NBS CIRCULAR 574

Amplitude and Phase Curves for Ground-Wave Propagation in the Band 200 Cycles per Second to 500 Kilocycles

UNITED STATES DEPARTMENT OF COMMERCE

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National Bureau of Standards Circular 574
Issued May 21, 1956

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.
Price 20 cents

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James R. Wait and H. Herbert Howe [1]

After making several extensions to the formulas of Van der Pol and Bremmer, field strength and phase values of the very low frequency ground wave from a short vertical antenna are computed. The ground conductivity values chosen are 4, 0.01, and 0.001 mho per meter. The distances considered range from 1 to 1,500 miles.

1. Introduction

Considerable interest has been shown recently in the propagation characteristics of very low frequency radio waves [1, 2, 3].[2] Electromagnetic energy is present in lightning discharges at frequencies down to 100 cps and less [4]. There has also been a revived interest in very low frequencies (VLF) for communication purposes because they are only slightly affected by ionospheric disturbances for ranges up to 200 miles or so. VLF has been found particularly suitable for navigational systems using a phase-comparison technique [5, 6]. In such a system, two continuous-wave signals are converted in the receiver to a common comparison frequency, which is displayed on a phase meter. Lines of equiphase-difference form the position lines from which this navigational aid, known as "Decca", is based. Another promising application of VLF is to geophysical prospecting, where it is known that the propagation of the ground wave is dependent on the upper strata of the earth's crust. At frequencies of the order of 100 kc, the substrata at depths down to several hundred feet will influence the attenuation and phase velocity of the ground wave [7].

In view of the present VLF developments in the above-mentioned fields, it seems very desirable to present detailed information on the ground-wave characteristics of an electric dipole source. The frequencies considered are 200 cps to 500 kc. The conductivity between the source and the observer is considered to be essentially homogeneous, with conductivity σ in mhos per meter. The values of σ chosen are 4, 10^{-2}, and 10^{-3}, corresponding to sea water, well-conducting land, and poorly conducting land, respectively. In this frequency range the displacement currents in the ground can be neglected compared to the conduction current, so the dielectric-constant value need not be considered. The validity of this approximation was examined in detail elsewhere [1, 8]. The distances in miles from the source to the receiver are taken from 1 to 1,500, inclusive. To account for normal atmospheric refraction [9], the effective earth radius is taken to be k times the actual radius. It should be pointed out, however, that the effective earth-radius concept is not strictly valid at very low radio frequencies. For most of the calculations, k is set equal to 4/3 to conform with standard practice.

[1] The contribution of the second author consisted of devising the program for the automatic computer. The responsibility for the technical content of this paper rests completely on the first author.

[2] Figures in brackets indicate the literature references at the end of this paper

2. Some Theoretical Considerations

For any usual source of VLF waves, the equivalent antenna will have a height that is small compared to the wavelength. Furthermore, for most cases the distance of the observer to the source will be large compared to the height of antenna or vertical column of current. In this case, the fields are proportional to the product of the average current I and the height l of the antenna. The solution for the field of a vertical dipole of moment Il over a spherical homogeneous earth was first carried out successfully by Watson [10]. Methods for obtaining numerical values for application to radio-wave propagation have been developed by Van der Pol and Bremmer [11], Norton [12], and others. The application of these methods is very straightforward for the purpose of calculating the amplitude of the ground wave for medium and high radiofrequencies. Some special consideration is required, however, at very low radiofrequencies where the induction and static fields of the antenna are not negligible. This is particularly true from the standpoint of the phase variations with distance.

Choosing a spherical coordinate system (r,θ,ϕ), the earth's surface is defined by $r=a$, and the source is a radially oriented electric dipole of moment Il located on the earth at $\theta=0$. A time factor $\exp(i\omega t)$ is implied throughout. The radial electric-field component E_r in the air can be derived from a scalar function Π by

$$E_r=-\frac{\dfrac{\partial}{\partial\theta}\left(\sin\theta\,\dfrac{\partial\Pi}{\partial\theta}\right)}{r\sin\theta}. \quad (1)$$

The vertical component E of the electric field at a distance D measured along the surface is then given by

$$E=-\frac{1}{D}\frac{\partial}{\partial D}\left(D\frac{\partial}{\partial D}\Pi\right),\qquad (2)$$

since $\theta=D/a$ is small compared to unity, Van der Pol and Bremmer [11] give the following expression for Π:

$$\Pi=\frac{Il}{2\pi i\omega\epsilon_0 D}e^{-i\beta D(1+1/2\beta ak)}W,\qquad (3)$$

where $\beta=2\pi$/free-space wavelength, and $\epsilon_0=8.854\times10^{-12}$ farad/m. W is the now famous residue series given by

$$W=[2\pi(\beta a)^{1/3}(D/a)k^{-2/3}]^{1/2}e^{-i\pi/4}$$

$$\cdot\sum_{s=0}^{\infty}\frac{\exp[-i\tau_s(\beta a)^{1/3}(D/a)k^{-2/3}]}{2\tau_s-1/\delta^2}\qquad (4)$$

with

$$\delta=e^{-i3\pi/4}\left(\frac{\sigma}{\epsilon_0\omega}\right)^{1/2}(k\beta a)^{-1/3},$$

and where the τ_s are solutions of the equation

$$\frac{H^{(2)}_{2/3}\left[\frac{1}{3}(-2\tau_s)^{3/2}\right]}{H^{(2)}_{1/3}\left[\frac{1}{3}(-2\tau_s)^{3/2}\right]}=-\frac{e^{-i\pi/3}}{\delta\sqrt{-2\tau_s}},\qquad (5)$$

where the left-hand side is the ratio of two Hankel functions of the second kind. Van der Pol and Bremmer develop methods for obtaining numerical values of τ_s [3, 11].

The "residue series" representation for W is quite suitable for computation when D is large enough for the series to converge rapidly. At shorter distances (down to 1 mile) and very low radio frequencies the series would become impractical, since thousands of terms would be required to secure convergence. At these very short distances, however, the earth can be considered nearly flat and another expression for $W(D)$ is more suitable, which is given by

$$W=1-i(\pi\rho)^{1/2}-2\rho+i(\pi)^{1/2}\left(1-\frac{\delta^3}{2}\right)\rho^{3/2}$$

$$+\frac{4}{3}(1-\delta^3)\rho^2-i\frac{(\pi)^{1/2}}{2}\left(1-\frac{3}{2}\delta^3\right)\rho^{5/2}$$

$$-\frac{1}{15}(8-16\delta^3+7\delta^6)\rho^3+\ldots,\qquad (6)$$

where ρ is Sommerfeld's numerical distance, defined by

$$\rho=\frac{\beta D}{2}\left(\frac{\epsilon_0\omega}{\sigma}\right).\qquad (7)$$

The above series formula for W was proposed by Bremmer.[3] It can be verified that when $\delta=0$, corresponding to a flat earth, the right-hand side corresponds to the series expansion for the well-known Sommerfeld flat-earth propagation function $y(\rho)$, which has now been tabulated extensively [12]. It has also been shown recently that the general formula for W can be extended to a curved stratified ground with elevated antnn as [13].

Using (2) and (3), the electric field is now given by

$$E=\frac{-i\bar{\beta}^2Il}{2\pi\omega\epsilon_0}\frac{e^{-i\bar{\beta}D}}{D}\left[W+\frac{1}{i\bar{\beta}D}\left(W-2D\frac{\partial W}{\partial D}\right)\right.$$

$$\left.+\frac{1}{(i\bar{\beta}D)^2}\left(W+D^2\frac{\partial^2W}{\partial D^2}-D\frac{\partial W}{\partial D}\right)\right],\qquad (8)$$

where $\bar{\beta}=\beta(1+1/2\beta ak)$.

The three principal terms in the above expression, proportional to $1/D$, $1/D^2$, and $1/D^3$, can be identified with the radiation, induction, and static fields, respectively. It is clear from the above development that the effect of finite conductivity and earth curvature will affect to some extent all three components of the field. At very short distances where $\rho^{1/2}\beta D$, $\rho^{3/2}\delta^2\beta D$, and D/a are small compared to one, the expression for the field simplifies considerably to

$$E=\frac{-i\mu\omega Il}{2\pi D}e^{-i\beta D}\left[1-\frac{i}{\beta D}-\frac{1}{\beta^2D^2}\right],\qquad (9)$$

which, of course, is identical to the field of an electric dipole on a flat perfectly conducting plane. At large distances where $\beta D\gg1$, the expression for E is given by

$$E=\frac{-i\mu\omega Ile^{-i\bar{\beta}D}}{2\pi D}W,\qquad (10)$$

where W is defined by eq (4) or (6). At intermediate distances where neither of the above two special cases is valid, it appears to be necessary, in general, to employ the above ρ series formula for E. For most purposes, however, it is sufficiently accurate to use the following formula for the whole range of D.

$$E=\frac{-i\mu\omega Ile^{-i\bar{\beta}D}}{2\pi D}\left[1-\frac{i}{\beta D}-\frac{1}{\beta^2D^2}\right]W.\qquad (11)$$

[3] Private communication.

The maximum error in this formula occurs where βD is in the vicinity of unity and for the range of parameters chosen in this report will affect the amplitude to within 1 percent and the phase by less than 2 degrees. Therefore, eq (11) can be used for all the calculations.

3. The Numerical Results

The major numerical task in this work was the evaluation of the residues series for W. In applications to field-strength calculation at medium radiofrequencies it is usually sufficient to use only one term of the residue series and then to interpolate with the flat-earth formula, which is valid at short distances. The procedure for making rapid calculations of this type has been developed by Norton [12]. For amplitude and phase calculations at VLF, the interpolation method is not of sufficient accuracy, and more terms of the residue series are required. In fact, it was considered desirable at the outset of this work to carry out calculations from the residues series formula at distances down to 37 miles, where no interpolation with the ρ series formula is required. In fact, there was a region of overlap in the calculations that provided a convenient check on the computations. Due to the excessively large number (up to 1,000) of terms required to secure convergence of the residue series at short distances and frequencies as low as 200 cps, it was summed on the SEAC (the National Bureau of Standards electronic automatic computer). Values of the amplitude and phase of W are provided in the tables 1 and 2, respectively, for several conductivity values and various distances. There are several gaps in the tables that correspond to situations where it was not possible to devise a convenient program. In some cases this occurred where the Bremmer-type [3] series formulas for δ did not converge.

TABLE 1. *Amplitude of W ($k=4/3$)*

Frequency	Distance (miles)					
	37.7	75.3	150.6	377	753	1,506
			$\sigma=4$			
kc						
0.2	1.00060	-------	0.99407	0.97581	0.93220	0.81890
.5	.99900	-------	.99047	.96176	.89458	.72915
1.0	.99850	-------	.98641	.94614	.85400	.64030
2.0	.99779	0.99334	.98069	.92445	.79981	.53391
5.0	.99638	.98931	.96939	.88286	.70267	.37563
10.0	.99478	.98478	.95680	.83827	.60807	.25677
20.0	.99252	.97839	.93925	.77923	.49714	.15426
50.0	.98802	.96582	.90541	.67493	.33772	.06105
100.0	.98296	.95183	.86886	.57555	.22340	.02377
200.0	.97578	.93237	.81996	.46216	.12951	.00714
500.0	.96130	.89459	.73171	.30572	.04907	.00089
			$\sigma=10^{-2}$			
0.2	1.00060	0.99873	0.99401	0.97582	0.93223	0.81899
.5	-------	-------	.99040	.96179	.89468	.72946
1.0	-------	-------	.98634	.94620	.85425	.64104
2.0	.99843	.99333	.98062	.92461	.80044	.53554
5.0	.99914	.98927	.96930	.88333	.70458	.37952
10.0	.99532	-------	.95652	.83914	.61196	.26304
20.0	.99252	.97783	.93797	.77999	.50364	.16225
50.0	.98508	.95997	.89577	.66732	.34198	.06646
100.0	.96884	.92669	.82821	.53124	.20598	.02256
200.0	.92053	.83428	.66792	.30708	.06808	.00243
500.0	-------	-------	-------	-------	-------	.00000
			$\sigma=10^{-3}$			
0.2	1.00060	0.99873	0.99434	0.97583	0.93228	0.81916
.5	-------	.99732	.99062	.96182	.89485	.73010
1.0	.99936	.99572	.98650	.94625	.85465	.64246
2.0	.99838	.99324	.98056	.92458	.80121	.53834
5.0	.99653	.98866	.96837	.88212	.70525	.38440
10.0	.99365	.98207	.95224	.83247	.60808	.26627
20.0	.98687	.96739	.92012	.75019	.47724	.15362
50.0	.95220	.89869	.79018	.50525	.21268	.02962
100.0	-------	-------	-------	-------	-------	-------
200.0	-------	-------	-------	-------	.00091	.00000
500.0	-------	.02701	.00912	.00069	.00001	.00000
	Distance (kilometers)					
	60.6	121	242	606	1,211	2,420

TABLE 2. *Phase of W ($k=4/3$)*

(The values are all negative)

Frequency	Distance (miles)					
	37.7	75.3	150.6	377	753	1,506
			$\sigma=4$			
kc						
0.2	0.050	-----	0.364	1.431	4.021	11.111
.5	.079	-----	.579	2.267	6.323	17.222
1.0	.112	-----	.825	3.209	8.883	23.829
2.0	.162	0.429	1.178	4.541	12.445	32.717
5.0	.273	.704	1.898	7.183	19.319	49.020
10.0	.414	1.036	2.738	10.160	26.792	65.744
20.0	.641	1.546	3.978	14.377	36.939	87.334
50.0	1.190	2.693	6.616	22.793	55.986	126.041
100.0	1.965	4.205	9.879	32.423	76.410	166.527
200.0	3.341	6.740	15.031	46.466	104.711	222.115
500.0	7.052	13.162	27.188	76.567	162.995	335.992
			$\sigma=10^{-2}$			
0.2	0.087	0.193	0.437	1.546	4.184	11.347
.5	----	-----	.761	2.555	6.735	17.823
1.0	----	-----	1.189	3.787	9.714	25.054
2.0	.536	.943	1.907	5.701	14.121	35.229
5.0	1.232	1.989	3.721	10.104	23.590	55.601
10.0	2.241	-----	6.389	16.047	35.511	79.551
20.0	4.284	6.695	11.298	26.273	54.866	116.592
50.0	10.277	15.572	24.990	53.082	102.976	205.783
100.0	20.107	29.943	46.675	93.804	173.888	335.889
200.0	39.462	57.839	87.843	168.225	300.818	207.051
500.0	---------	---------	---------	---------	---------	250.022
			$\sigma=10^{-3}$			
0.2	0.169	0.309	0.613	1.808	4.558	11.883
.5	----	.640	1.184	3.212	7.674	19.189
1.0	.729	1.151	2.024	5.103	11.603	27.839
2.0	1.361	2.112	3.562	8.343	17.937	40.945
5.0	3.256	4.913	7.886	16.749	33.306	70.570
10.0	6.369	9.455	14.690	29.420	55.301	110.389
20.0	12.533	18.377	27.897	53.146	95.179	181.992
50.0	30.804	44.539	65.916	118.842	202.064	9.181
100.0	----	-----	-----	-----	-----	-------
200.0	----	-----	-----	-----	65.632	9.896
500.0	----	192.925	220.429	320.516	146.157	159.609
	Distance (kilometers)					
	60.6	121	242	606	1,211	2,420

It is believed that these tabulated values of W are of considerable intrinsic theoretical interest, since they can be used to verify the validity of operational methods used to calculate the field of a dipole near gently curved surfaces. The ρ series formula for W was derived by such a method.

Using eq (8) and (11) in conjunction with the tabulated values of W, the amplitude E and phase ϕ of the vertical electric field are plotted in figures 1 to 16. The frequencies chosen are 0.2, 0.5, 1.0, 2, 5, 10, 20, 50, 100, 200, and 500 kc. The ground-conductivity values are 4, 10^{-2}, and 10^{-3} mho/m, corresponding to sea water, well-conducting land, and poorly conducting land, respectively. The values shown in these curves can be compared directly with some earlier calculated results [1], using essentially Norton's method [12] for the frequency range 15 to 500 kc. The agreement is usually within 2 percent for the cases that could be compared directly. It appeared that the maximum discrepancies occurred in regions where the earlier curves were interpolated between the flat-earth values and the first term of the residue series. As mentioned above, no interpolation is required in the present curves, so they are believed to be more accurate.

To conform with standard practice [9], the ratio k of the effective to the actual radius of the earth is taken to be 4/3 in the calculations. To illustrate the effect of modifying the earth's radius, factor k from 1.0 to 4/3 is illustrated in figures 17 and 18. The amplitude E and the phase lag ϕ are plotted as a function of frequency for distances of 124 and 621 miles with a ground conductivity of 10^{-2} mho/m. The difference between the two sets of curves for $k=1$ and 4/3 is not insignificant, particularly from the standpoint of phase. However, at distances less than 150 miles or so, and at frequencies less than 100 kc, the amplitude difference is within 1 percent, and the phase difference is less than 2 degrees.

4. Conclusion

It is believed that the curves presented in this report will provide basic information on ground-wave propagation in the low, very low, and ultra-low frequency bands. It should be emphasized that the ionosphere has not been considered here, and consequently the total field at larger ranges (>200 miles) can be modified. It is possible, however, by using special techniques, to eliminate or reduce the "sky wave" even at large distances so that the computed values of the ground-wave propagated fields should have some significance even out to 1,000 miles.

We thank Loris Perry for assistance with the calculations, and A. G. Jean and J. R. Johler for their helpful suggestions.

5. References

[1] J. R. Wait and L. L. Campbell, Transmission curves for ground wave propagation at V. L. F., Defense Research Telecommunications Establishment Report (no. R. 1), Ottawa, Canada (1952).

[2] K. G. Budden, The Propagation of low frequency waves to great distances, Phil. Mag. **44**, 504–513 (1953).

[3] H. Bremmer, Terrestial radio waves (Elsevier Publishing Co., New York, N. Y., 1949).

[4] H. Norinder, Variations of the electric field in the vicinity of lightning discharges, Arkiv. Geopysik, Bd. 2, No. 20, Stockholm (1952)

[5] A. B. Schneider, Phase variations with range of the ground wave from C. W. transmitters in the 70–130 kc/s band, J. British Inst. Radio Engrs. **12**, 181–191 (1952).

[6] B. G. Pressey, G. E. Ashwell, and C. S. Fowler, The measurement of the phase velocity of ground wave propagation at V. L. F. over a land path, Proc. Inst. Elec. Engrs. **100**, pt. III, 73–84 (1953).

[7] J. R. Wait, Theory of electromagnetic surface waves over geological conductors, Geofisica Pura e Applicata **28**, 47–56 (1954).

[8] J. R. Wait and L. L. Campbell, Effect of a large dielectric constant on ground wave propagation, Can. J. Phys. **31**, 456–457 (1953).

[9] S. S. Atwood and C. R. Burrows, Radio wave propagation, OSRD, Summ. Tech. Report, Committee on Propagation (1946).

[10] G. N. Watson, The diffraction of radio waves by the earth. Proc. Royal Soc. (London) [A] **95**, 546–563 (1919).

[11] B. Van der Pol and H. Bremmer, The diffraction of electromagnetic waves from an electrical point source round a finitely conducting sphere, Phil. Mag. **24**, 141–175 and 825–864 (1937); **25**, 817–834 (1937); **27**, 261–275 (1939).

[12] K. A. Norton, Calculation of the ground wave field intensity, Proc. Inst. Radio Engrs. **29**, 623–643 (1941).

[13] J. R. Wait, Radiation from a vertical antenna over a curved stratified earth, J. Research NBS **56**, (1956) RP2671.

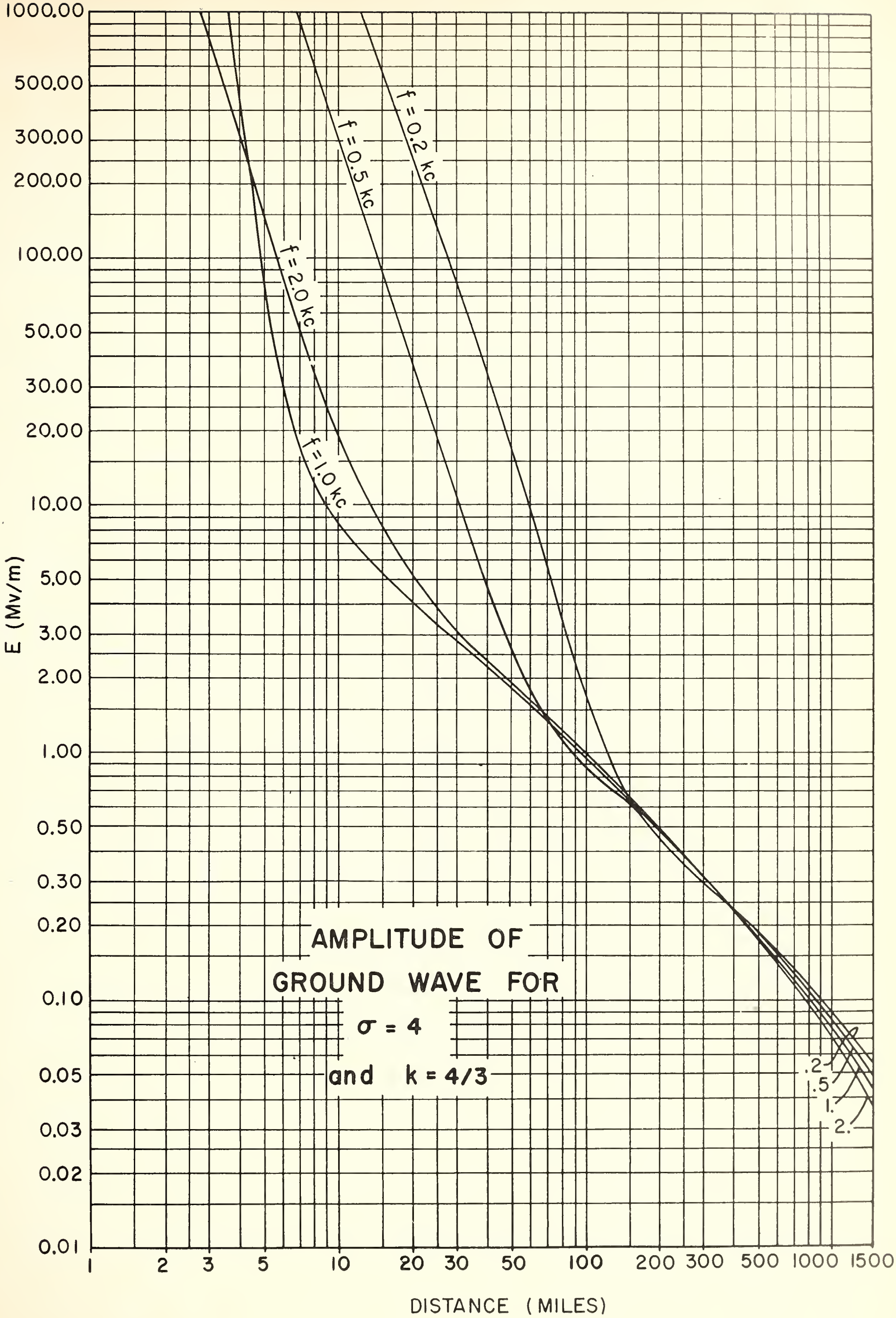

FIGURE 1.

Amplitude of the electric field as a function of distance for various frequencies normalized so that the radiation field is 100 mv/m at $D=1$ mile

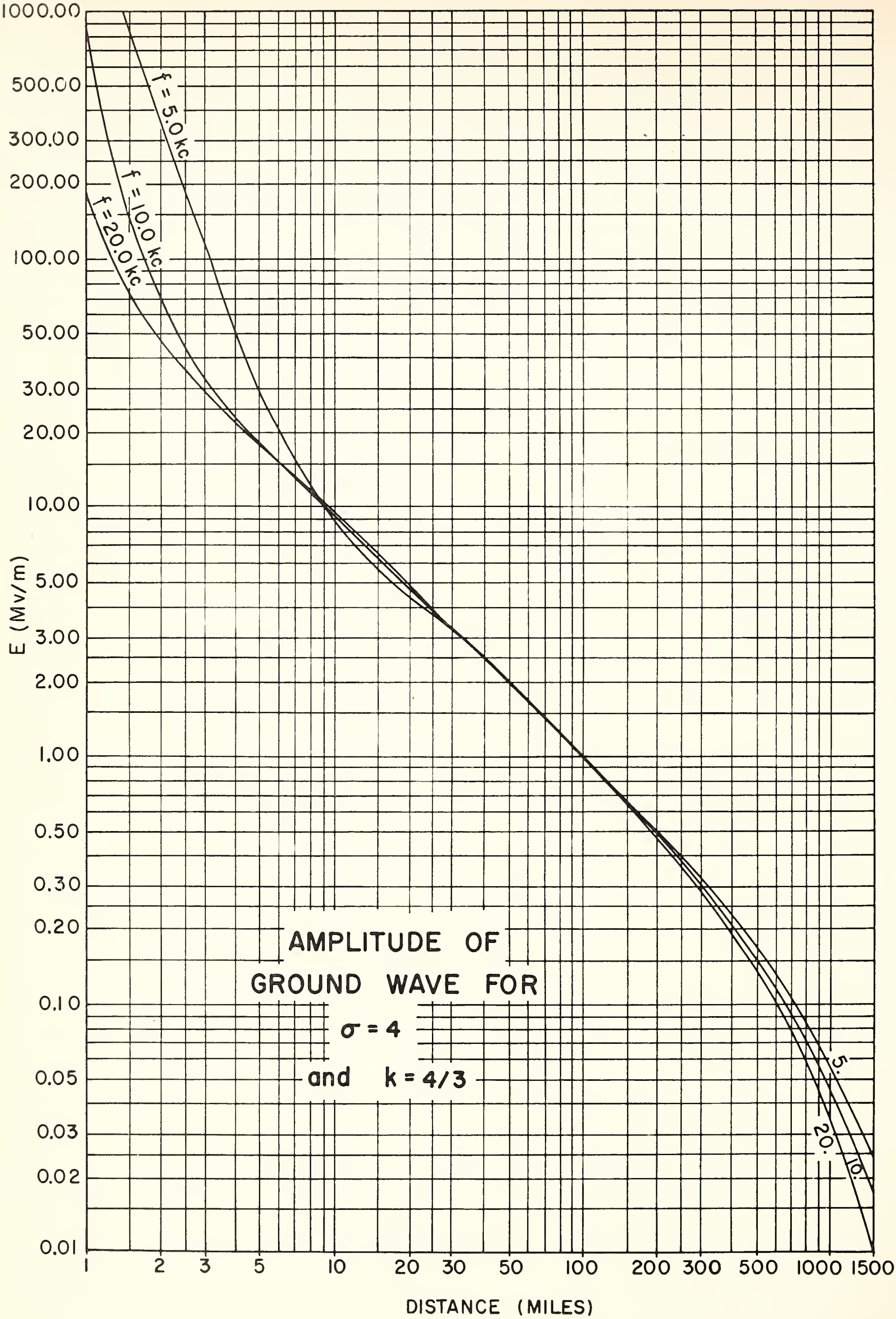

FIGURE 2.

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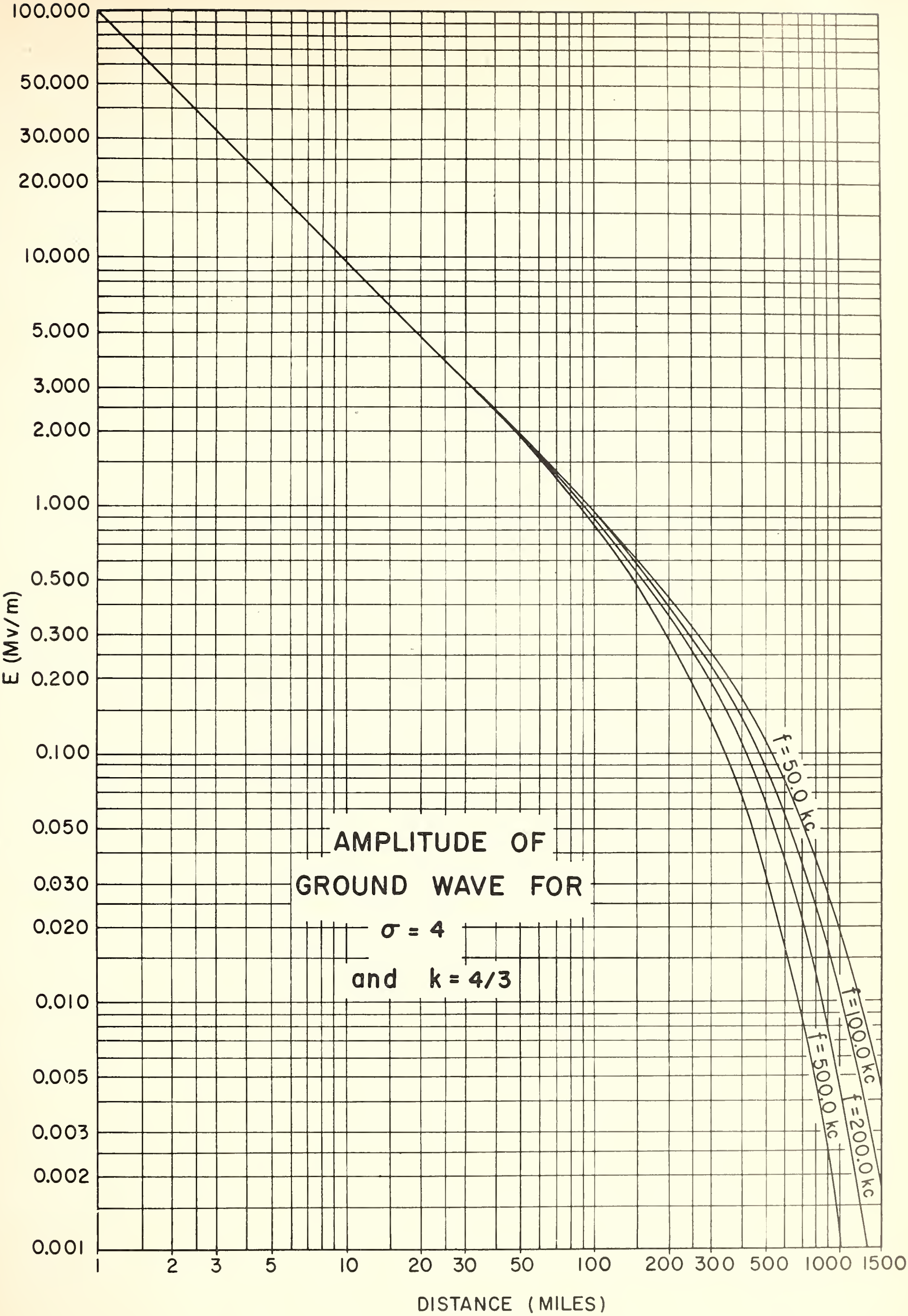

FIGURE 3.

See figure 1 legend.

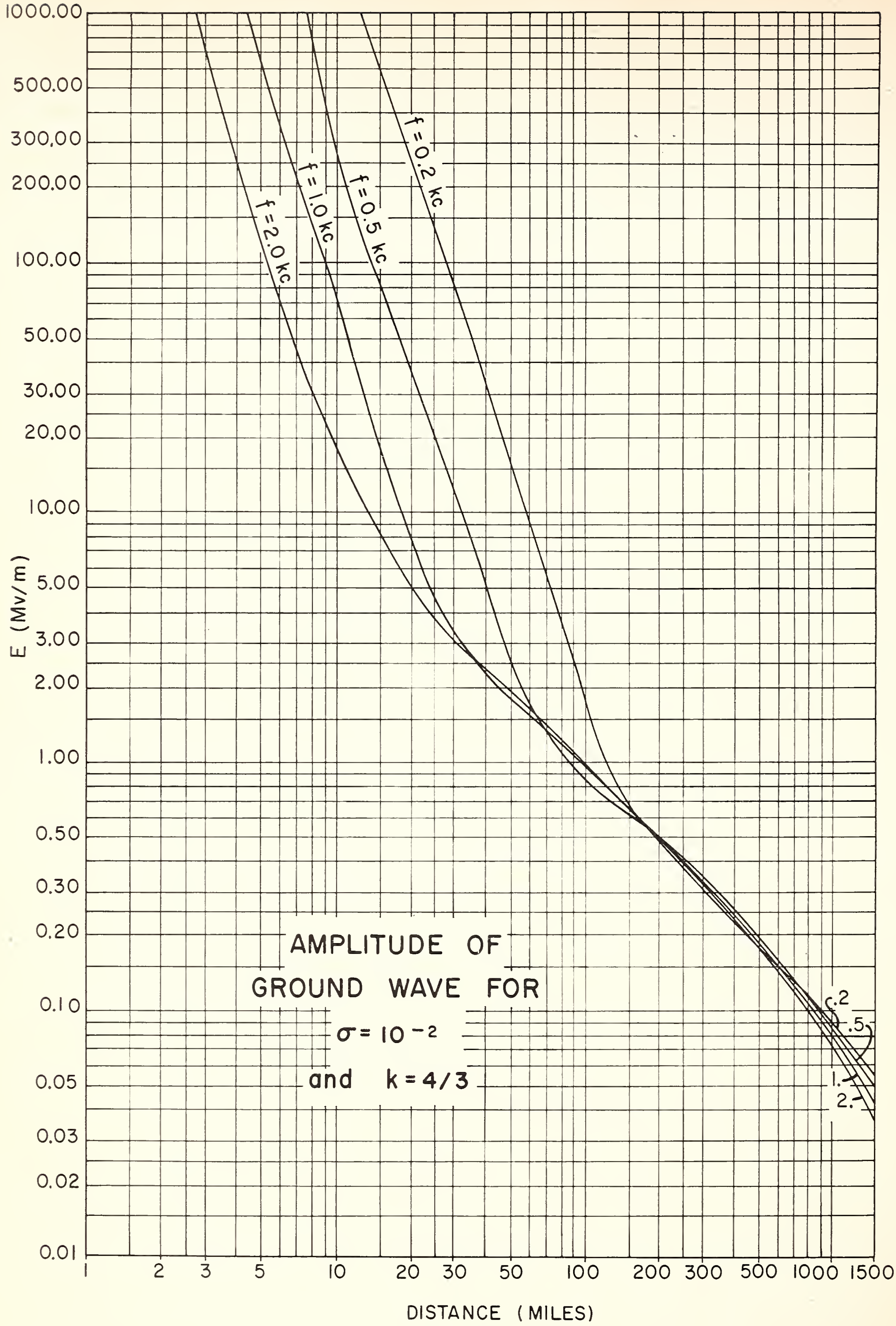

FIGURE 4.

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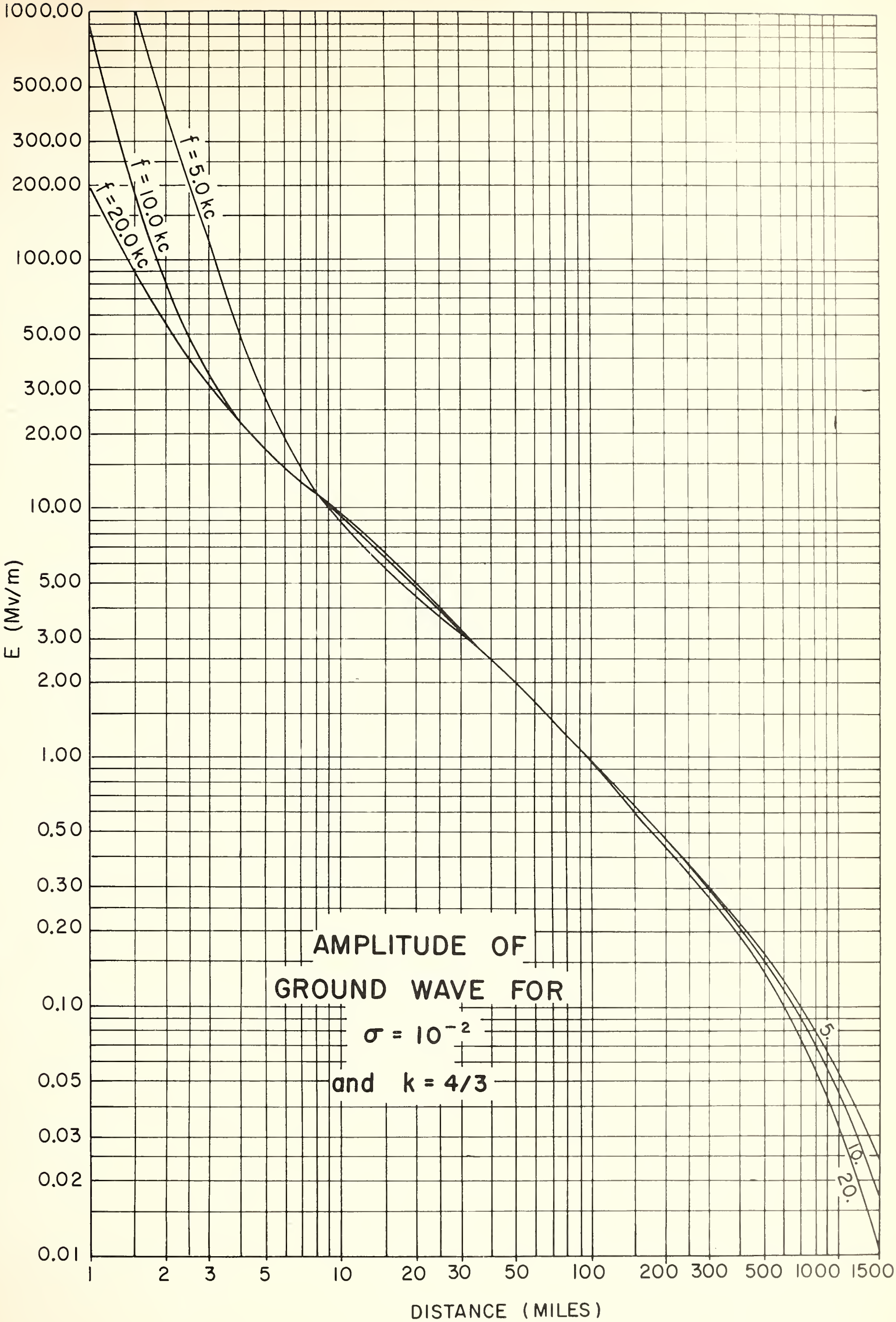

Figure 5.

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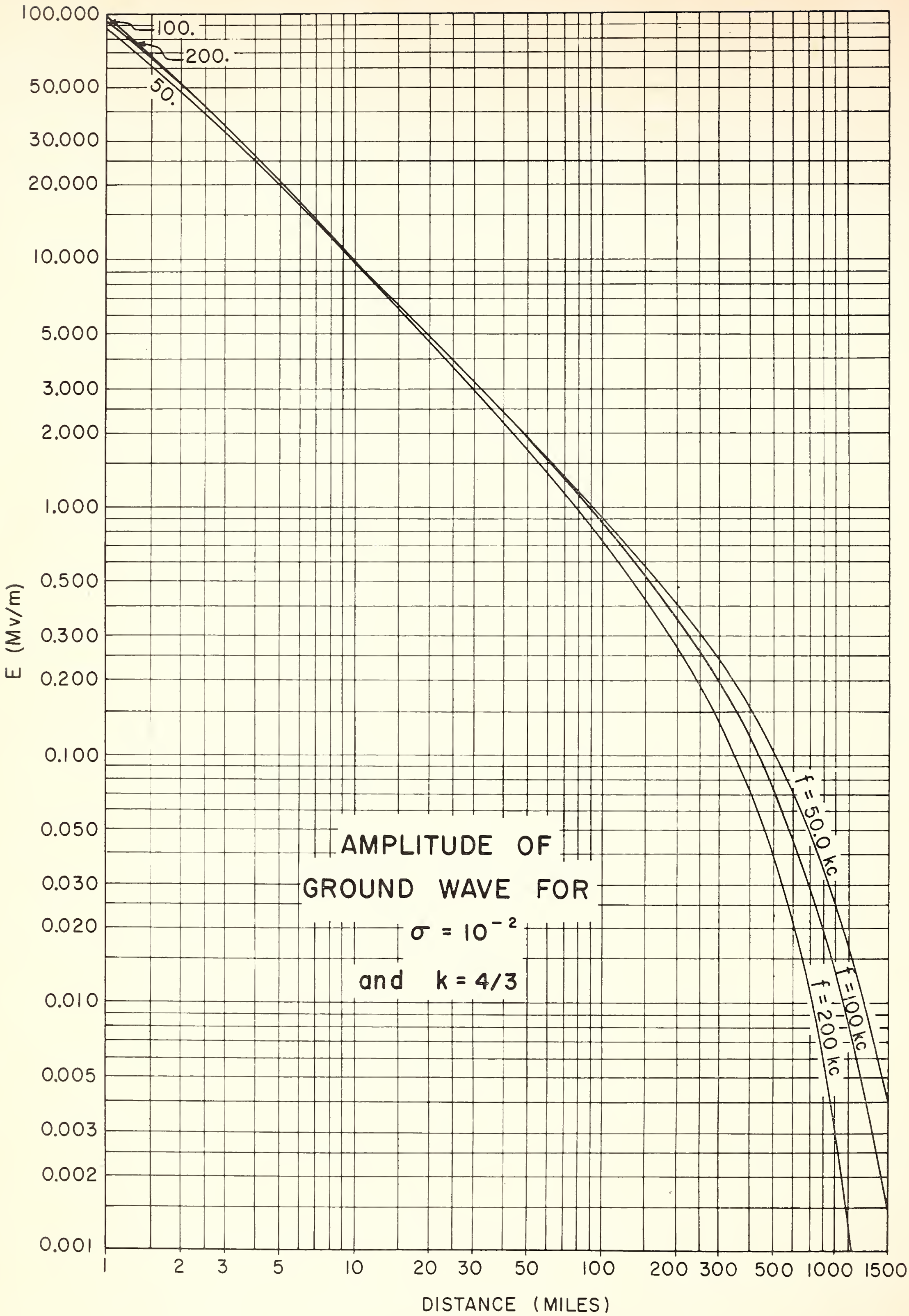

Figure 6.

See figure 1 legend.

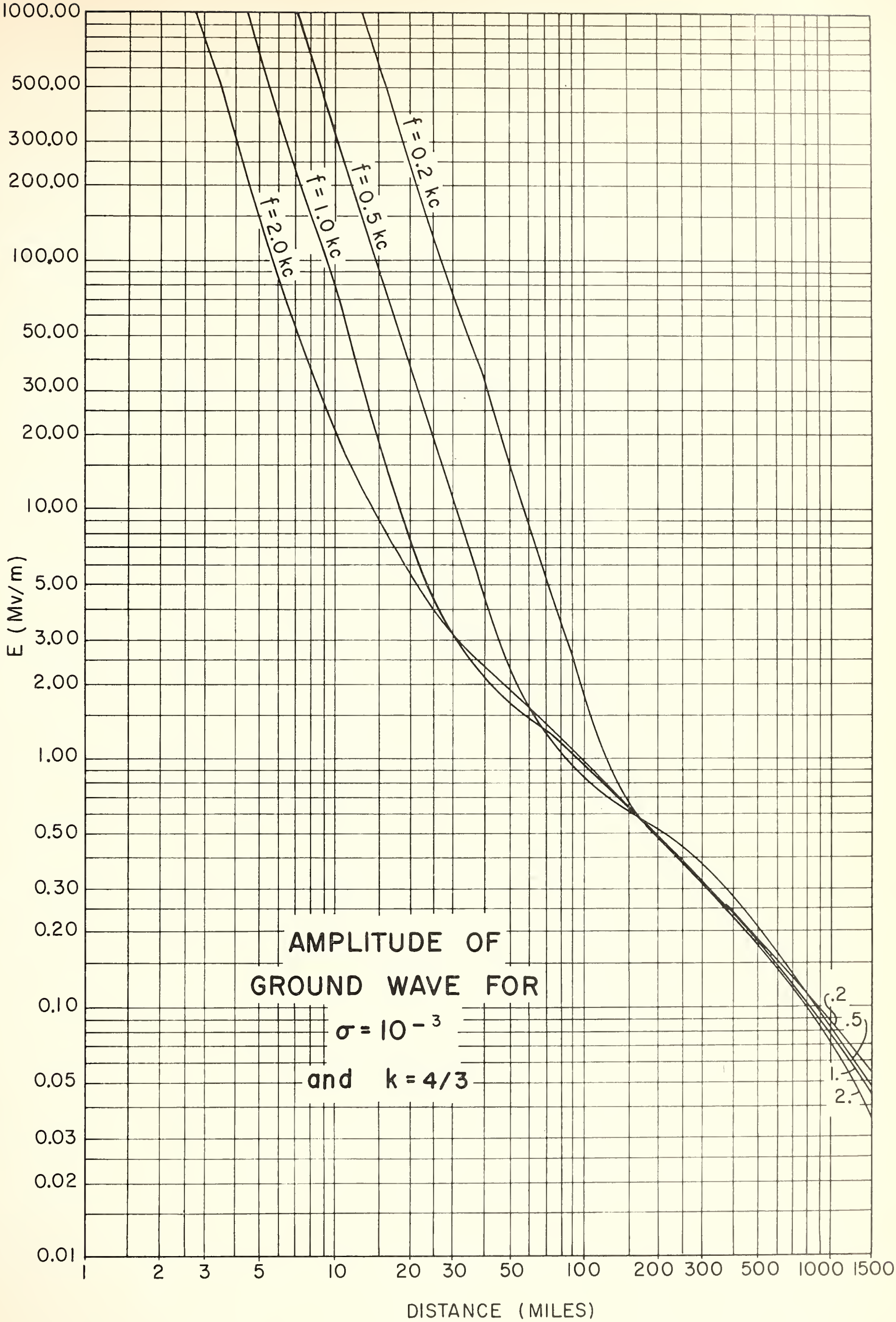

FIGURE 7.

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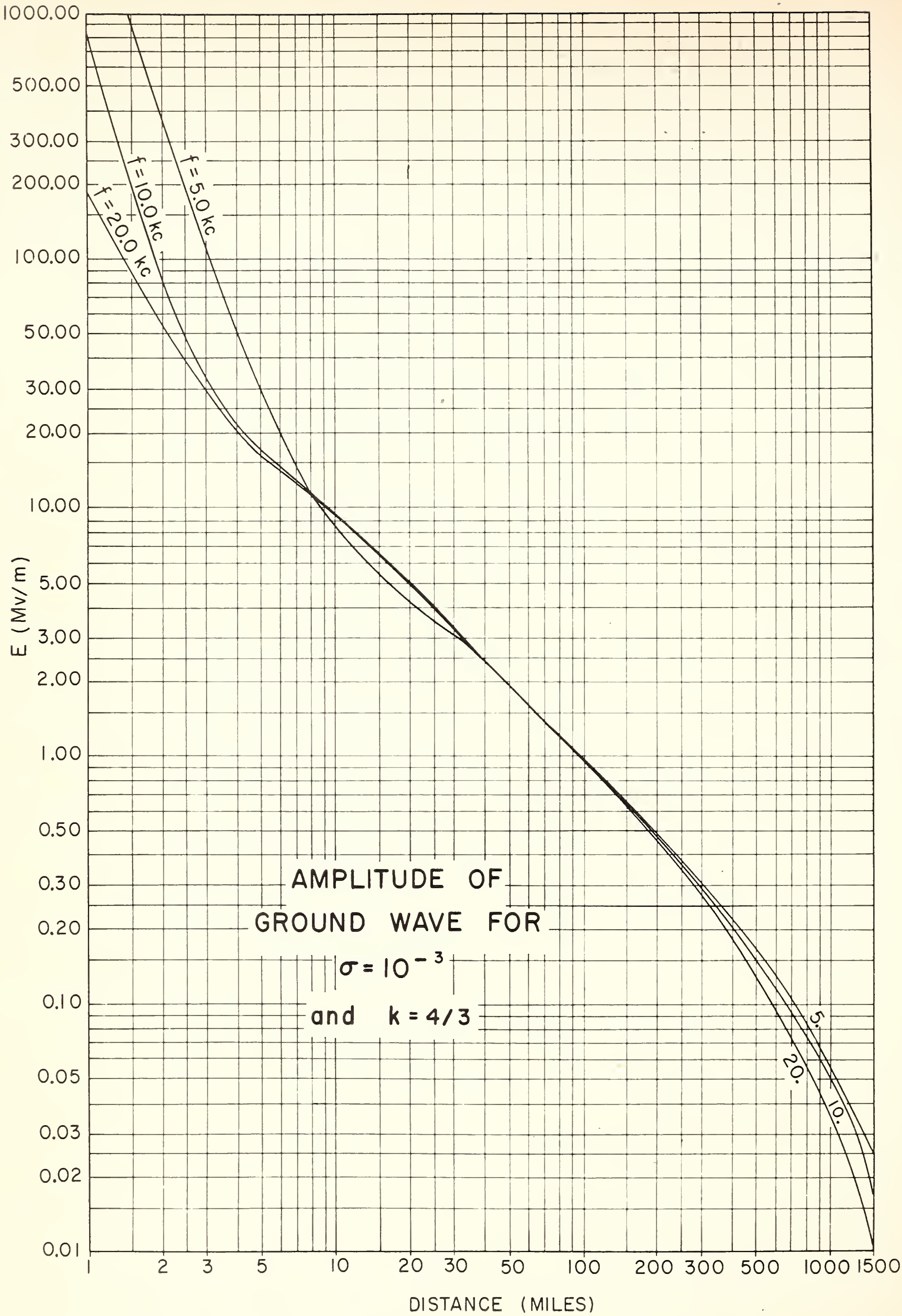

FIGURE 8.

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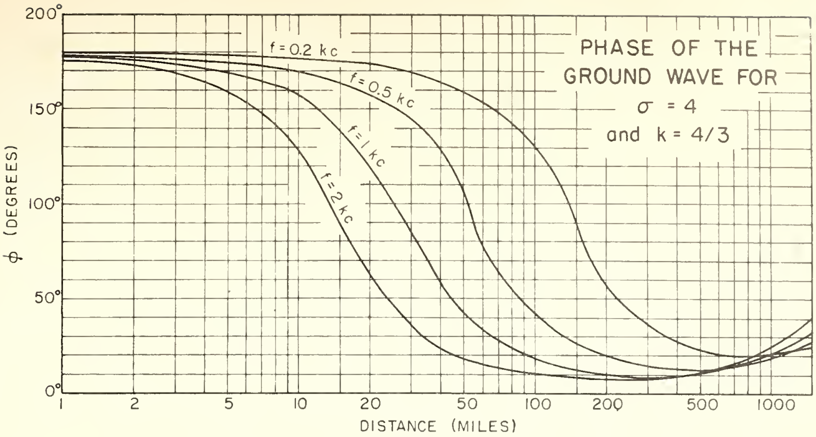

FIGURE 9.

Phase (lag) of the electric field as a function of distance for various frequencies normalized so that the phase is 180° as D approaches zero.

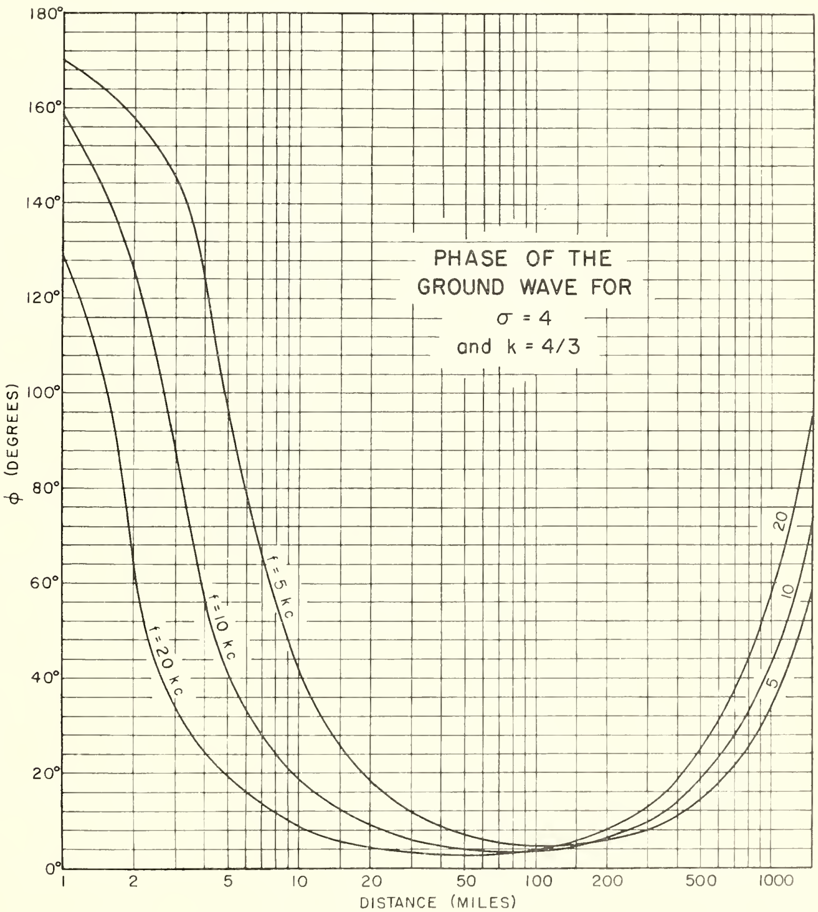

FIGURE 10.

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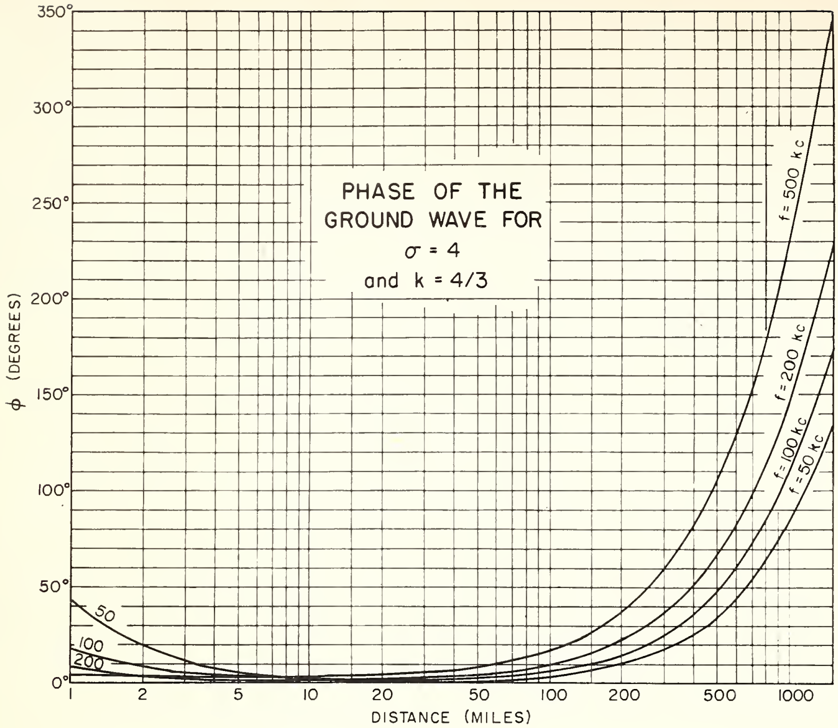

Figure 11.

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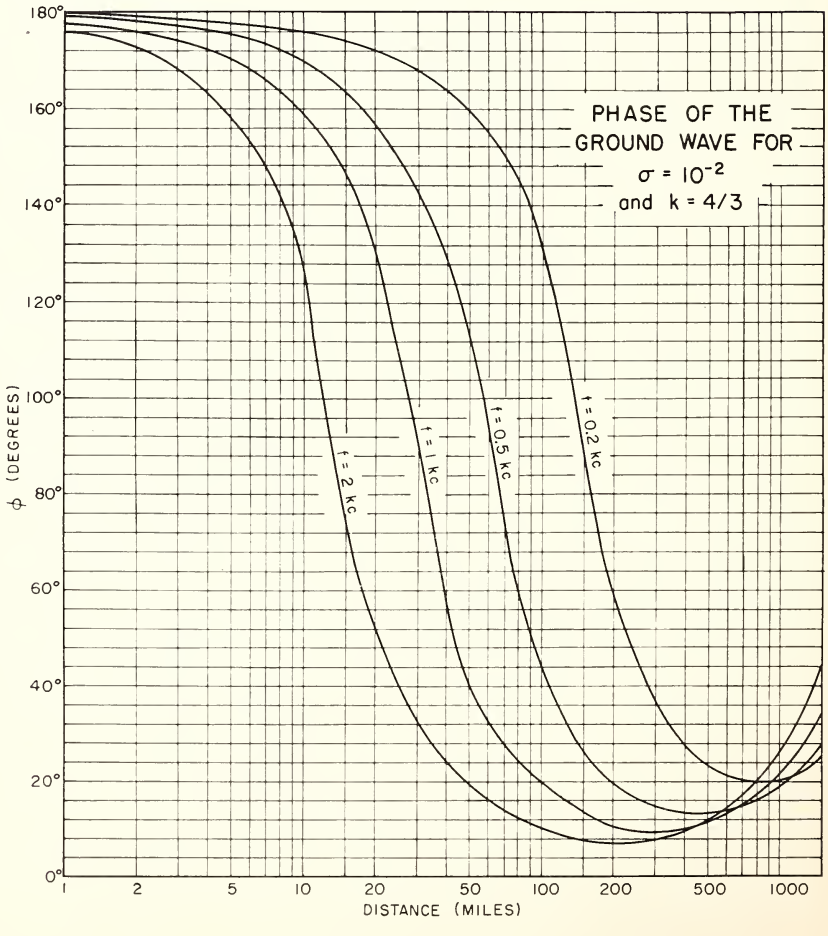

Figure 12.

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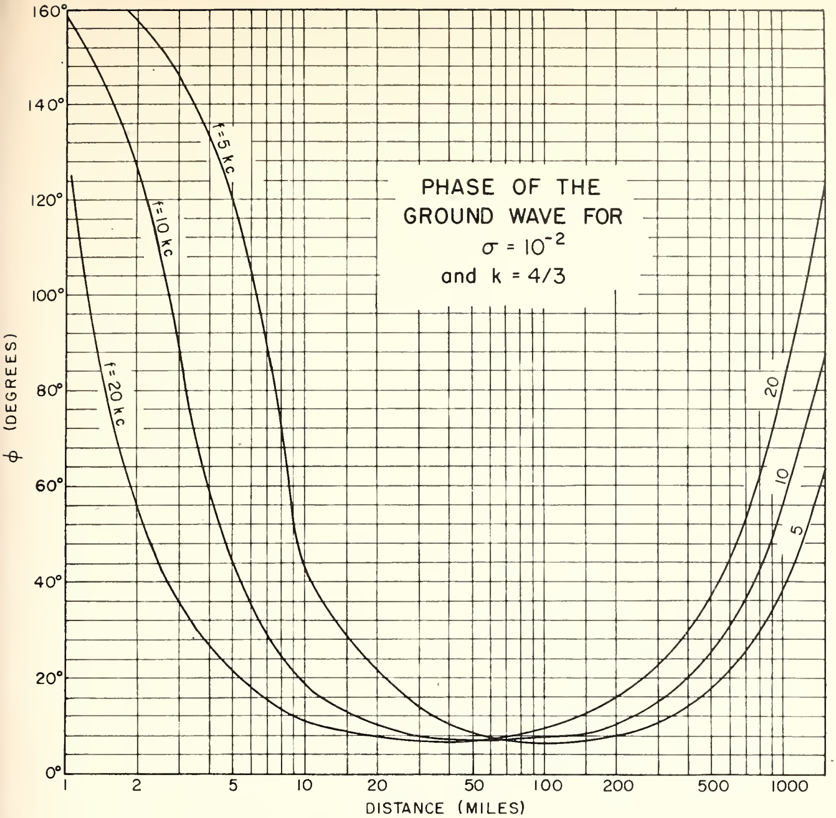

FIGURE 13.

See figure 9 legend.

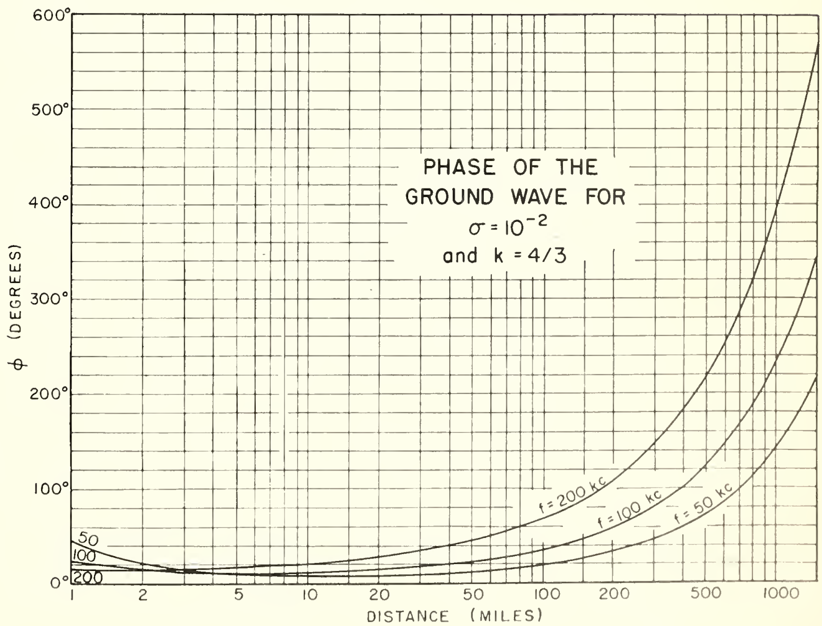

FIGURE 14.

See figure 9 legend.

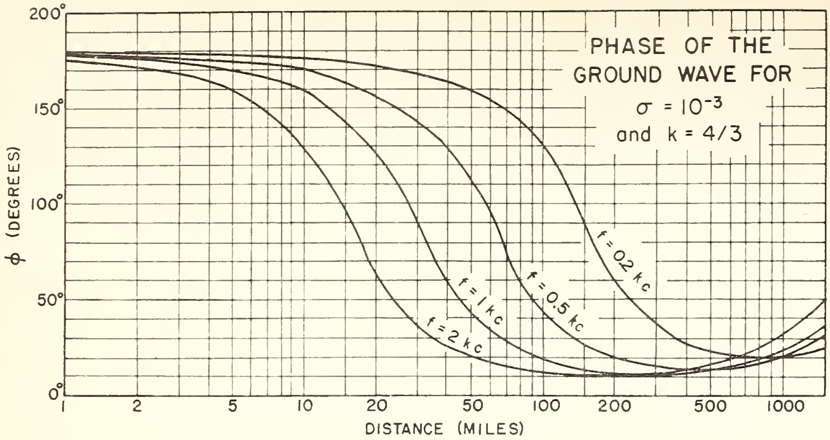

FIGURE 15.

See figure 9 legend.

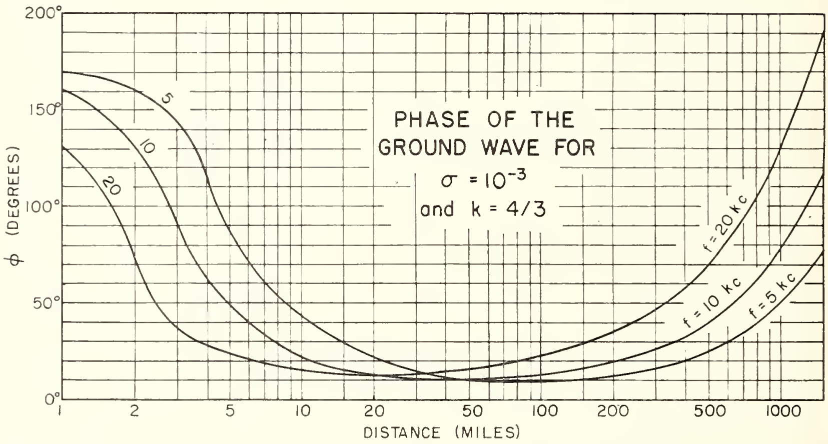

FIGURE 16.

See figure 9 legend

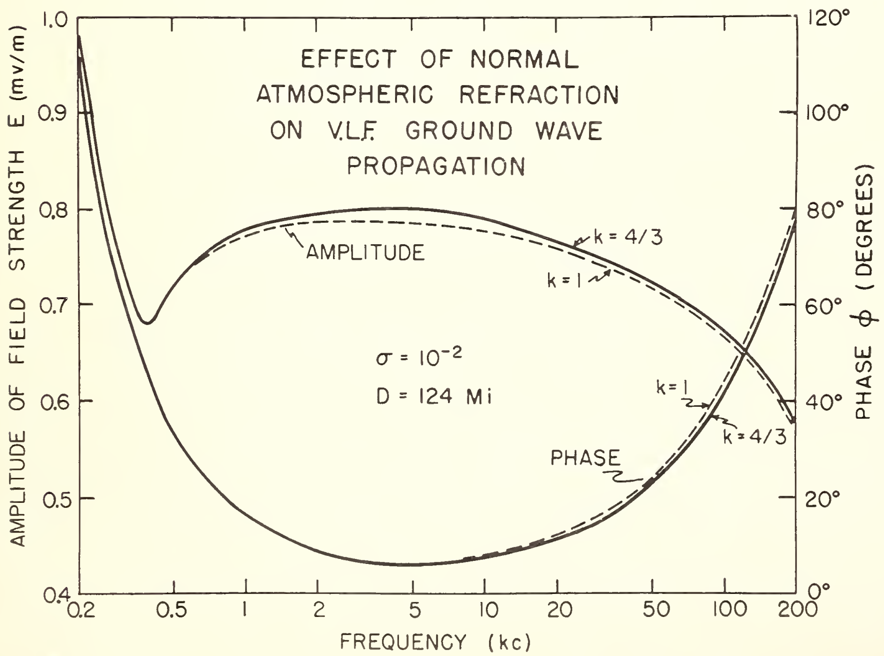

FIGURE 17.

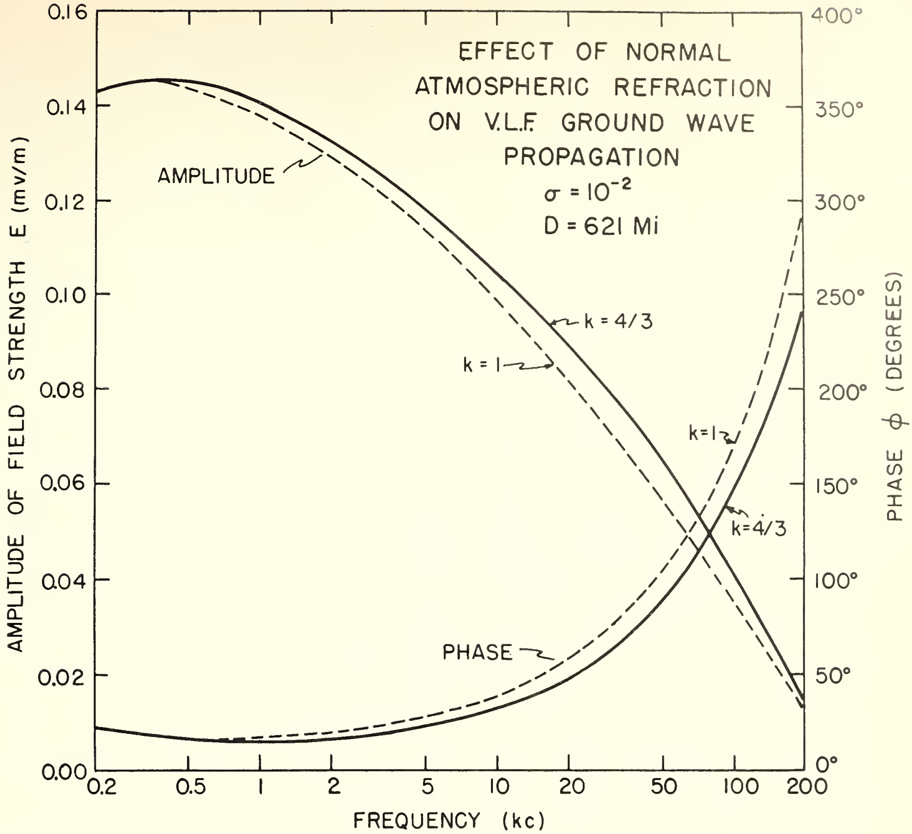

FIGURE 18.

BOULDER, January 19, 1956.

U. S. GOVERNMENT PRINTING OFFICE · 1956

DEPARTMENT OF COMMERCE
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