SOME EFFECTS OF THE DISTRIBUTED CAPACITY BETWEEN INDUCTANCE COILS AND THE GROUND

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ABSTRACT

It is well known that the effective capacity of two condensers which are connected in series is $\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}$ if $\mathfrak{C}_1$, $\mathfrak{C}_2$ are the capacities of the two condensers. In the paper here presented it is shown that this rule does not always hold, and it is found that another more general relation is true. This relation depends upon the method of measuring the effective capacity. If the method used is that of resonance adjustment and if a coil symmetrical as to its two terminals is used in the resonance experiment, then it is shown mathematically and verified experimentally that $\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}$ is a linear function of $\frac{1}{\mathfrak{C}_1+\mathfrak{C}_2}$.

When an inductance coil is used in electrical circuits, its distributed capacity affects its behavior in various ways. Most of these are due to the capacity between various parts of the coil as well as to the capacity of the coil to ground. The two are usually indistinguishable. Thus, the effective capacity of a coil depends both on the capacity between parts of the coil and on the capacity of parts of the coil to ground. The same is true for the resistance. There appears to be one effect, however, which depends on the effective capacity of the coil to ground, primarily. This effect is observed when two condensers connected in series are connected across the coil terminals and when the common terminal of the two condensers is grounded.

Let $\mathfrak{C}_1$, $\mathfrak{C}_2$ be the capacities of the two condensers. Vary either $\mathfrak{C}_1$ or $\mathfrak{C}_2$ until resonance is obtained. If the coil had no capacity to ground, resonance would be obtained whenever $\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}$ had a proper value, depending on the frequency and on the inductance of the coil, but independent of the particular values of $\mathfrak{C}_1$ and $\mathfrak{C}_2$.

If the coil has capacity to ground, this is no longer true. In this case another more complicated relation takes the place of the relation

$$\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}=\text{const.}$$

The nature of this relation may be found by representing the distributed capacity to ground by a series of capacities, as in Fig. 1. The whole coil is here divided into n sections, and the capacity of each to ground is represented by a lumped capacity

at its end, as e. g., by C_2. The line GH represents the ground. The inductances of the various sections are written, respectively, as L_1, L_2, $\cdots$ L_n. The resistances of the same sections are denoted by R_1, R_2, $\cdots$ R_n. The emfs induced in the sections are written as $E_1\epsilon^{j\omega t}$, $E_2\epsilon^{j\omega t}$, $\cdots$ $E_n\epsilon^{j\omega t}$, where E_1, E_2 $\cdots$ E_n are supposed to be independent of the time, where ϵ is the base of natural logarithms and where $j=\sqrt{-1}$. It is understood in writing the emfs in the complex form just used that the real part of the complex quantity written is considered only, and that in all the equations that follow the real part only of both sides will be considered. The mutual inductance between the sections having inductances L_1, L_2 will be written M_{12} and a similar notation will be used for all other sections.

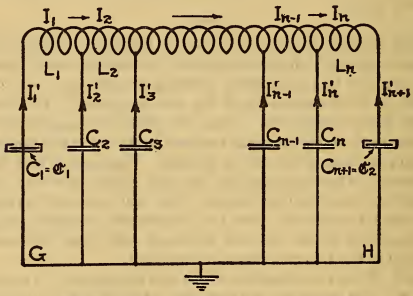

FIG. 1.—*Equivalent circuit for coil and its distributed capacity to ground*

The currents in L_1, L_2, $\cdots$ L_n are written

$$I_1\epsilon^{j\omega t},\ I_2\epsilon^{j\omega t},\ \cdots\ I_n\epsilon^{j\omega t}$$

The currents in C_1, C_2, $\cdots$ C_{n+1} are written as

$$I'_1\epsilon^{j\omega t},\ I'_2\epsilon^{j\omega t},\ \cdots\ I'_{n+1}\epsilon^{j\omega t}$$

Kirchhoff's laws, when applied to the network of Fig. 1, give

$$\begin{cases} I'_1=I_1 \\ I'_2+I_1=I_2 \\ I'_3+I_2=I_3 \\ - - - - - \\ - - - - - \\ I'_n+I_{n-1}=I_n \\ I_n=-I'_{n+1} \end{cases} \tag{1}$$

and also

$$
\left\{
\begin{aligned}
&(L_1 j\omega + R_1) I_1 + M_{12} j\omega I_2 + M_{13} j\omega I_3 + \cdots + M_{1n} j\omega I_n \\
&\qquad - \frac{I'_2}{jC_2\omega} + \frac{I'_1}{jC_1\omega} = E_1 \\
&M_{12} j\omega I_2 + (L_2 j\omega + R_2) I_2 + M_{23} j\omega I_3 + \cdots + M_{2n} j\omega I_n \\
&\qquad - \frac{I'_3}{jC_3\omega} + \frac{I'_2}{jC_3\omega} = E_2 \\
&M_n j\omega I_1 + \cdots\cdots + (L_n j\omega + R_n) I_n \\
&\qquad - \frac{I'_{n+1}}{jC_{n+1}\omega} + \frac{I'_n}{jC_n\omega E_n} = E_n
\end{aligned}
\right. \tag{2}
$$

or using (1)

$$
\left\{
\begin{aligned}
&\left(L_1 j\omega + R_1 + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}\right) I_1 + \left(M_{12} j\omega - \frac{1}{j\omega C_2}\right) I_2 + M_{13} j\omega I_3 \\
&\qquad + \cdots + M_{1n} j\omega I_n = E_1 \\
&\left(M_{12} j\omega - \frac{1}{j\omega C_2}\right) I_1 + \left(L_2 j\omega + R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_3}\right) I_2 \\
&\qquad + \left(M_{23} j\omega - \frac{1}{j\omega C_3}\right) I_3 + \cdots + M_{2n} j\omega I_n = E_2 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&M_{n1} j\omega I_1 + \cdots + \left(M_{n,n-1} j\omega - \frac{1}{j\omega C_n}\right) I_{n-1} \\
&\qquad + \left(L_n j\omega + R_n + \frac{1}{j\omega C_n} + \frac{1}{j\omega C_{n+1}}\right) I_n = E_n
\end{aligned}
\right.
$$

Resonance occurs when the determinant of this system of equations vanishes, provided in that determinant all the resistances are set equal to zero. This is only approximately true, but is a good approximation if the resistances are small, because for small values of the resistances the position of resonance is independent of the resistances. Thus, the condition for resonance is

$$
\begin{vmatrix}
L_1 - \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{\omega^2}, & M_{12} + \frac{1}{C_2\omega^2}, & M_{13}, \cdots\cdots & M_{1n} \\
M_{12} + \frac{1}{C_2\omega^2}, & L_2 - \left(\frac{1}{C_2} + \frac{1}{C_3}\right)\frac{1}{\omega^2}, & M_{23} + \frac{1}{C_3\omega^2}, \cdots & M_{2n} \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
M_{1n} \cdots\cdots & \cdots\cdots & \cdots\cdots & , L_n - \left(\frac{1}{C_n} + \frac{1}{C_{n+1}}\right)\frac{1}{\omega^2}
\end{vmatrix} = 0
$$

As an equation in C_1 and C_{n+1} it has the form

$$\frac{A}{C_1}+\frac{B}{C_{n+1}}+\frac{D}{C_1C_{n+1}}+F=0 \tag{3}$$

In particular, if the surroundings of the coil are symmetrical with respect to the two ends

$$A=B,$$

and hence,

$$A+\frac{D}{C_1+C_{n+1}}+F\frac{C_1C_{n+1}}{C_1+C_{n+1}}=0 \tag{4}$$

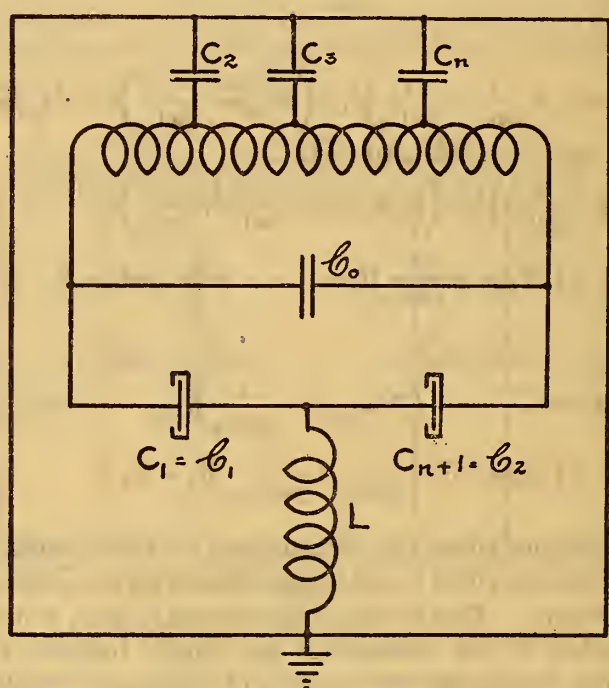

FIG. 2.—*Equivalent circuit for coil, its self-capacity, and its distributed capacity to ground*

Therefore, if for different values of C_1 proper values of C_{n+1} be found so as to secure resonance, and if $\frac{C_1C_{n+1}}{C_1+C_{n+1}}$ be calculated, a constant number will not be obtained. However, if its value be plotted against the value of

$$\frac{1}{C_1+C_{n+1}}$$

a straight line must result according to (4).

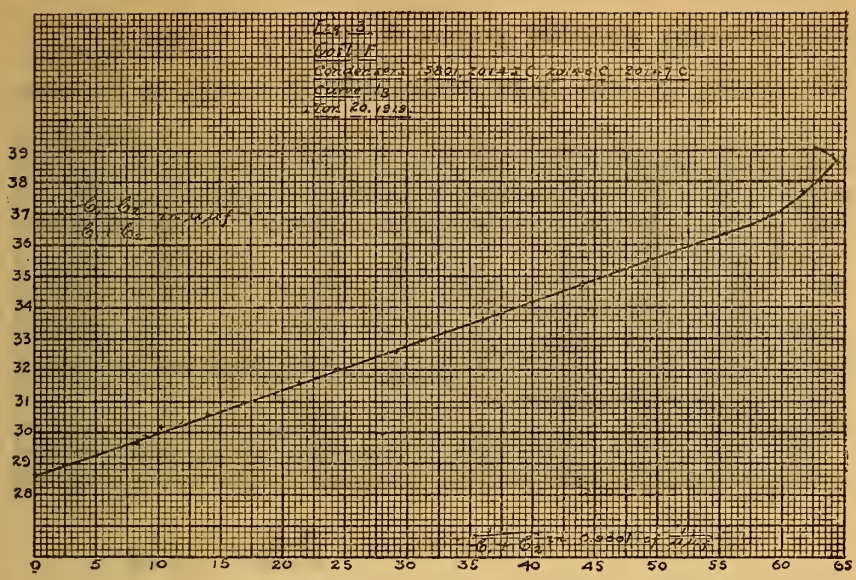

FIG. 3.—*Observed results for variation of* $\frac{\mathfrak{C}_1\,\mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}$ *with* $\frac{1}{\mathfrak{C}_1+\mathfrak{C}_2}$

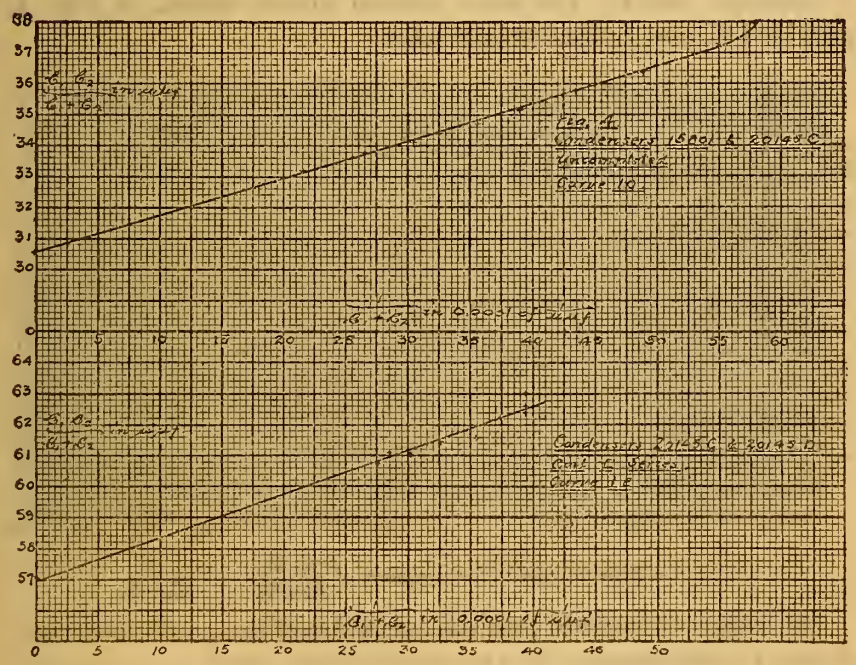

FIG. 4.—*Observed results for variation of* $\frac{\mathfrak{C}_1\,\mathfrak{C}_2}{\mathfrak{C}_1+\mathfrak{C}_2}$ *with* $\frac{1}{\mathfrak{C}_1+\mathfrak{C}_2}$

Curves given as IA, IB, IC, ID, IE (Figs. 3, 4, 5, and 6) give experimentally obtained graphs of $\frac{1}{C_1+C_{n+1}}$ against $\frac{C_1\ C_{n+1}}{C_1+C_{n+1}}$. The graphs are seen to be nearly straight except at the ends,

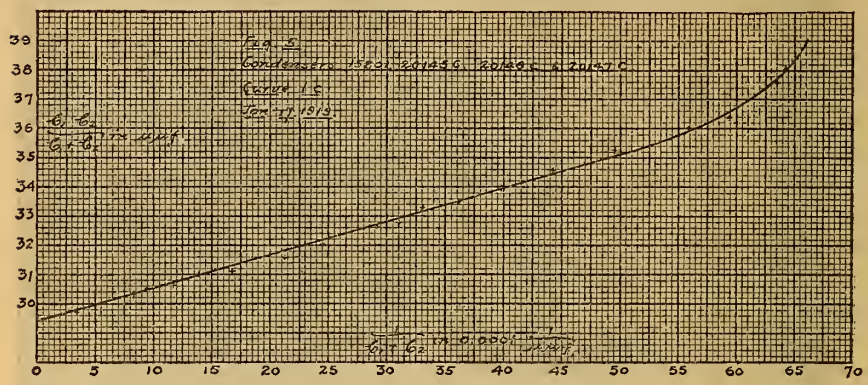

FIG. 5.—*Observed results for variation of* $\frac{\mathbb{C}_1\,\mathbb{C}_2}{\mathbb{C}_1+\mathbb{C}_2}$ *with* $\frac{1}{\mathbb{C}_1+\mathbb{C}_2}$

where they have a bend. This bend is probably due to a dissymmetry in the surroundings.

By similar reasoning it is found that a relation of the form (3) is satisfied also for the more general circuit shown in Fig. 2, provided $\mathbb{C}_0$ and L are kept constant.

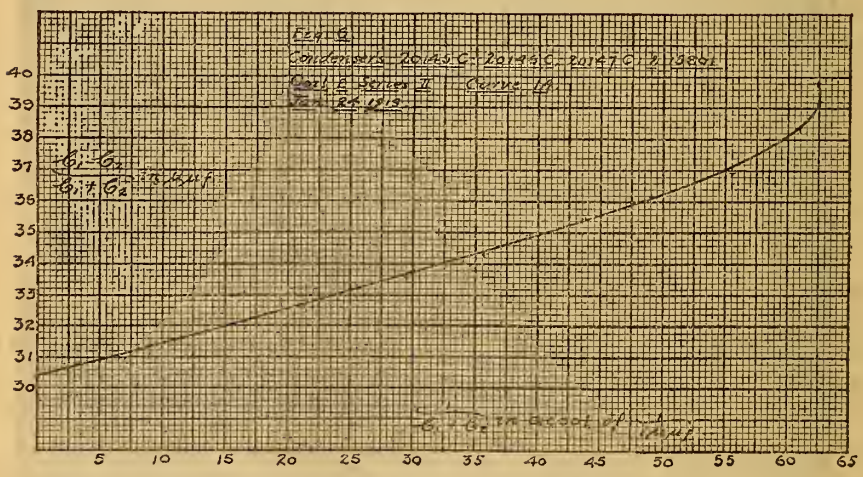

FIG. 6.—*Observed results for variation of* $\frac{\mathbb{C}_1\,\mathbb{C}_2}{\mathbb{C}_1+\mathbb{C}_2}$ *with* $\frac{1}{\mathbb{C}_1+\mathbb{C}_2}$

This seems to indicate that the curvature of the graphs observed is due to dissymmetry in the surroundings, because distributed capacity effects between different parts of the coil are probably taken account of by the above-mentioned calculation.

SUMMARY

When two condensers connected in series with common terminal grounded are connected across the terminals of an inductance coil, their effective capacity in series is not equal to their effective capacity so far as resonance of the coil is concerned. If the capacity of one condenser is $\mathfrak{C}_1$ and of the other $\mathfrak{C}_2$, the quantity

$$\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1 + \mathfrak{C}_2}$$

is the effective capacity of the two condensers in series. As stated, it does not stay constant when $\mathfrak{C}_1$ is changed arbitrarily and $\mathfrak{C}_2$ is readjusted for resonance. However, $\frac{\mathfrak{C}_1 \mathfrak{C}_2}{\mathfrak{C}_1 + \mathfrak{C}_2}$ is linearly related to $\frac{1}{\mathfrak{C}_1 + \mathfrak{C}_2}$.

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