution curve for that temperature.)[7]

[7] Compare also, Jour. Op. Soc. Am., 4, pp. 389–401, 1920. B. S. Sci. Papers, No. 417, 17, p. 234, 1921.

2. λ_c has been computed for a Planckian radiator at various temperatures and plotted as a function of temperature as shown in Figure 3. These computations have been made by arithmetic throughout by the formula

$$\lambda_c = \frac{\Sigma V \cdot E \cdot \lambda}{\Sigma V \cdot E}$$

taking values of V, E, and λ at intervals of 10 millimicrons, and are more accurate than the *graphic* integrations used in previous papers.[8]

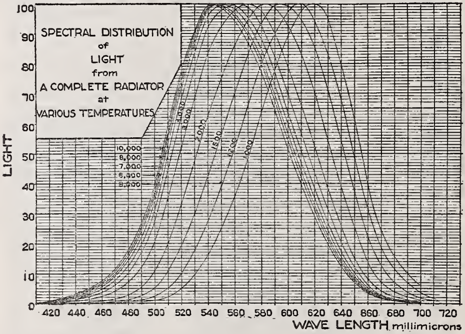

Fig. 2.—*Spectral distribution of light, Planckian radiator at various temperatures.*

Energy by Planck's Formula (C_2=14350).
Visibility:
λ, 560–650.
H. E Ives, Phil. Mag. Dec. 1912, p. 859.
λ, 410–550 and 660–710.
Hyde, Forsythe & Cady, Jour. Frank.
Inst. 48, p. 87.
Numbers attached to curves indicate temperatures in degrees K.

3. λ_c has likewise been computed in the same way for the original experimental data on the spectral distribution of energy from the lamp, and this value of λ_c used to derive the color temperature from the relation between λ_c and color temperature shown in Figure 3. The color temperature so derived is[9]

2848° K.

[8] J. Op. Soc. Am., 4, pp. 389–401; 1920. B. S. Sci. Papers, 417, 17, pp. 234–235, 1921.

[9] It is to be observed that while the visibility of energy enters into the formulas used, it does *not* enter in such a way as to affect the temperature found so long as the same values of visibility known to be approximately correct are used in determining all values of λ_c considered and the spectral distribution approximates Planck's formula. The values of visibility actually used throughout the present paper are shown by the solid curve in Fig. 8, J. Op. Soc. Am., 4, p. 471.

This value is the weighted mean of three separate computations, and from their agreement it is estimated that the uncertainty of this result due to approximations in computation is less than 3°.

The sensibility and accuracy of this method are clearly demonstrated by the consistency of the several points determining the curve at about 2850° K, Figure 3. Judging from this, the uncertainty is less than 5°.

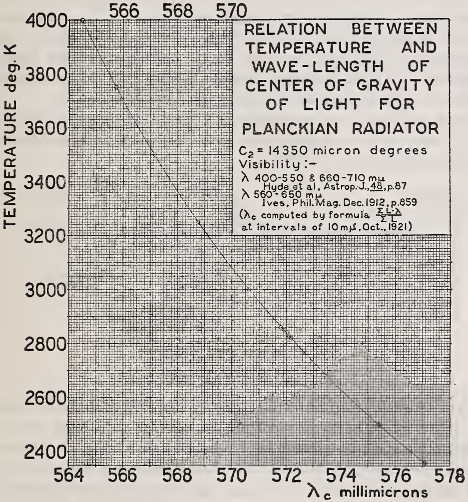

FIG. 3.—*Relation between temperature and wave-length of center of gravity, Planckian radiator*

3. STANDARDIZATION BY COLOR MATCH WITH PLANCKIAN RADIATOR (NELA LABORATORY)

In order to compare this standard with the color temperature scale of the Nela Research Laboratory, a 500-watt gas-filled lamp of the type now used as photometric standards was accurately color-matched with B. S. Lamp 1717 at 118.0 volts by a substitution method. The voltage for color match was determined by 20 settings by *each* of four observers. The resulting mean voltage was 101.0 v, for which the current was 4.097 a. The lamp was then sent to the Nela Research Laboratory and its color temperature by color match with a "black body" was found to be[10]

2848° K at 101.0 v, 4.099 a.

4. CONCLUSION AS TO STANDARD

On the basis of the good agreement between the color temperature derived from Coblentz's isothermal data and that independently found by color matching with a "black body" at the Nela Research Laboratory, we may define our standard for future reference in a more fundamental way than by referring to a particular lamp, as we have at the beginning of this discussion.

Our standard source is accordingly a source closely approximating the Planckian spectral distribution in the visible spectrum and having a color temperature of 2848° K.

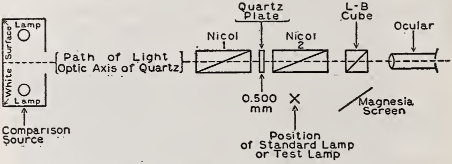

Fig. 4.—*Essential parts of apparatus*

V. EXPERIMENTAL METHOD

The essential feature of the rotatory dispersion method is this: A quartz plate between nicol prisms, serving as a light filter of adjustable spectral transmission,[11] is used to modify the color of a comparison source so as to match the unknown, the constants of the apparatus being chosen so that the spectral distribution of the light emerging from the quartz-nicol train is always represented by the Planckian formula. Colorimetrically, the experiment is equivalent to varying the temperature of a "black body" until it is color matched with the lamp in question and then noting the temperature. The essential parts and arrangement of the apparatus are shown in Figure 4. The experimental procedure is then as follows: The standard lamp of known spectral distribution is placed at X so as to illuminate part of the photometric

[10] Letter, W. E. Forsythe, Nela Lab., to I. G. Priest, Aug. 11, 1921.

[11] J. Op. Soc. Am., 4, pp. 485–486.

field. The quartz plate being removed, the current in the comparison lamps in the box is adjusted to give a color match in the photometric field. This current is thenceforth maintained constant. The source whose color temperature is to be measured is then substituted for the standard lamp; the quartz plate is inserted between the nicols and nicol No. 2 is rotated (angle, ϕ) to produce a match of color quality. (A brilliance match is of course simultaneously made by other nicols, not shown in Fig. 4.)

The actual apparatus used was the Arons Chromoscope.[12] The Lummer-Brodhun cube is set so that the field has the form shown in Figure 5. The visual angle of the circle (comparison light) is about 3.5°. This form of field appeared to be somewhat more sensitive than the *concentric* field for matching of color quality, although its particular odd shape is not to be recommended.

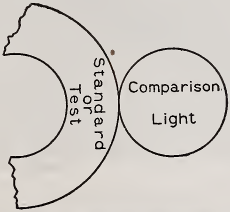

Fig. 5.—*Form of photometric field*

VI. METHOD OF CONSTRUCTING THE TEMPERATURE SCALE

Let ϕ (measured from extinction position with quartz removed, and in same direction as the rotation by the quartz) be the angle through which nicol No. 2 is rotated to obtain a color match.

The method of constructing the temperature scale corresponding to the instrument reading (ϕ) is a refinement and extension of that previously published.[13]

The spectral distributions of energy corresponding to different values of ϕ are shown in Figure 6, together with the spectral distributions of a Planckian radiator at various temperatures.

[12] Ann. der Phys. (4), **39**, pp. 545–568; 1912.
[13] J. Op. Soc. Am., **5**, pp. 178–183; March, 1921.

Inspection of this figure shows:

1. The distributions obtained by rotatory dispersion approximate very closely to the theoretical distributions by the Planckian formula.

2. The temperature corresponding to any value of ϕ can be inferred *approximately* from simple inspection of this figure, although this method of establishing the relation between ϕ and temperature is not sufficiently precise for our present purpose.[14]

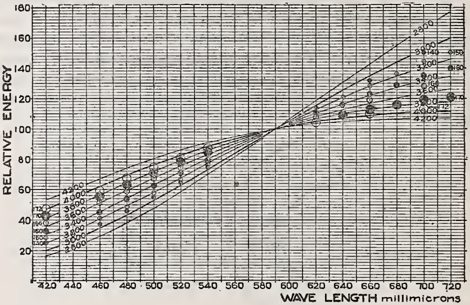

FIG. 6.—*Spectral distributions of energy, Planckian radiator at various temperatures compared with distributions obtained by rotatory dispersion*

The solid black curves represent Planck's formula with $C_2=14350$. The numbers attached to these curves indicate temperatures in degrees K.

The various circles represent distributions obtained by the arrangement shown in Fig. 4. Each different style and size of circle refers to a particular value of ϕ; and the numbers attached to the circles indicate values of ϕ in circular degrees.

In all cases, energy =100.0 at wave lengths 590 (arbitrary convention).

The precise relation between ϕ and temperature has been obtained as follows:

1. λ_c has been computed for the spectral distributions corresponding to different values of ϕ (Fig. 6) in the same way as for the standard lamp and the Planckian radiator as described above.

2. Temperatures corresponding to these values of λ_c have been read from Figure 3 and plotted as a function of ϕ in Figure 7.

Figure 7 thus obtained now serves as a calibration curve for deriving color temperature from experimentally observed values[15] *of* ϕ.

[14] The curve shown in Fig. 6, J. Op. Soc. Am., 5, p. 182, was obtained by this simple process of inspection.

[15] Cf. "Experimental method" above.

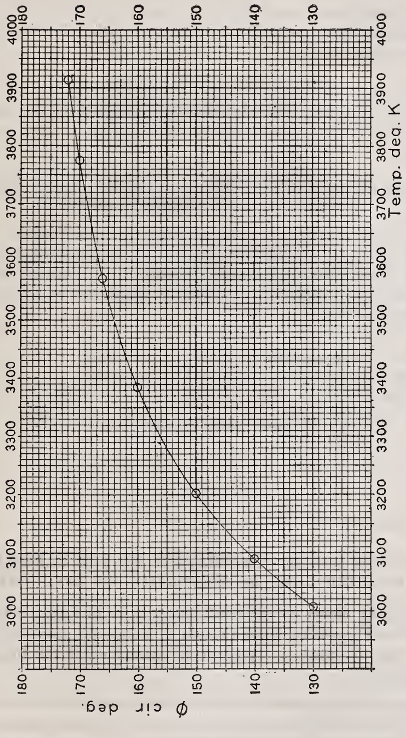

Fig. 7.—*Relation between ϕ and temperature*

VII. CHECK MEASUREMENTS

1. CHECK OF THE METHOD WITH RADIOMETRIC DETERMINATIONS

The color temperature of B. S. Lamp 1716 (a 500-watt gas-filled stereopticon lamp, like 1717) at 22.0 l.p.w., has been found by this method to be

3082° K (mean of 30 observations).

The temperature derived by means [16] of λ_c from the spectral energy distribution determined by Coblentz [17] is

3086° K.

2. CHECK OF THE METHOD WITH COLOR TEMPERATURE DETERMINATIONS BY THE NELA RESEARCH LABORATORY

The color temperature of a 900-watt gas-filled "Movie" lamp at 22.7 l.p.w., has been independently determined by Forsythe at the Nela Research Laboratory, using their methods, and by the author at the Bureau of Standards, using the present method. The results follow.[18]

	°K
Nela:	
Before B. S. measurement	3091
After B. S. measurement	3083
Mean	3087
Bureau of Standards (each value is mean of 10 observations)	3090
	3095
	3067
	3087
	3093
	3079
Mean	3085

VIII. THE COLOR TEMPERATURE OF THE GAS-FILLED TUNGSTEN LAMP AS A FUNCTION OF EFFICIENCY

The data shown by the small open circles in Figure 8 refer to a 500-watt gas-filled lamp (Mazda C National Lamp Works) of the type now used as a photometric standard at the Bureau of Standards.

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[16] By the same method as described above for deriving the color temperature of the standard lamp No. 1717 from the radiometric data.

[17] Coblentz's determinations on Lamp 1717 were made in April, 1917. His determinations on Lamp 1716 were made in December, 1918, after readjusting his apparatus.

[18] Letter, Forsythe to Priest, July 29, 1921.

These data were obtained in the following way:*

1. Two lamps of nearly identical characteristics (equal efficiencies at equal voltages) were selected.

2. One of these (B. S. 3261) was used to determine efficiency as a function of voltage for increasing voltage until the filament failed.[19] This filament failed at 200 volts, the efficiency at 195 v being 38.2 l.p.w.

3. The other (B. S. 3260) was used to determine color temperature as a function of voltage at increasing voltages until the filament failed, at 206 volts.

4. Correlating the data on the two lamps, color temperature is shown as a function of efficiency in Figure 8.

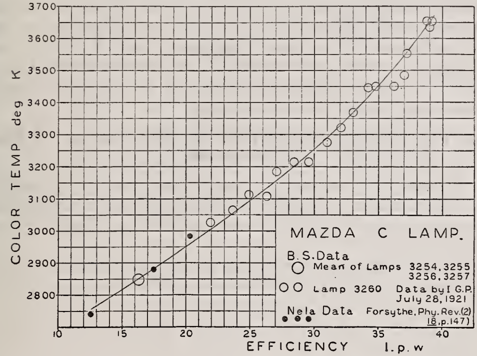

FIG. 8.—*Color temperature of 500-watt gas-filled photometric standard lamp as a function of efficiency*

Some of Forsythe's previously published data[20] are also plotted in this figure. The agreement is as good as could be expected considering the different lamps involved.

In order to avoid burning the lamp longer than absolutely necessary at any one voltage (which would have shortened its life and forestalled observations at the highest temperatures), accuracy was sacrified for speed in these observations. The observations of temperature were made as rapidly as possible and only five at

[19] These determinations were made by Ben S. Willis, Photometric Section of this Bureau.

[20] Phy. Rev. (2), 18, p. 147; August, 1921.

each voltage. On this account the points (Fig. 8) depart from a smooth curve. It is believed nevertheless that the curve which has been drawn through them is not in error, on this account, by more than 10° at any point. These data are, however, presented as a preliminary roughing out of the color temperature—efficiency relation rather than a precision determination.

At the highest efficiency attained (39.2 l.p.w.) the color temperature observed was 3644° K. The accepted value [21] for the true temperature of the melting point of tungsten is 3673° K. No precise relation between the true temperature and the color temperature of gas-filled lamps can be stated; it appears, however, that the present determination is in as close accord with the accepted melting point as could be expected.[22] After the filament failed, a spectroscopic analysis of it was made by the spectroscopy section of the Bureau of Standards. The report states that "so far as can be determined spectroscopically, the filament is pure tungsten." The lamp makers also state that the filaments of such lamps are "substantially pure tungsten."

IX. THE COLOR TEMPERATURE OF THE CRATER OF THE CARBON ARC

Previous work has shown[23] that the temperature of the crater of the arc varies by nearly 200° C, dependent upon the carbons particularly, and upon the current and other conditions to a less extent.

The most reliable of our data by the rotatory dispersion method indicate color temperatures as follows for the crater of a 65-volt, 10-ampere arc: Solid carbons 3780° K (mean of 50 observations); cored carbons 3420° K (mean of 50 observations). These means are considered uncertain by about 50°.

So far as we know there are no previous determinations of "color temperature" of the arc with which to compare these results. Waidner and Burgess[24] give 3680° to 3720° as "black body brightness temperature."

The method described would be convenient and suitable to use in an extensive determination of the temperature of the arc under various conditions.

WASHINGTON, November 4, 1921.

[21] Worthing, Phys. Rev. (2), **10**, p. 392; 1917.
[22] Cf. Forsythe, Phys. Rev. (2), **18**, p. 147; 1921.
[23] Waidner and Burgess, B. S. Bull., **1**, pp. 109–124; 1904.
[24] B. S. Bull., **1**, p. 123.