AIR FORCES ON CIRCULAR CYLINDERS, AXES NORMAL TO THE WIND, WITH SPECIAL REFERENCE TO DYNAMICAL SIMILARITY

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I. INTRODUCTION

One of the most important questions in present-day aerodynamics is that of finding a satisfactory method for passing, by computation alone, from the forces observed on models held at rest in a wind stream to the forces which will act on the full-sized bodies moving into free still air. Two separate questions are involved: (1) What is the effect on the force on a particular model of the change from the confined and somewhat turbulent current in a wind tunnel to free still air? (2) What is the effect of a mere change of size, either in still air or in the wind tunnel? The latter question is the more important because most of our quantitative experiments are made in wind tunnels. A good deal of work has been done on the subject with reference to aerofoils and other shapes of immediate practical interest, but no extensive investigation has been published on bodies of simple geometrical shape within a large range of speed and size, and we have little information on the difference between free-air and wind-tunnel experiments without change of size, the two questions being usually treated together without any attempt to distinguish between them. The present paper presents the results of experiments in the 54-inch wind tunnel of the Bureau of Standards on the resistances of cylinders with their axes normal to the wind; it is therefore a contribution to our knowledge of the effects of change of size of the body in a particular wind tunnel.

II. THE STILL-AIR RESISTANCE EQUATION

Let us suppose that a rigid body of given shape and in a given attitude is moving forward into free still air at the constant speed V. The forces exerted by the air on the body—for example, the head resistance R—will evidently depend on the speed V and on the size of the body, which may be specified by the value of some linear dimension L, such as the diameter, if the body is a cylinder. The only other quantities the forces can depend on are the properties of the air. Of these, the density ρ and viscosity μ may evidently be of importance. If the speed is high enough to cause much compression of the air in front of the body, the compressibility must also be taken into consideration. It is commonly assumed that no other physical properties of still free air have any sensible influence on the problem in hand, and if this assumption is correct, the head resistance is determined by the other quantities enumerated and is connected with them by an equation containing no further quantities except dimensionless, numerical constants.

It is convenient to replace the viscosity μ by the kinematic viscosity $\nu = \mu/\rho$, and to specify the compressibility by the speed of sound S, which is determined by the density and the compressibility. With this notation, the assumption just mentioned amounts to saying that the equation

$$R = F(V, L, \rho, \nu, S) \tag{1}$$

is complete.

But if equation (1) is complete, the requirement that all its terms must be of the same dimensions enables us to say, as a mathematical certainty,[1] that it is reducible to the form

$$R = \rho L^2 V^2 f\left(\frac{VL}{\nu}, \frac{V}{S}\right). \tag{2}$$

[1] We shall assume that the reader who is interested in the deduction of the equations used in the present paper will, if not already familiar with the dimensional method, look it up in some of the references to be given below, as a detailed account of the method lies outside the scope of this paper. The best discussion of the subject in its relation to aerodynamics may be found in the reports of the British Advisory Committee for Aeronautics, **3**, p. 30; 1911–12. Valuable notes by Rayleigh also appear in **1**, p. 38; 1909–10, and **2** p. 26; 1910–11; and many instructive examples of the use of dimensional reasoning may be found scattered through these reports, as well as through Rayleigh's papers. Some very illumininating remarks on the use of the dimensional method may be found in Rayleigh's paper on "The Principle of Similitude," in Nature of March, 1915. A more detailed discussion of the algebraic aspect of the subject is given in a paper by Riabouchinski in No. 4 of the Koutchino Bulletins, 1912, quoted from L'Aèrophile of September 1, 1911. For the purpose of the present paper, reference to the excellent presentation by Buckingham in Trans. Am. Soc. Mech. Eng., **37**, p. 263, 1915, is adequate.

If air were incompressible, S would disappear from equation (1), and equation (2) would have the simpler form

$$R=\rho L^2 V^2 f\left(\frac{VL}{\nu}\right). \tag{3}$$

At low speeds, where the pressure differences set up by the motion are very small, it is evident that compressibility can not have any appreciable effect; and if the assumptions mentioned above are justified, equation (3) will be valid for motion into free still air. At high speeds—for instance, for projectiles—compressibility becomes an important element in the phenomena of air resistance; but it appears from experiment and is commonly assumed to be true that at airplane speeds or lower the effects of compressibility are negligible, so that equation (3) is equivalent to equation (2). Adopting this assumption and setting

$$f\left(\frac{VL}{\nu}\right)=C', \tag{4}$$

we have the familiar form of air-resistance equation:[2]

$$R=C'\rho L^2 V^2, \tag{5}$$

which may also be written

$$R=C\rho A V^2, \tag{6}$$

where A is the area of the projection of the body on a plane normal to the direction of motion. For a given value of $\frac{VL}{\nu}$ the resistance coefficient C or C' is characteristic of the given shape and attitude. If the circumstances are such that viscosity plays no part in determining the resistance, $f\left(\frac{VL}{\nu}\right)$ does not depend on ν and is therefore a mere constant, as is C. Otherwise C will vary with $\frac{VL}{\nu}$.

If equation (3) is satisfied, equation (6) evidently enables us to determine the value of C from experiments on a small body or model and then compute from this value the resistance of a larger body at the same value of $\frac{VL}{\nu}$, the two bodies being said to be dynamically similar when $\frac{VL}{\nu}$ is made the same for both. We have not enough accurate data on the resistances of bodies in free still air to enable us to test the accuracy of the foregoing air-resistance equation generally, and so judge the adequacy of the

[2] Helmholtz, Monatsberichte der Kgl. Akademie der Wissenschaften zu Berlin; June, 1873.

assumptions from which it follows, although with our present experimental knowledge there seems little reason to doubt that the assumptions are sensibly correct or that the equation would be pretty closely confirmed.

For lack of anything better, the still-air resistance equation has often been used to pass from model experiments made in a wind tunnel to the full-scale body moving into free still air. It can not be expected a priori that computations made in this way will give accurate results, because the conditions in the confined and somewhat turbulent air stream in the tunnel are not the same as those for which the equation was deduced; and, in fact, we know by experiment that the results of such computation are not very satisfactory. We have, therefore, to reconsider the problem and deduce the equation for this more general case.

III. THE WIND-TUNNEL RESISTANCE EQUATION

We suppose, as before, that the bodies under consideration are all of the same shape and are presented to the wind in the same way. We shall also, from the start, suppose that the wind speed is not high enough to make compressibility of any importance; the air force R on the body will then evidently depend on V, L, ρ, and ν as in free still air. But there are now other things to be considered, and an equation containing only the foregoing quantities would certainly not be complete. The nature of the flow about the body, and therefore the magnitude of the forces acting on it, may depend on the size and shape of the tunnel, not only in the immediate vicinity of the body, but at some distance away, especially ahead. For the turbulence of the air stream is influenced by the honeycomb, perhaps even by the shape and size of the wind-tunnel room or the design and speed of the suction fan. And we know that in some cases [3] a change of turbulence has a very marked effect.

In order to be complete, the equation corresponding to (1) must take account of all these new circumstances which are peculiar to the wind tunnel, and, in particular, it must include a number of lengths l', l'', l''', etc., which are linear dimensions sufficient to specify the shape and size of the balance arm, tunnel (including possible roughness of its walls), honeycomb, suction fan, wind-tunnel room, etc. The rate of revolution of the fan is another thing that may have to come in.

It is conceivable, moreover, that when vortex motion is present, as is the case behind cylinders, some further physical property

[3] Wieselsberger, Zeitschrifte für flügtechnik, 5, p. 140 ff; 1914.

of the air, not yet recognized because it does not manifest itself in ordinary experiments with gases, may come into the problem and influence the phenomenon; and the results of the present investigation have led the author to consider this possibility seriously. For the present, however, we shall ignore this possibility, and we shall also assume that the speed of the fan does not enter into the problem except as it influences the air speed V.

Under these conditions the new or additional circumstances which differentiate experiments in the wind tunnel from experiments in free still air may be specified by the values of the lengths l and the equation which corresponds to (1) may be written

$$R=F\ (V, L, \rho, \nu, l', l'', \text{etc.}). \tag{7}$$

As before, the principle of dimensional homogeneity requires that if equation (7) is complete, as it is by our hypothesis, it must be reducible to the form

$$R=\rho L^2V^2\ \mathrm{F}'\left(\frac{VL}{\nu}, r'\ r'', \text{etc.}\right), \tag{8}$$

in which r', r'', etc., represent the length ratios l/L, l'/L, etc. And just as we passed from equation (3) to the abbreviated form (6), so we may abbreviate by setting

$$F''\left(\frac{VL}{\nu}, r', r'', \text{etc.}\right)=C'_1, \tag{9}$$

and again get the familiar form of resistance equation

$$R=C_1\rho A V^2 \tag{10}$$

The difference is that, whereas in the case of free still air we have reason to suppose that the resistance coefficient C depends only on $\frac{VL}{\nu}$, in the case of the wind tunnel the coefficient C must be expected to depend, as indicated by equation (9), on a number of other quantities r', r'', etc., which vary inversely as the size L of the body under experiment, even though everything about the tunnel remains entirely unchanged.

Now although, a priori, we have no means of knowing anything definite about the form of F—that is, how the various circumstances may influence the resistance coefficient C quantitatively—physical common sense gives us some qualitative information. For example, in experiments on spheres one of the r's, say r', represents the ratio of the diameter of the tunnel to the diameter of the sphere. If this ratio is very large we may be sure that its precise value is of little importance, whereas if the sphere nearly fills the tunnel so that r' is not much above unity, the value of r'

will evidently be very important. In other words, it is obvious that if we want to avoid interference by the walls we must keep the model small; and that if we do keep it small enough r' will have no sensible effect on the resistance and may just as well be omitted from among the arguments of F in equation (9).

Other r's will refer to the dimensions of the wind-tunnel room and of the fan, and it is quite safe to assume that these have much less effect on the resistance than such other things as the nature of the balance arm and its shield, or the size and proximity of the honeycomb. Thus some of the r's will have no sensible effect on the value of C, while others may or may not, according to whether they are large or small.

As a first approximation we therefore assume that none of the circumstances to which the r's refer has any sensible effect on the air resistance; and to see how good the approximation is, we plot the observed values of the resistance coefficient $C_1 = R/\rho A V^2$ against the simultaneous values of $\frac{VL}{\nu}$. If we find that the points all lie on a single curve within the observational errors, we know that within the range of these experiments none of the r's was of any importance and that the simplified equation

$$C_1 = f\left(\frac{VL}{\nu}\right)$$

is as accurate as the experiments. But if, on the other hand, we do not find this agreement, we know that some one of our assumptions is wrong. Either we can not assume the effect of the circumstances to which the r's refer to be negligible, or, as a remote possibility, some further physical property of the air, unrecognized in ordinary experiments but manifesting itself in the vortex motion which exists behind many bodies, may affect the force. We must then investigate further. For, if there is such a property of the air, it will enter even in the case of motion in free still air.

IV. GENERAL RESULTS OF THE AUTHOR'S EXPERIMENTS AND COMPARISON WITH PREVIOUS WORK

The previous work on wires and cylinders (axes normal to the wind) is very meager. In fact, but three investigations in any way complete have been found; namely, that of Föppl[4] at Göttingen, that of Morris and Thurston[5, 6] at East London College, and that carried out by Melville Jones and other members of

[4] Jahrbuch der Motorluftschiff-Studiengesellschaft, 4, p. 85; 1910–11.

[5] Aeronautical journal, Great Britain, No. 58, p. 69; April, 1911.

[6] Lepère, Bulletin de l'Institut Aèrotechnique de l' Université de Paris, 2: 1912.

the aeronautics staff of the National Physical Laboratory of Great Britain.[7] The results are plotted in Fig. 1*A*. It is seen that there are marked differences. Eiffel[8] gives results for two cylinders only, and no correction is made for the ends, so that no comparison can be made. Some work was also done by Huff at the Massachusetts Institute of Technology,[9] but here also no correction was made for the ends. All of the investigators mentioned consider the range only from $\frac{VL}{\nu}$ equal to 1000 to 25 000, approximately, L being taken as the diameter of the cylinder. Thus the need of some more extended work is evident.

The present work was carried out chiefly at higher values of $\frac{VL}{\nu}$, though there is some overlap. Cylinders of 1, 1¼, 1½, 1¾, 2, 2½, 3, 4, 4½, 5, 5½, and 6 inches (0.0254 to 0.1524 m) were used with velocities from 15 to (in the case of the smaller cylinders) 80 miles per hour (25 to 130 km per hour, approximately). The range of values of $\frac{VL}{\nu}$ was from 10 000 to 185 000. The cylinders were made of wood, with the exception of those less than 2 inches in diameter, which were of brass; but an additional 1-inch wood cylinder and 4-inch brass cylinder were also used. The results are expressed by plotting the coefficient C against $\frac{VL}{\nu}$ as a base.

The detailed observations are plotted in Figs. 2, 3, 4, and 5. Though the points are rather scattered, a curve more or less well defined is drawn in. As may be seen by a study of the points, the shape of the ends of the curves is not as certain as the shape of the middle portion, so that extrapolation is dangerous. In the case of the cylinders of 3-inch and greater diameter the differences were not great enough to justify a separate curve for each one. These curves are collected for comparison in Fig. 1*B*. The measurements were all taken on the same balance, so that the results are comparable. The first noticeable thing is that for cylinders of diameter less than 3 inches, the resistance coefficient depends not only on the parameter $\frac{VL}{\nu}$, but also on L. The coefficient for a 1-inch cylinder is half as large again as that for a 3-inch cylinder at the same value of $\frac{VL}{\nu}$, thus showing clearly the failure to satisfy accurately the still-air resistance equation.

[7] Technical report of the Advisory Committee for Aeronautics, Great Britain, p. 40, 1910–11, p. 126, 1912–13.
[8] Eiffel, The Resistance of the Air and Aviation, translation by Hunsaker, p. 67.
[9] Aviation and Aeronautical Engineering, p. 395; June, 1917.

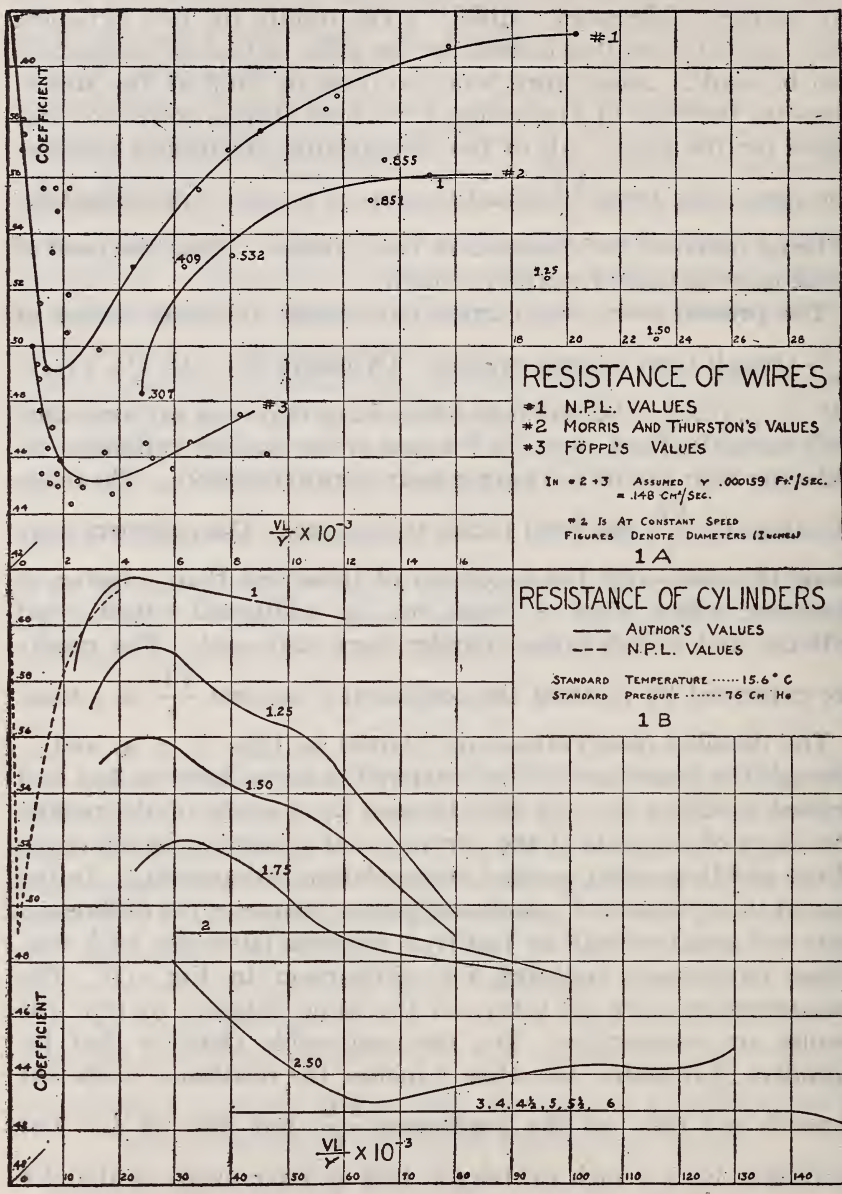

FIG. 1.—*Air resistance coefficients for cylinders, axes normal to the wind, showing summary of results by author and others*

A, Comparison of values obtained by other investigators; *B*, Values obtained by the author

The forms of the curves for different values of L show marked changes in the coefficient. It might be pointed out that these facts are not entirely new. For looking at the figures given by Morris and Thurston (some are shown in Fig. 1A) it is noticeable that the 1¼-inch cylinder gives a coefficient much less than the

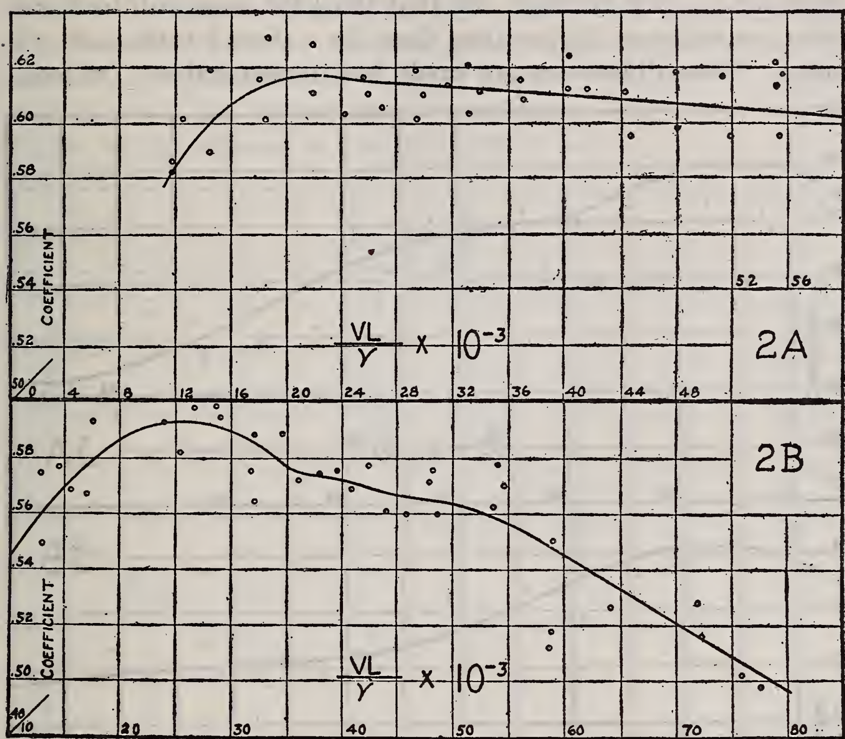

FIG. 2.—*Observed values of the resistance of cylinders*

A, For a 1-inch brass cylinder; dimensions, 0.999 by 16.88 inches; three tests. *B*, For a 1¼-inch brass cylinder; dimensions, 1.251 by 18.00 inches; two tests

1-inch, and the 1½-inch less than the 1¼-inch, and so on. Their values and the author's are as follows:

TABLE 1.—Comparison of Author's Results with Those of Morris and Thurston

L (in inches)	$\frac{VL}{\nu}$	Coefficients as measured by Morris and Thurston	Coefficients as measured by the author
	Absolute units	Absolute units	Absolute units
1	15 000	0.56	0.61
1¼	19 250	.54	.585
1½	23 100	.50	.56
1¾	26 900	.46	.52
2	30 700	.44	.49

The agreement between the relative values is striking; the absolute values differ by 10 per cent. Furthermore, although the work by Huff at the Massachusetts Institute of Technology can not be directly compared, the results obtained show that the coefficients for a three-fourths-inch cylinder are definitely higher than those for a 1-inch cylinder, and that those for a one-half-inch cylinder are definitely higher than those for a three-fourths-inch cylinder. These differences are small, but unmistakable. On look-

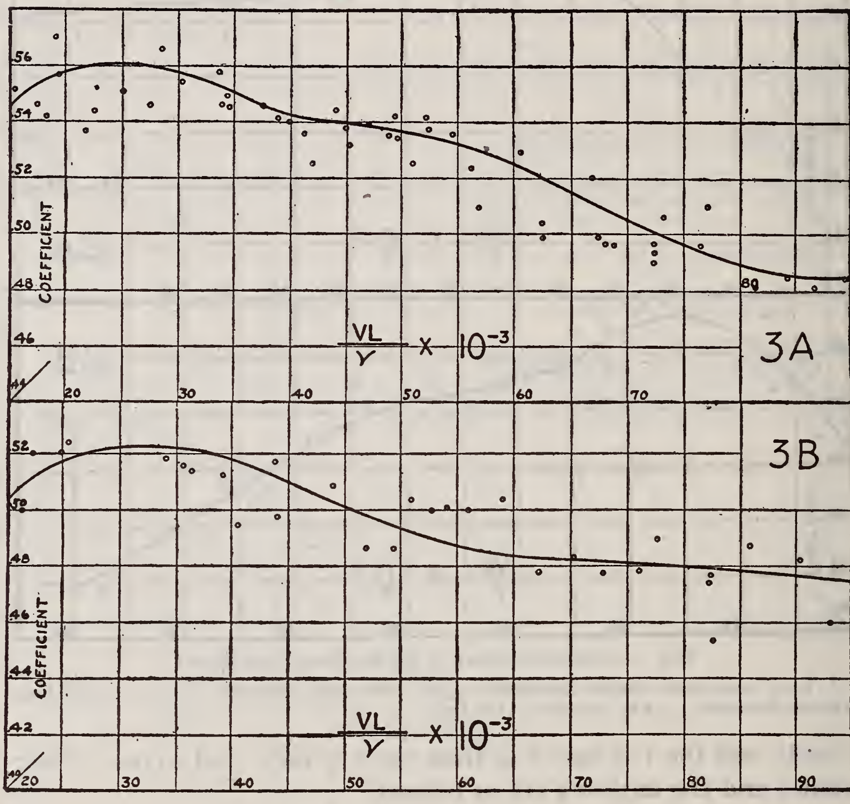

FIG. 3.—*Observed values of the resistance of cylinders*

A, For a 1½-inch brass cylinder; dimensions, 1.503 by 18.11 inches; three tests. *B*, For a 1¾-inch brass cylinder; dimensions, 1.772 by 17.97 inches; two tests

ing at my curves in the region of values of $\frac{VL}{\nu}$ in which they are working, it is seen that, if there is any difference for the smaller diameters, we should expect it to be small in that region. We can not attain high enough speeds in our tunnels to extend the experiments on the small cylinders to higher values of $\frac{VL}{\nu}$. Thus the dependence of the resistance coefficient on size at identical values of $\frac{VL}{\nu}$ has been indicated before, though no one has definitely pointed it out.

Finally, for values of $\frac{VL}{\nu}$ in excess of 150 000, all of the curves with the exception possibly of the 4-inch show a definite drop. On the 4-inch we have only two points in this region, so that we can not say with certainty whether there is or is not. These two points are at the highest speeds obtainable. Thus there is indication of the presence of a second critical velocity.

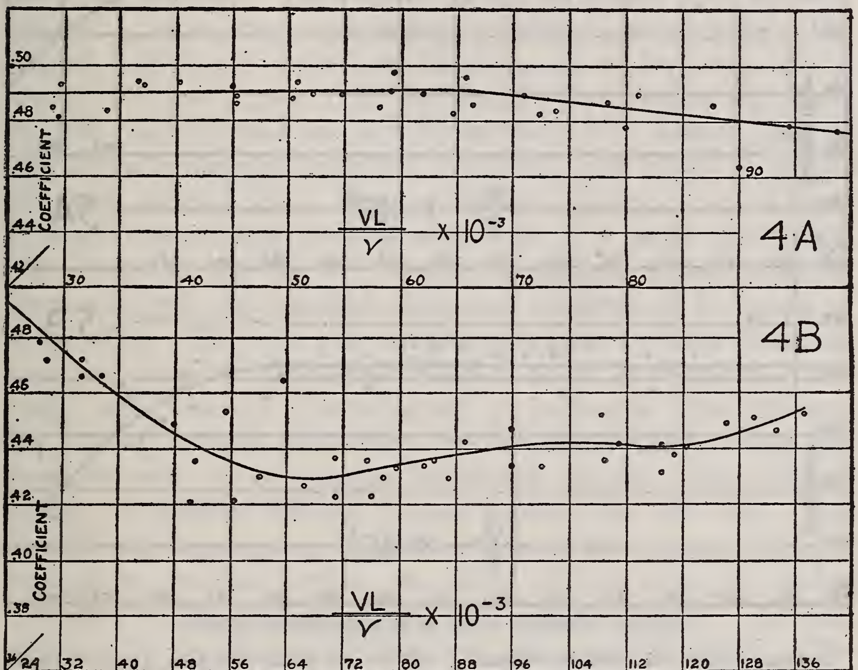

FIG. 4.—*Observed values of the resistance of cylinders*

A, For a 2-inch wood cylinder; dimensions, 1.990 by 17.94 inches; three tests. *B*, For a 2½-inch wood cylinder; dimensions, 2.510 by 17.91 inches; two tests

The results obtained by the aeronautics staff of the National Physical Laboratory and others at the lower values of $\frac{VL}{\nu}$ may be here noted. There were found curves as shown in Fig. 1*A*. For comparison the N. P. L. curve is plotted with my results in Fig. 1*B*. At very low values of $\frac{VL}{\nu}$ the coefficient is high, decreasing rapidly to a minimum at a value of $\frac{VL}{\nu}$ approximately equal to 1 570.

The coefficient then rises rapidly. Their models were all of a diameter of 1 inch or less. In the overlapping region my values agree very well with theirs.

Thus it is seen clearly that the still-air resistance equation is not satisfied even approximately by wind-tunnel experiments on cylinders. We must investigate further to find what particular

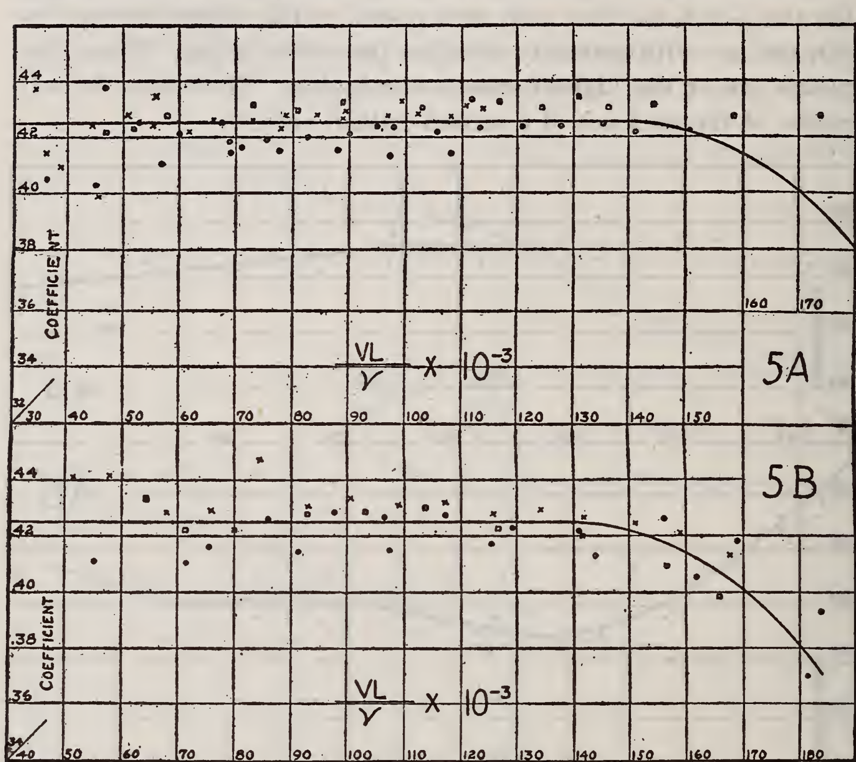

FIG. 5.—*Observed values of the resistance of cylinders*

A, Crosses—3-inch wood cylinder; dimensions, 2.993 by 17.91 inches; three tests. Crosses in circles; 3-inch wood cylinder; surface waxed. Circles—4-inch wood cylinder; dimensions, 3.990 by 17.97 inches; one test. Squares—4-inch brass cylinder; dimensions, 4.002 by 18.06 inches; one test. *B*, Crosses—4½-inch wood cylinder; dimensions, 4.486 by 17.97 inches; one test. Circles—5-inch wood cylinder; dimensions, 4.995 by 18.00 inches; one test. Squares—5½-inch wood cylinder; dimensions, 5.484 by 17.97 inches; one test. Crosses in circles—6-inch wood cylinder; dimensions, 5.991 by 17.97 inches; one test.

element involved in wind-tunnel experiments, if any, causes this discrepancy. The author's views are presented in the discussion at the end of the paper. Whatever view we may take as to an explanation of the results, the experiments show that extrapolation from wind-tunnel results to free-air results based on the test of a small model, and the use of the still-air resistance equation is fraught with serious danger.

V. FORCE MEASUREMENTS

1. APPARATUS

(*a*) THE TUNNEL.—The entire wind-tunnel facilities of the Bureau of Standards were placed at my disposal by Dr. Briggs. As there is no published description of the tunnel, it may not be out of place to give here a brief account of its principal features. The tunnel, similar to those at the National Physical Laboratory, is contained in a large room, the air being drawn through the tunnel by a four-bladed 9-foot (2.745 m) propeller and returning through the room. The room is 69 feet 10 inches (21.3 m) long, 18 feet (5.49 m) high, and 30 feet 4 inches (9.28 m) wide. The tunnel itself has its axis along the long axis of the room, and is 45 feet 6 inches (13.88 m) in total length, the propeller tips being 13 feet (3.97 m) from one end of the room. The working part of the tunnel is straight, octagonal in section, 53¾ inches (1.365 m) between opposite faces, and 25 feet 4½ inches (7.74 m) long. This portion is built of wood supported by a metal framework. The entrance consists of a wooden framework 4 feet (1.22 m) long, covered with airplane cloth and rounded off to admit of easy inflow. The exit end consists of a cone 15 feet 1½ inches (4.61 m) long and 9 feet 1¾ inches (2.79 m) in diameter at the outer end; that is, approximately 9° half-angle after allowing for a small straight part where the junction is made. This exit cone is built up of wooden framework covered with airplane cloth. A wooden diffuser is used around the exit end. Two honeycombs are used to straighten the air flow, one at the exit end of the working portion, the other 2½ feet (0.763 m) downstream from the entrance end of the working portion. The propeller is driven by a 100 horsepower, direct-current motor. The motor is controlled by rheostats in both field and armature circuits and may be run with the armature either on 110 or 220 volts. The line voltage is only fairly constant, fluctuating considerably on some days, while it is very steady on others. A traverse of the tunnel showed variations in velocity of as much as 2 per cent, although for the most part they were less than this, a fair average for the part occupied by the models being 0.6 per cent. This traverse was taken at one section only, and, due to the stress of war work, no others have been made.

(*b*) THE BALANCES.—The tunnel is equipped with two balances—one for aerofoils and the other for heavier work. The first balance is similar to the National Physical Laboratory balance[10] in every way except as to the means of making torque measurements. It is sensitive to 0.0001 pound (45 mg). The largest forces which can be measured on it are 3 pounds (1.36 kg). Hence it was impossible to use this balance for all the cylinders, and, as it was desired to obtain results whose relative values were accurate, measurements on this balance were discarded in plotting the curves. The other balance consists of a system suspended by two thin steel strips, and measures, primarily, the force in the direction of the wind. If lift measurements are desired, moments can be taken about a second pair of strips, and from these the lift force may be computed. In the present work this second pair was not used. The balance is sensitive to 0.001 pound (454 mg), and will take forces up to the strength of the strips. All of the points plotted were obtained on this balance. It must be remembered that both balances measure moments only, and actual forces are obtained by assuming the force to act at the center of symmetry of the body; hence only bodies with a horizontal plane of symmetry can be used.

(*c*) SPEED MEASUREMENT.—The wind speed was measured by means of the usual static plate and inclined gage. This particular gage was constructed with unusual care. The glass tube was a pyrex tube straight to 0.002 inch (0.05 mm). The ledge on which it rested was straight when the tube was put on, to 0.002 inch (0.05 mm), and the tube was held down on the ledge to within 0.005 inch (0.13 mm). The liquid used was benzene, as this obviates the trouble from dirt and grease which is always present with water. Since benzene attacks rubber, it was necessary to use special connectors, and, since it has a high coefficient of expansion, it was necessary to observe the temperature and to make a correction. This gage was standardized against a U tube of large diameter containing benzene and read with a cathetometer. The static plate was calibrated against a standard Pitot tube placed at the center of the tunnel. The calibration was a remarkably good one, and it is certain that the static plate gives the Pitot velocity at that particular place to a few tenths of 1 per cent. The gage was very steady except at very high speeds, the oscillations as a rule being slow enough to permit of reading the gage well within 1 per cent. At very low speeds the accuracy is, of course, not so great.

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[10] Cowley and Levy, Aeronautics, p. 9.

It must be remembered that a Pitot tube gives us the quantity $\frac{1}{2}\rho V^2$ only, where ρ is the density of the air. It is not usual in aerodynamic work to make the calculation for V every time. Since the forces are assumed to vary directly as ρV^2, this quantity alone is computed. If a value of V is given, it is determined from this, using "standard" density.[11] Thus the true value of V is not obtained. The method consists really in a comparison of the force on the body with the pressure given by a Pitot tube placed in the same air stream. This is unfortunate when we consider the general formula for the force $= \rho L^2 V^2 \, f\left(\frac{VL}{\nu}\right)$. For a change of temperature does several things, the effect of which may best be shown by an example. Suppose we take measurements at 25° C instead of at our standard 15° C. Making the Pitot reading identical in the two cases, $\frac{1}{2}\rho V^2$ is the same for both temperatures. On the other hand ρ has changed. V has actually been increased by about 2 per cent (the density being less by about 4 per cent). The kinematic viscosity, ν, equal to the viscosity divided by the density, has also changed. The viscosity has increased by about 3½ per cent, the density decreased by 4 per cent, hence the kinematic viscosity as a whole increased by 7½ per cent, approximately. Hence $\frac{VL}{\nu}$ has been decreased by 5½ per cent. The importance of this fact was not at first appreciated, so that, although records of the air temperature had been kept, no barometric readings were made. These were, however, obtained from a recording barograph with sufficient accuracy to compute $\frac{VL}{\nu}$ to 1 per cent. The other investigators quoted do not state whether they made any corrections of this sort or not.

(*d*) THE MODELS.—The models have already been described in a general way. The wooden ones were turned by a patternmaker. They were accurate to 0.01 inch (¼ mm), both as to being circular and as to being straight. The wooden models were of white pine, and the surfaces were coated with shellac. The brass ones were made of commercial brass tubing and were accurate to 0.005 inch (⅛ mm). All were approximately 18 inches (0.453 m) long. They were held on the balance arm by means of a five-sixteenths inch steel spindle.

[11] The "standard" density used at the Bureau of Standards and also at the National Physical Laboratory is 0.1223 g/cc, corresponding to the density of dry air at 15.°6 C, 760 mm pressure.

2. METHODS OF MEASUREMENT

It was advisable from the theoretical standpoint to obtain results applicable to infinite cylinders. To secure this result the "guard-ring" principle was used. Two short cylinders were placed in line with the cylinder on the balance, one being suspended from the roof of the tunnel and the other being on the balance spindle. The top guard was in line with the cylinder when the balance was in its zero position and cleared by just enough to allow the necessary play. These guards were 3 inches long. Though this length is rather arbitrary, it was found experimentally to be quite sufficient. Measurements were taken as follows (two observers; see Fig. 10 for sketches of the two set-ups required): With the cylinder and one guard on the balance arm, as described, the observer of velocity signaled when the velocity had reached some predetermined value. The observer at the balance adjusted the weights so that the beam was, on the average, in its zero position. This was repeated for the whole series of wind velocities. Then a second set of readings was taken with only the lower guard on the balance to obtain the spindle correction, the cylinder being suspended from the roof over the guard, so as to secure the same flow. The second set of readings was taken at approximately the same speed as the first. In computing, the second set was reduced to exactly the same speed as the first, assuming the square law over this short range. The difference between the readings at the same speed gives the resistance of the cylinder at that speed. Due allowance was made for the length of the balance arm, as explained in the section on balances. No windshield was used, as it was thought best to avoid any possible interference caused by it. In many cases conditions permitted one observer to make both balance and gage readings with fair accuracy.

3. RESULTS OF FORCE MEASUREMENTS

The general results have already been discussed, but certain others may be noted. The 3-inch cylinder was freshly coated with a wax mixture, and a test was made immediately afterward. The resistance dropped by approximately 3 per cent. After standing a week, a second run was made, and the resistance was found to have its original value. Since 3 per cent is not much beyond the errors of experiment, it would therefore seem that waxing the surface has little effect. The wood and brass cylinders check within the experimental error, and this result is in conformity with Zahm's conclusion [12] as to the independence of skin friction on the surface so long as the surface is not visibly rough.

In investigating the accuracy of the results, some measurements of the "end effect" were made. On the 1-inch cylinder it was found that the omission of the guards decreased the force by approximately 10 per cent. On the 4-inch cylinder, on the other hand, the omission of the guards made practically no difference, the effect being less than 1 per cent. This was tried with the cylinder both vertical and horizontal. It was thought at first that, since the balance measures moments only, the change in the force might be large, although the change in the moment is small. The test with the cylinder horizontal shows that this is not the case, for with the cylinder horizontal we know that the force acts in the same horizontal plane whether the guard is present or not. The values of the coefficient derived for this cylinder with no guards agree very well with the values with the guard. Now the N. P. L. investigators found [13] by varying the length of the wires that for small wires correction could be made for the ends by using in the calculations a value of the length of the wire 4 diameters shorter than the actual length. Thus it appears that for a given length the end correction does not increase proportionally to the diameter, but that changes take place in the flow about the ends. Thurston,[14] on the other hand, found for a cylinder of diameter 0.85 inch (2.16 cm) by the same method of differences a correction of 2.6 times the diameter; so that, further, it seems that the end correction depends largely on the tunnel.

This matter of end effects is extremely complicated by the character of the ends of the model. Owing to the three dimensional flow around the end of a cylinder of finite length, the resistance per unit length obtained from measurements on such a cylinder will be less than the value for an infinite cylinder, because the air tends to "spill" around the end. But if the end is sharp, turbulence is produced by the sharp edge, and hence the force is larger than if the ends were capped with, say, hemispheres. Thus it may happen that turbulence makes up for the loss due to the change in direction of the stream lines near the end. This is probably the explanation of the peculiar result observed by the author in the case of the 4-inch cylinder; and the variation in the form and roughness of the ends of the models may explain the variation in the magnitude of the end effect as measured by different observers.

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[12] Phil. Mag., 8, p. 58; July, 1904.

[13] Technical Report of the Advisory Committee for Aeronautics, Great Britain, p. 40; 1910–11.

[14] Engineering, 97, Jan., 1914.

4. ACCURACY OF FORCE MEASUREMENTS

The errors of the measurements may be considered from three points of view, namely, (1) as to their effect on the relative values of the forces on different cylinders; (2) as to their effect on the absolute values of the forces on all the cylinders; and (3) as to their effect on a single observation. It will be found that the relative values as given by the final curves are correct to approximately 2 per cent; that the absolute values are correct to 10 per cent or possibly less; and that the extreme range of error of any one observation is 5 per cent. It is because of the greater accuracy of the relative values that so much stress has been laid on relative characteristics and relative values.

In the first place, there is an error affecting each individual observation due to the very nature of the quantity to be measured. The effect of the vortex flow behind the cylinder is to produce a force the magnitude of which is continually changing. Under ideal conditions, with vortices formed at a uniform rate, the force would be periodic, and it ought not be hard to detect the periodicity at low speeds. But great complications are introduced by the fluctuations of speed in the tunnel. These entirely disrupt the periodicity and cause what may best be described as an irregularly varying flow. Thus what we attempt to measure is a time average of the force. Now, in such cases increased sensitiveness of the measuring apparatus is of no advantage beyond a certain point, and too great sensitiveness may be undesirable. To make the measurement more troublesome, the amplitude of this irregular variation is by no means negligible in comparison with the absolute value of the average force, the conditions of measurement being much like those prevailing at the "burble" point in aerofoil measurements. Hence the range of error of any one observation is large, and the only ground for claiming 5 per cent is that the measurements repeat and fit a smooth curve within that limit of error. Of course, according to the theory of errors, the probable error due to this cause in the absolute value of any point of the curves is less, probably as small as 1 or 2 per cent.

In the second place, the question of guards offers difficulty. It has already been mentioned that the length of the guards was found by experiment to be entirely sufficient; doubling the guard length had no effect on the measured force. A second question is as to their alignment. It is naturally impossible to align the guards exactly, and the flexure of the cylinder with increase of speed soon changes any accurate alignment which may have been made with the air at rest. Hence, since the 1-inch cylinder had already shown the greatest end effect, the effect of lack of alignment of guards was tried with it. It was found that moving the guard toward the direction from which the wind was blowing decreased the force; moving it in the opposite direction increased the force; the total change on moving the guard from a position one-fourth inch toward the front to a position one-fourth inch toward the back being 3 or 4 per cent of the total force. Thus in the actual experiment the error owing to lack of alignment can not be greater than 1 per cent, since a shift of one-eighth inch would be unusual.

In the third place, there is an error introduced by the presence of the gaps between the guards and the cylinder. Now, Thurston found (see footnote 14) that, when there was a gap at *each* end of one-sixteenth inch between a cylinder and fixed guards, the force was 4 per cent less than the value for no gaps; with a gap of one-fourth inch there was a diminution of 12 per cent. Thus the error is quite serious. However, my method of measurement largely eliminates this error. For whatever error is made in the first measurement when the cylinder and one guard in contact with the cylinder are on the balance is partly repeated in the second measurement with the guard and spindle alone on the balance. If the error in the force due to the gap is e, and the moment produced by the spindle and lower guard is S, the balance in the first measurement records a moment equal to $36F+45e+S$, approximately. In the second case the moment is $S+27e$. (The length of the balance arm is 36 inches and the length of the model 18 inches.) Thus on subtraction we have a moment $36F+18e$, which is interpreted as a force of amount $F+\frac{1}{2}e$ acting on the cylinder alone. Thus my method of measurement halves the error due to the presence of the gap. In all cases the gap was one-sixteenth inch or less, so that according to Thurston's results the maximum error in the absolute values of the forces is 2 per cent. Since the width of the gap could vary by only a small amount, the error in relative values is much less.

There is also an error due to the fact that the tunnel limits the air stream. The larger models make the area of the tunnel smaller at that section, and hence increase the speed. This is, of course, not equivalent to placing the model in a stream having this increased speed; for in front of and behind the model the air is still at the low speed. We should, however, expect the effect of the interference to be an increase in the force. The effects which we are interested in are decreases in the force with increase of size, so that the effect of the error would be to decrease the differences we obtain, and if it were eliminated the differences would be still larger than those actually observed.

The error due to the flexure of the cylinder must also be considered. Owing to flexure, the center of gravity of the body will be shifted so that the weight produces an additional moment which we interpret as an increased resistance. The error due to this cause is slight, since the weight of the cylinders is not great and the flexure is small. Again, there is an error if the axis of the cylinder is not normal to the wind. This is extremely small for any inclinations possible in these experiments. Then, there are the usual avoidable mistakes in reading or computing.

An error affecting chiefly the accuracy of the absolute values may arise from the velocity characteristics of the tunnel. First, the velocity distribution in the tunnel and the effect of the distribution on the force is uncertain. The Pitot calibration gives the velocity only at some one point, and it is possible for the distribution of the velocity to be of such a nature that large errors may be introduced. For we measure moments only, and if the irregularities are such that the velocity is low on both sides or high on both sides, as is frequently the case, our resultant moment will be incorrect. The error due to this cause might amount to as much as 4 per cent in an extreme case. But, in addition to this primary effect of the variation of the velocity across the section, there is the secondary effect on the flow and the consequent effect on the force. Of this we know nothing, but it is possible that the flow may be so modified as to cause large errors in the force.

Second, there is the effect of turbulence in the tunnel. There may be time variations in the velocity introduced by the tunnel wall or the honeycomb. Wieselsberger,[15] in measuring the resistance of spheres in the region of the critical velocity, found on introducing an artificial turbulence that the critical velocity was lower and that there were deviations from the simple resistance equation (6). However, he introduced the turbulence by means of a screen of 5 cm mesh placed 60 cm away from the model. In my experiments the balance was approximately 450 cm away from the honeycomb, consisting of hexagonal cells 7.62 cm between opposite faces. If turbulence does extend so far back, it must be very small. In his case the effect persisted for all sizes; in my case the large cylinders showed no such effect, agreeing with the simple resistance equation very well. There is, however, a possibility of error from this cause.

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[15] Zeitschrifte für flügtechnik, 5, p. 140 ff; 1914.

Turbulence does not appear to affect the relative values of the resistance coefficient C. For in the tunnel at East London College used by Morris and Thurston, a fine-wire gauze is placed 3 feet (0.92 m) in front of the model, and the tunnel is rectangular, 4 (1.22 m) by 1 feet (0.31 m), so that the turbulence is probably different from that in the Bureau of Standards wind tunnel. Notwithstanding this, the relative values of the forces on the same sized cylinders at the same speeds agree very closely.

The lack of agreement in the absolute values obtained by the above-mentioned investigators is undoubtedly partly due to the velocity distribution, and this error can not be estimated or allowed for. Even if a very accurate traverse of the tunnel is made, one is sure that the model has changed the distribution. In conclusion, it may be said that the relative accuracy of the present results is well within 5 per cent, but that although the absolute values in the overlapping region check the N. P. L. values well within this amount, they are not certain to closer than 10 per cent.

VI. MEASUREMENTS OF PRESSURE DISTRIBUTION

1. METHOD OF PRESSURE MEASUREMENTS

Owing to the peculiar variations of the resistance coefficients of the cylinders, it was felt advisable to undertake measurements of the pressure distribution over the cylinders to see if they would not throw some light on these variations. The measurements were only rough, but they yield some interesting results. The method adopted was very simple. A single small hole was drilled in the cylinder at a distance of about 6 inches from one end, and this hole could be placed in any position relative to the wind stream by mere rotation of the cylinder. A special fitting was made, consisting of an iron pipe with the cylinder spindle screwed in the top and a bearing in which the pipe could turn. A pointer was fixed to the bearing and a divided head to the pipe, so that the angular setting could be read off. The hole was connected by means of a rubber tube passing through the top of the tunnel to one side of a slant gage (the same as was used for speed measurements in the work on forces). The other side of this gage was connected to the static opening of a Pitot tube placed in the tunnel a little below its axis, so as not to interfere with the flow about the hole. Readings were thus obtained of the difference between the actual pressure at any point of the cylinder and the pressure that would prevail there if there were no cylinder present.

The speed can, of course, be determined from the maximum increase in pressure on the front of the cylinder, which is $\frac{1}{2}\rho V^2$, but readings were taken by means of the static plate connected to a second inclined manometer. This second manometer had a comparatively large slope and contained water, so that its indications were not very accurate. Nevertheless, it served to enable the observer to keep the speed fairly constant.

It is to be noted that this method of making pressure readings by means of a hole in the surface does not necessarily give us the true pressure on the cylinder, because we have no idea as to what modifications the hole may introduce in the flow. We know that in our case the surface of the body is covered with a vortex layer, which causes the phenomenon of skin friction. Now vortex layers may constitute the boundary between regions of very different pressures, as, for instance, in the case of the vortex layer bounding the slip stream of a propeller. Whether the hole breaks through this vortex layer and gives us the pressure outside it, or whether it gives some other pressure, we do not know, but in any event this method will give us some indication as to whether the flow changes or not, and that was its purpose in this instance.

The diameter of the hole was in all cases one-sixteenth of an inch, so that we obtained simply an average over a certain section. Smaller holes were not used, because the greater resistance introduced made the readings uncertain and slow. The cylinders used were the 1-inch, 3-inch, 4½-inch and 6-inch wood cylinders; of these the 1-inch was run at four speeds, the 3-inch at two speeds, and the others at one speed. In all cases evidence of the fluctuations of the flow was present in the fluctuation of the gage.

2. RESULTS OF PRESSURE MEASUREMENTS

Since the results are not very accurate, the readings were not reduced to absolute pressures, but the gage reading itself was used, it being proportional to the pressure. It is usual in plotting such results to plot from a circle as base, laying off the pressures at the various points along the radii through these points, negative differences being toward the center. In this paper, to permit of the use of a larger scale, the negative pressure is plotted outward the same as the positive one, but no confusion need arise if it is remembered that on the front (toward the wind) the pressure is greater than the static pressure, while on the back (and also part way on the front), it is less than the static pressure. Sample curves are shown in Figs. 6 and 7, the ordinates being

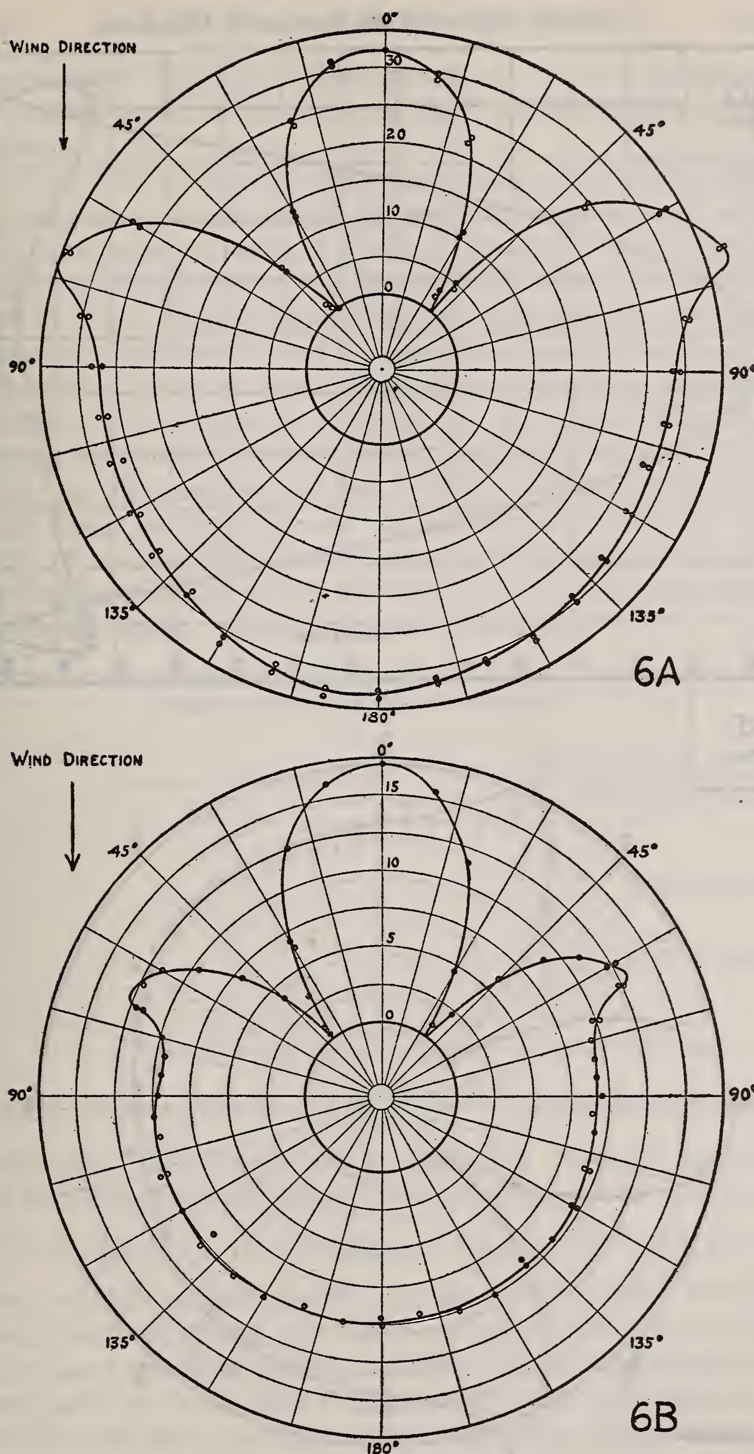

FIG. 6.—*Pressure distribution around cylinders*

A, One-inch cylinder at 63 miles per hour; *B*, Three-inch cylinder at 46.6 miles per hour

NOTE.—The radial distance from the circle marked o is proportional to the difference between the pressure on the cylinder and the static pressure in the tunnel. From 40° (approximately) to 180° the pressure on the cylinder is less than the static pressure; from 0° to 40° it is greater than the static pressure. Both positive and negative differences are plotted out for convenience.

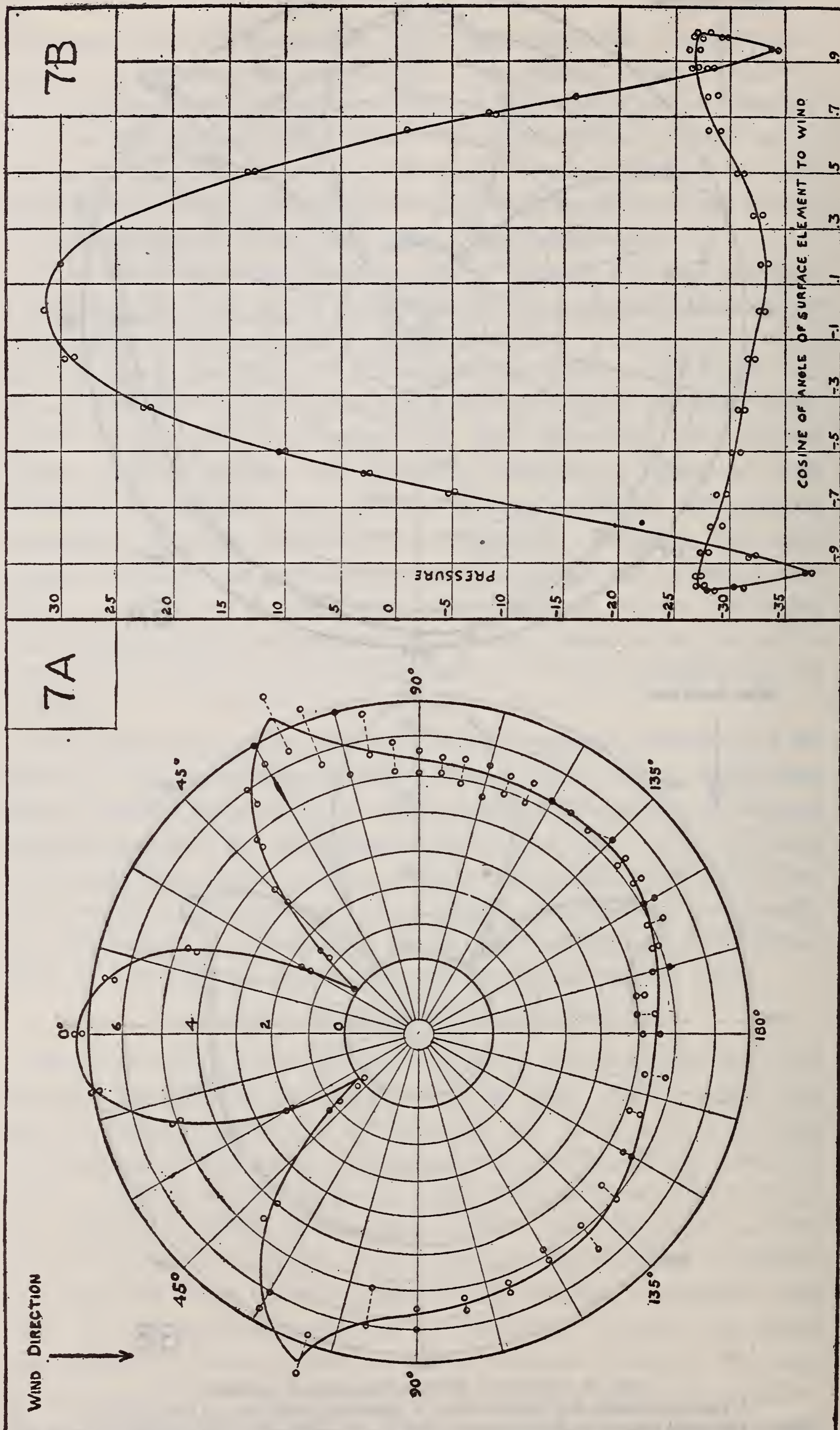

FIG. 7.—*Pressure on cylinders*

A, Six-inch cylinder at 30.1 miles per hour (see note to fig. 6); *B*, One-inch cylinder at 63 miles per hour gage readings. To get absolute values, it is only necessary to take the pressure on the front as $\frac{1}{2}\rho V^2$, and the others in proportion.

At first sight no essential difference appears between the different cylinders. On all of them the pressure becomes equal to the static pressure at an angle of about 40° from the wind direction; the pressure drops below static very quickly afterward, the maximum having a characteristic form and occurring at nearly the same angle, between 65° and 70°; and the pressure drop over the back is very nearly uniform. Apparently there is no very great difference in the character of the flow. Yet if we examine the curves more closely, inaccurate and irregular as are some of the points, one fact becomes evident. This is that the relative size of the hump on the front and the hump on the back, in other words, the ratio of the pressure increase on the front to the pressure drop on the back, is very different for the 1-inch cylinder from that for the others. The figures are as follows:

TABLE 2.—Ratio of Maximum Increase in Pressure on Front to Average Decrease in Pressure on Back for Various Cylinders

Diameter of cylinder (in inches)	Ratio of maximum increase in pressure on front to average decrease in pressure on back
1	1.10 : 1 1.19 : 1 1.20 : 1 1.05 : 1 (at different speeds)
3	1.74 : 1 1.64 : 1 (at different speeds)
4½	1.47 : 1
6	1.54 : 1

These values are at different values of $\frac{VL}{\nu}$, and hence not strictly comparable; yet a comparison is interesting. Though the variation with speed for any one cylinder is great, due to the change in $\frac{VL}{\nu}$ (15 per cent), yet the difference between the 1-inch and the other cylinders is much greater, 30 per cent or more. Furthermore, the three others are within 20 per cent of one another. Fig. 6 shows the character of this difference in a striking way, and there can be no doubt that the high value of the resistance coefficient for the small cylinders arises from some difference in the flow at the rear of the cylinder which causes the pressure on the rear to be further reduced below the static pressure. It may be noted that this is consistent with the results on the end effect. For, if the pressure at the back is decreased, we should expect the disturbance around the ends to be greater, and this was found to be the case.

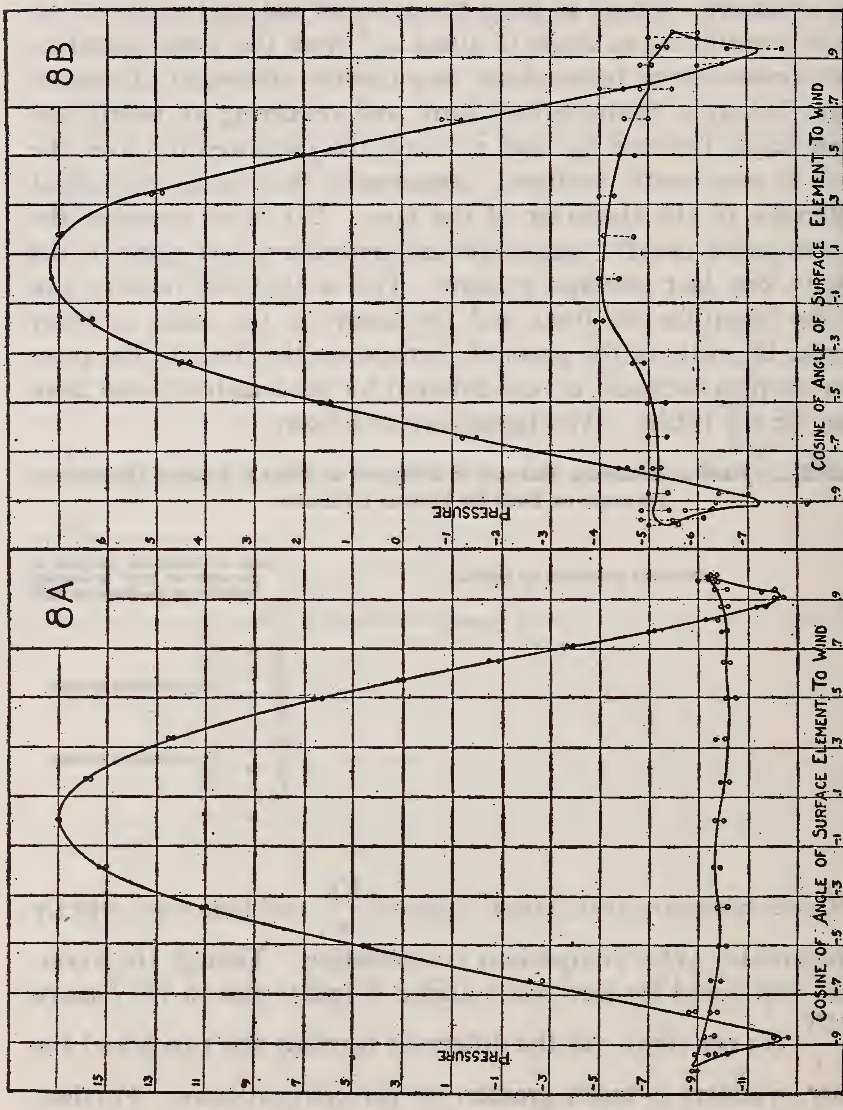

Fig. 8.—*Pressure on cylinders*

A, Three-inch cylinder at 46.6 miles per hour; *B*, Six-inch cylinder at 30.1 miles per hour

In the case of the cylinders of larger diameter the pressure curves show us that although the square law with a constant coefficient is a good approximation to the truth, it is only an approximation. Thus, comparing the 3-inch and 6-inch cylinders, although the force coefficients are identical, the pressures making up the force are distributed differently in the two cases. In the case of the 6-inch the hump on the front at an angle of 65° is larger, but this is compensated for by an increase in the size of the hump on the back. If the assumptions involved in the deduction of the free-air resistance equation were satisfied, we should expect geometrically similar figures; and since the figures are not similar, the equation is only a good approximation.

It was decided to integrate the pressure over the surface and see how well this checked the measured force. For this purpose a second series of curves was plotted (Figs. 7 and 8). Let us suppose that at any point on the cylinder the pressure is P. Its contribution to the component of the force in the direction of the wind is $P\,ds \cos \theta$ (see Fig. 9). But $ds \cos \theta$ equals dy. Hence the total force is $\int P\,dy$. Thus, if we plot P against y, or what is the same thing, against $\sin \theta$, or what is again the same thing, against the cosine of the angle of the surface element to the wind, the area of our curve, introducing the proper constants, will be the total force in the direction of the wind. These areas were measured with a planimeter, and the force computed. Little can be inferred from the results except that the skin friction is small. In two cases the pressure observations were not complete, so that the integration could not be performed. The agreement is as follows:

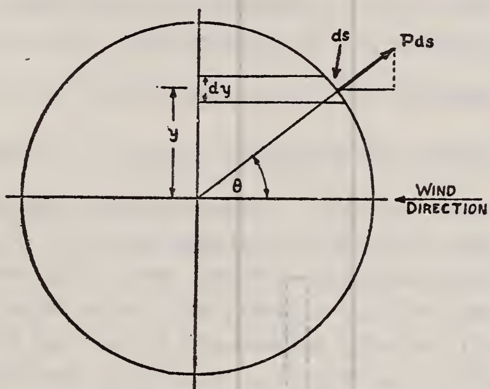

Fig. 9.—*Method of computing the resistance from the pressure measurements*

TABLE 3.—Comparison of Measured Values of Force Coefficients with Values Computed from Pressure Measurements

Diameter (in inches)	Speed	Calculated coefficient	Observed coefficient
	Miles per hour	Absolute units	Absolute units
1	16.36	0.57	0.59
	63.0	.59	.61
3	30.1	.42	.43
	46.6	.42	.43
4½	31.0	.43	.43
6	29.4	.43	.43

3. ACCURACY OF PRESSURE MEASUREMENTS

It is apparent from the irregularity of the points that the pressure measurements do not have a high degree of accuracy. In the first place, there is again the uncertainty of the velocity distribution across the section and its effect on the pressure. Then the

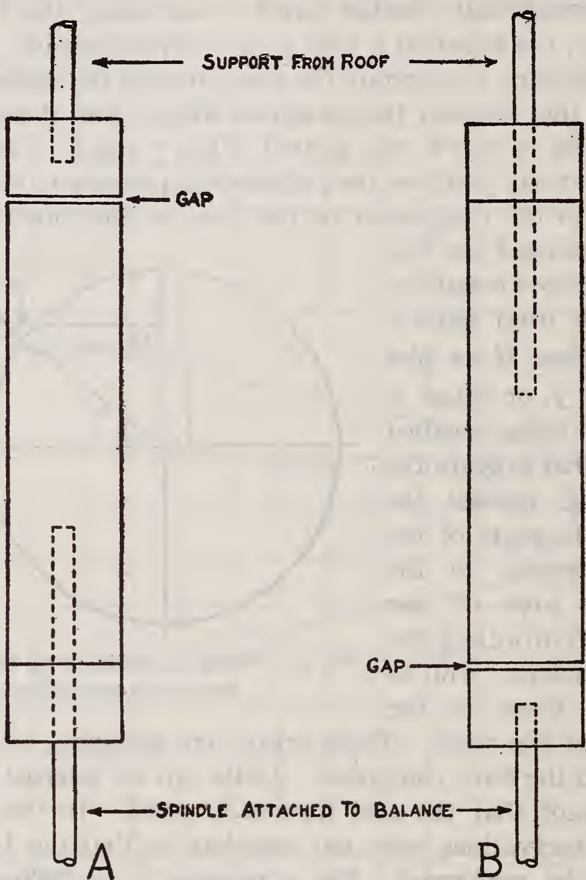

FIG. 10.—*Method of eliminating the effect of the ends of the cylinder*
A (Observation 1), Cylinder and lower guard on the balance; *B* (Observation 2), lower guard on balance

pressure behaves as does the force and varies irregularly, especially at the point of maximum pressure decrease and on the back. Sometimes it seems that there is a definite period to these changes, but the speed does not remain constant very long, so that this is not certain. Often there seems to be a sudden change. Because of the inaccuracies we can not place any great dependence on the pressure integration but must regard the curves simply as giving us a general idea of the distribution. The prominent features are the general shape of the curves, the difference in this shape in the case of large and small cylinders, and the very great unsteadiness of the 6-inch as showing the approach to a critical velocity.

VII. CONCLUSION

The result of the experimental work which has now been described is to show that our first approximation,

$$R = \rho A V^2 f\left(\frac{VL}{\nu}\right),$$

to the general wind-tunnel resistance equation,

$$R = \rho A V^2 f\left(\frac{VL}{\nu}, r', r'', \text{etc.}\right),$$

is not satisfied by these wind-tunnel experiments. The resistance coefficient is a function of L, the diameter of the cylinder, as well as of $\frac{VL}{\nu}$ and the velocity fields about cylinders of different diameters at identical values of $\frac{VL}{\nu}$ are not geometrically similar. This is confirmed by the pressure measurements. The flow about the different cylinders is of the same general nature, since the pressure distributions show the same general characteristics; but it is not exactly the same, since the pressure distributions are not exactly similar. In the case of the smaller cylinders the decrease in pressure on the back is greater in proportion to the increase in pressure on the front than in the case of the larger cylinders. Hence either some one of the arguments r of the function f, which we have omitted in our first approximation, must have an appreciable effect on the force, or some further property of the air must be involved, so that the equation given above as the "wind-tunnel resistance equation" is itself not complete.

Most of the circumstances to which the arguments r refer have already been considered in the discussion of the errors. Thus, the presence of the tunnel wall does make the force on the cylinder different from what it would be in the free air, but the force in the tunnel is greater and the magnitude of the difference increases with increase in the ratio of the diameter of the cylinder to the diameter of the tunnel. The effects of the guards have been investigated; their length is found sufficient and errors due to the gap and to lack of alignment small. The surfaces of the cylinders were shellaced and fairly smooth. Speeds are so low that compressibility does not enter. Indeed, the fact that the experiments of Morris and Thurston made in a tunnel whose form, honeycomb, balance, guards, etc., were all different from those of the Bureau of Standards tunnel gave results of the same character as mine, indicates strongly that none of the circumstances to which the *r*'s refer are responsible for the observed results.

The most serious possibility, however, is that the honeycomb may have an appreciable effect. For, as has already been mentioned, turbulence, in the case of spheres, has a marked effect. Wieselsberger (see footnote 15) found deviations from the still-air resistance equation such as those observed here, when he placed a net in front of the model, thus producing a turbulent-air stream. However, his net consisted of 1 mm threads with a mesh of 5 cm, and was only 60 cm in front of the model. The deviations were present for all his spheres, which ranged in diameter from 70 to 280 mm. In my experiments the turbulence is produced by a honeycomb 450 cm in front of the model. The cells of the honeycomb are hexagonal, 7.62 cm between opposite faces. The deviations in my case are not present for cylinders of diameter 7.62 cm or larger. Thus, if turbulence is the circumstance which affects the force by an appreciable amount, cylinders are many times more sensitive than spheres to the presence of turbulence. Furthermore, the turbulence in the tunnel at East London College used by Morris and Thurston was produced by a fine-wire gauze 92 cm in front of the cylinders, and we must conclude that, if turbulence affects the results appreciably, the character and amount of the turbulence are of little importance. Thus, to explain the deviations as due to conditions in the tunnel which do not exist in still free air, we must make assumptions, which, though possible, are a little forced.

If we believe these necessary assumptions to be improbable, there is only one alternative, namely, that, in the case of the vortex motion existing behind the cylinders, some hitherto unknown property of the fluid manifests itself. Since we have no other experiments indicating the existence of such a new property, we can not at present formulate any statement of its exact nature. If there is such a property, it exists even in the case of the motion of cylinders in still free air, and should appear in the still-air resistance equation.

There does seem to be one element in the flow about a cylinder which may involve such a new property. This element is the size of the vortices formed behind the cylinders. For, when two circular vortices are close enough together, the separate parts of each vortex will have different velocities, the vortex will, therefore, be distorted, and we can no longer consider its effect as being the effect of a filament. Thus, if our cylinder is made smaller and smaller, the vortices, if their size depends on a new property of the fluid and does not change exactly in the same ratio as the diameter of the cylinder, will be brought close enough together for this action to take place, and the character of the flow will be altered. We do know that vortices are of finite size when formed in a real fluid, for it is a property of vortices that in their motion they consist always of the same fluid particles, and in the case of a smoke ring we see that the ring moving forward is of comparatively large thickness. We also observe that smoke rings having ring diameters in a given ratio do not always have cross-section diameters in the same ratio, so that probably in the case of the columnar vortices behind cylinders the ratio of the cross-section diameter to their distance apart does not remain constant. The whole question might be settled by taking photographs of air flow past different sized cylinders.

This assumption of a new property of the fluid seems to satisfy most of the general requirements. It will presumably give a flow which is only slightly different from one where only filaments are present. When the body is large the distortion disappears, because the vortices are not close enough together, but as we make the body smaller the interaction becomes greater and greater. As it requires additional energy to distort the vortices, it would seem that the effect would be to increase the force. Thus, such an assumption would account for the general nature of the observed variations of the resistance coefficient C.

Acknowledgment is made to Dr. L. J. Briggs and to Dr. E. Buckingham, who assisted in the preparation of the paper. Much credit is also due to Alfred McMurdie and Gregory Breit, who assisted in the observations, for the care with which they did their work.

WASHINGTON, April 17, 1920.

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