METHODS FOR COMPUTING AND INTERCOMPARING RADIATION DATA

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I. INTRODUCTORY STATEMENT

This paper gives a simple method for computing spectral energy curves, using the Planck formula. In order to facilitate the computations, a table of values of log $(e^u - 1)$ is given. Such data do not appear to have been published heretofore. By means of this table, a table of logarithms, and a slide rule, one can compute a complete spectral energy curve in an hour.

The paper gives also a chart for intercomparing the thermal radiation constants, with similar data which may be obtained indirectly from theoretical considerations, using Planck's universal constant, h, and data obtained from ionization-potential, photoelectric, and X-ray work. Of course it is to be understood that whatever the true (theoretical) value of the constant, h, and the spectral radiation constant, c_2, may be, the value of c_2, as obtained from measurements on black-body radiators should be used in optical pyrometry and radiation work.

During the past few years the writer [1] has made frequent use of this table in computing spectral energy data and has found it sufficiently extensive for ordinary use. The chart is the outcome of numerous intercomparisons [2] of spectral radiation data with similar data resulting from photoelectric and other experiments, supplemented by the very complete computations and tabulations made by Dushman.[3]

II. COMPUTATION OF SPECTRAL RADIATION DATA

The Planck radiation formula, as usually written, is

$$E_\lambda = c_1 \lambda^{-5} (e^{c_2/\lambda T} - 1)^{-1} \qquad (1)$$

The function, $e^u - 1$, does not appear to have been tabulated heretofore. Some years ago a system of computation was

[1] Bulletin, Bureau of Standards, 13, p. 471, 1916.

[2] Coblentz, Phys. Rev., 32, p. 613, 1911 (On the Elementary Electrical Charge); also Bulletin, Bureau of Standards, 12, p. 553, 1916.

[3] Dushman, Gen. Elect. Rev., 18, p. 1167, 1915.

adopted whereby the value of λ was so chosen that the exponent $u = c_2/\lambda T$ resulted in a number which is given in exponential tables[4] of e^u. This, of course, is possible, whatever values of c_2 may be used. Hence, the present table is of permanent value. In view of the uncertainty of the exact value of c_2, the time does not yet seem ripe for tabulating E_λ, for different temperatures (say, in steps of 50°) and wave lengths.

By the system of calculation herein employed, spectral intensity data may be quickly computed. For this purpose the value of $\log c_1 = 5.00000$ is employed, the resulting values of E_λ being, of course, in arbitrary units. The Planck formula then becomes

$$\log E_\lambda = \log k - 5 \log \lambda \qquad (2)$$

in which values of $\log k = \log c_1 - \log (e^u - 1)$ are given in the fourth column of Table 1.

As already mentioned, the usefulness of the method lies in selecting the exponent

$$u = c_2/\lambda T \qquad (3)$$

so that values of u may be taken from the table. The value of λ usually comes out some uneven number, but this is unimportant.

Example.—The following data are given as an illustration of the computation[5] of the isothermal spectral energy curve for

$$T = 1596° K, \text{ using } c_2 = 14350.$$

Here

$$u = c_2/\lambda T = \frac{8.99}{\lambda}$$

From Table 1, for

$$u = 8.0, \lambda = 1.123\mu;$$

$$\begin{aligned} \log k &= 1.52579 \\ 5 \log \lambda &= .25190 \\ \hline \log E_\lambda &= 1.27389 \\ E &= 18.79 \text{ at } \lambda = 1.123\mu. \end{aligned}$$

[4] Smithsonian Mathematical Tables, Hyperbolic Functions.

[5] Bulletin, Bureau of Standards, 13, p. 476, 1916.

TABLE 1

u	e^u	$\log_{10}[e^u-1]$	$\log k = 5-\log[e^u-1]$
0.10	1.10 517	$\bar{1}$.021 932	5.97 807
.15	1.16 183	$\bar{1}$.209 069	5.79 094
.20	1.22 140	$\bar{1}$.345 177	5.65 483
.25	1.28 403	$\bar{1}$.453 357	5.54 665
.30	1.34 986	$\bar{1}$.543 894	5.45 611
0.35	1.41 907	$\bar{1}$.622 286	5.37 772
.40	1.49 183	$\bar{1}$.691 815	5.30 819
.45	1.56 831	$\bar{1}$.754 586	5.24 542
.50	1.64 872	$\bar{1}$.812 057	5.18 795
.55	1.73 325	$\bar{1}$.865 254	5.13 475
0.60	1.82 212	$\bar{1}$.914 935	5.08 507
.65	1.91 554	$\bar{1}$.961 677	5.03 833
.70	2.01 375	0.005 926	4.99 407
.75	2.11 700	.048 053	4.95 195
.80	2.22 554	.088 317	4.91 169
0.85	2.33 965	.126 971	4.87 303
.90	2.45 960	.164 234	4.83 577
.95	2.58 571	.200 221	4.79 978
1.00	2.71 828	.235 099	4.76 491
.05	2.85 765	.268 964	4.73 104
1.10	3.00 417	.301 935	4.69 808
.15	3.15 819	.334 089	4.66 591
.20	3.32 012	.365 509	4.63 449
.25	3.49 034	.396 257	4.60 374
.30	3.66 930	.426 397	4.57 361
1.35	3.85 743	.455 976	4.54 403
.40	4.05 520	.485 039	4.51 497
.45	4.26 312	.513 633	4.48 637
.50	4.48 169	.541 789	4.45 822
.55	4.71 147	.569 546	4.43 046
1.60	4.95 303	.596 929	4.40 308
.65	5.20 698	.623 970	4.37 603
.70	5.47 395	.650 691	4.34 931
.75	5.75 460	.677 114	4.32 289
.80	6.04 965	.703 261	4.29 674

TABLE 1—Continued

u	e^u	$\log_{10}[e^u-1]$	$\log k = 5-\log[e^u-1]$
1.83	6.23 389	.718 825	4.28 118
.85	6.35 982	.728 150	4.27 185
.90	6.68 589	.754 798	4.24 521
.93	6.88 951	.770 079	4.22 992
.95	7.02 869	.780 223	4.21 978
2.00	7.38 906	.805 437	4.19 456
.10	8.16 617	.855 287	4.14 471
.20	9.02 501	.904 446	4.09 555
.30	9.97 418	.952 995	4.04 701
.40	11.0 232	1.001 005	3.99 899
2.50	12.1 825	1.048 529	3.95 148
.60	13.4 637	1.095 647	3.90 435
.70	14.8 797	1.142 386	3.85 761
.80	16.4 446	1.188 779	3.81 122
.90	18.1 741	1.234 875	3.76 513
3.00	20.0 855	1.280 705	3.71 929
.10	22.1 980	1.326 293	3.67 371
.20	24.5 325	1.371 668	3.62 833
.30	27.1 126	1.416 851	3.58 315
.40	29.9 641	1.461 859	3.53 815
3.50	33.1 155	1.506 714	3.49 329
.60	36.5 982	1.551 428	3.44 857
.70	40.4 473	1.596 017	3.40 398
.80	44.7 012	1.640 493	3.35 951
.90	49.4 024	1.684 867	3.31 513
4.00	54.5 982	1.729 150	3.27 085
.10	60.3 403	1.773 349	3.22 664
.20	66.6 863	1.817 541	3.18 246
.30	73.6 998	1.861 532	3.13 846
.40	81.4 509	1.905 531	3.09 446
4.50	90.0 171	1.949 474	3.05 053
.60	99.4 843	1.993 367	3.00 663
.70	109. 947	2.037 215	2.96 278
.80	121. 510	2.081 023	2.91 898
.90	134. 290	2.124 798	2.87 520

TABLE 1—Continued

u	e^u	$\log_{10}[e^u-1]$	$\log k = 5-\log[e^u-1]$
5.00	148. 413	2.168 536	2.83 146
.10	164. 022	2.212 246	2.78 775
.20	181. 272	2.255 928	2.74 407
.30	200. 337	2.299 588	2.70 041
.40	221. 406	2.343 223	2.65 678
5.50	244. 692	2.386 841	2.61 316
.75	314. 191	2.495 809	2.50 419
6.00	403. 429	2.604 689	2.39 531
.50	665. 142	2.822 261	2.17 774
7.00	109 6.63	3.039 664	1.96 034
7.50	180 8.04	3.256 959	1.74 304
8.00	298 0.95	3.474 209	1.52 579
8.50	491 4.77	3.691 414	1.30 859
9.00	810 3.08	3.908 596	1.09 140
9.50	133 59.7	4.125 770	.87 423
10.0	220 26.5	4.342 926	.65 707
11.0	598 74.1	4.777 232	.22 277
12.0	162 755.	5.211 532	$\bar{1}$.78 847
13.0	442 413.	5.645 828	$\bar{1}$.35 417
14.0	1202 604.	6.080 123	$\bar{1}$.91 988

It is of course to be understood that after computing these data they must be multiplied by a factor to superpose them upon the observed spectral energy curve. This, as well as the determination of the different values of u, is easily accomplished with a slide rule.

III. INTERCOMPARISON OF RADIATION AND OTHER DATA

From Planck's radiation theory we have the following relations:

$$c_2 = c\,h\,k^{-1} = 4.9651\ \lambda_m\,T \qquad (4)$$

$$\sigma = \frac{ac}{4} = \frac{12\pi \times 1.0823\ k^4}{c^2\,h^3} \qquad (5)$$

$$\lambda_m T = \frac{c\,h}{4.9651\ k} \qquad (6)$$

$$k = \frac{eR}{cF} \qquad (7)$$

In these equations h is Planck's universal constant or constant of action; k is the Boltzmann gas constant, $k = 1.372 \times 10^{-16}$ erg. deg^{-1}; c is the velocity of light, $c = 2.9986 \times 10^{10}$ cm. sec.$^{-1}$; F

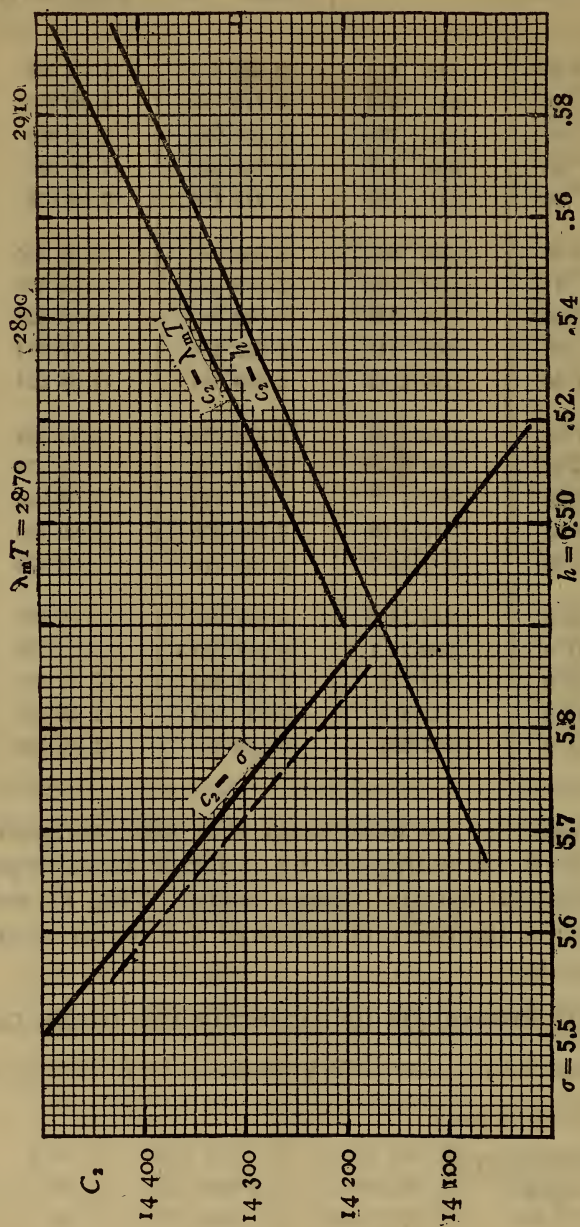

FIG. 1.—*Curves showing the relation between the radiation and other constants.*

is the Faraday constant, $F = 96500$ coulombs; R is the absolute gas constant, $R = 831.5$ erg. deg^{-1}; and e is the unit electric charge, $e = 4.774 \times 10^{-10}$ e. s. u. From equation (7) it may be

noticed that a change in the value of e affects the value of c_2 directly, while the value of σ is affected by e^4.

The data computed from the above-mentioned constants and formulæ are illustrated in Fig. 1, from which it is an easy matter to compare experimental data. For example, the writer's [6] value of the coefficient of total radiation is $\sigma = (5.72 \pm 0.012) \times 10^{-12}$ watt. cm^{-2}. deg^{-4}. This indicates a value of $c_2 = 14\ 320$ micron degrees and a value of $h = 6.55 \times 10^{-27}$ erg sec. The value of h, determined by Blake and Duane [7] by X rays, is $h = 6.555 \times 10^{-27}$ erg sec.; or an indicated value of $c_2 = 14\ 330$ micron degrees, which is close to experimental determinations of this constant.

Using the early determinations of c_2 and σ by Lummer and Pringsheim, and by Kurlbaum, and the above-mentioned relations in his radiation theory, Planck [8] deduced a value of $e = 4.69 \times 10^{-10}$ e. s. u. for the elementary charge of an electron. He calls attention to the fact that the value of this constant will depend upon the accuracy of the experimental determination of the constants of radiation, which in the meantime have been found quite different from the values used by him.

In the meantime the unit electric charge has been determined with high precision by Millikan,[9] and recent writers have been computing the radiation and other constants, using the value of $e = 4.774 \times 10^{-10}$ e. s. u. As shown by the dotted, c_2, σ – curve in Fig. 1, a change of about 0.1 per cent in e (using $e = 4.777$ as found in earlier experiments—1912) has an appreciable effect upon, and happens to bring a closer agreement between, the writer's values of c_2 and σ; though no particular significance is to be attached to this fact.

In view of the fact that several writers [10] have expressed the opinion that the radiation constants c_2 and σ could be derived more accurately from determinations of h by photoelectric and other data, it is important to emphasize that whatever the theoretical value of c_2 may be, the value of c_2 as determined on the best black bodies that can be constructed, using accepted experimental methods, is the one to be used in optical pyrometry and radiation work.

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[6] Proc. Nat. Acad. Sci., 3, p. 504, 1917.
[7] Blake and Duane, Phys. Rev. (2), 10, p. 624, 1917.
[8] Planck, Vorlesungen ü Warmestrahlung, p. 163; 1906.
[9] Millikan, Proc., Nat. Acad. Sci., 3, p. 231; 1917.
[10] Millikan, Phys. Rev., (2) 7, p. 378; 1916. Dellinger, Bulletin, Bureau of Standards, 13, p. 543; 1916.

From a recent recalculation and intercomparison by Birge [11] of the data on c_2, σ, and h, as determined by radiometric, photoelectric, X rays, and ionization potential measurements, it appears that the value of h, computed from radiometric data, compares favorably with that obtained by more direct measurement. The outstanding disagreement between all the observed and computed data appears to be of the order of 2 to 3 parts in 1000, whatever the method of experimentation. This is a very close agreement, considering the variety of the data and the difficulties involved in making the experiments. It seems to indicate something more than a fortuitous relation between properties of matter.

IV. SUMMARY

In this paper a simple method is given for computing spectral-energy curves, using the Planck formula. For this purpose a table of values of log $(e^u - 1)$ is given.

The paper gives also a chart for the intercomparison of thermal radiation constants with similar data, obtained indirectly from ionization potential, photoelectric, and X-ray measurements.

WASHINGTON, August 6, 1919.

[11] Birge, Phys. Rev., (2) 14, p. 361; 1919.