NBS CIRCULAR 515

Energy and Angle Distribution of the Photoprotons From Deuterium

UNITED STATES DEPARTMENT OF COMMERCE

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A valuable tool for nuclear physicists and engineers, radiochemists, biophysicists, and other workers in the rapidly expanding field of nuclear physics is available in the tables of *Nuclear Data* recently compiled by the National Bureau of Standards.

The initial volume of the tables, together with the supplements, will present a comprehensive collection of experimental values of half-lives, radiation energies, relative isotopic abundances, nuclear moments, and cross sections. Decay schemes and level diagrams, over 125 of which are included in the tables now ready, are to be provided wherever possible.

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UNITED STATES DEPARTMENT OF COMMERCE • Charles Sawyer, *Secretary*
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Energy and Angle Distribution of the Photoprotons From Deuterium

Martin Wiener

National Bureau of Standards Circular 515

Issued December 3, 1951

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.
Price 15 cents

Contents

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Formulas and graphs are given that describe the relationships between the energy of the photon which disintegrates a deuteron and the energy and direction of the resulting photoprotons. Data are also given for the conversion of angles from the laboratory to the center-of-mass system of reference. The formulas are valid for all photon energies, whereas the graphs extend to 200 Mev.

1. Introduction

The energy of each proton ejected when a deuteron is disintegrated by gamma rays is related in a simple manner to the energy of the photon which has caused the disintegration. In addition, the energy of the proton varies negligibly with the angle of emission for a given photon energy until the photon energy is of the order of 10 Mev. As the photon energy increases beyond this value, the photon momentum adds appreciably to the momentum of the protons ejected in the direction of the incident photon, and subtracts from the momentum of those ejected in the backward direction.

In general, there is a fixed relationship between the photon energy, the proton energy, and the angle between the directions of the photon and proton. Given any two of these quantities, one can find the third. Most frequently, one determines experimentally the energy and direction of a proton and one wants to determine the energy of the incident photon.

Graphical and analytical data on the above relationship were given by G. Scharff-Goldhaber [1],[1] who performed the analysis on a nonrelativistic basis and applied it to photon energies up to 100 Mev.

In the present paper relativistically exact formulas and a set of graphs are given.[2]

Most theoretical analyses are performed in the center-of-mass system of reference, whereas experimental data are collected in the laboratory system.[3] In order to reduce the experimental results to a theoretically meaningful form, additional relationships have been provided. These relationships pertain to the direction of the proton and to the solid angle measurements in the laboratory and in the center-of-mass systems.

The relationships are presented in graphical form as follows:

1. Proton energy versus photon energy at various constant values of the angle of proton emission in the laboratory system (figs. 2 to 8).
2. Angle of proton emission versus photon energy at various constant proton energies, in the laboratory system (figs. 9 to 12).
3. Proton energy versus angle of proton emission at various constant photon energies, in the laboratory system (figs. 13 to 14).
4. Angle of proton emission in the laboratory system versus angular shift of proton emission from the center-of-mass system at various constant photon energies (figs. 15 and 16).
5. Ratio of solid angles in the laboratory system and the center-of-mass system versus the angle of proton emission in the laboratory system at various constant photon energies (fig. 17).

2. Energy-Angle Relationships in the Laboratory System

The following symbols are used:

$h\nu$=energy of the incident photon
T_p=kinetic energy of the proton ejected
T_n=kinetic energy of the neutron ejected
P_p=momentum of the proton ejected
P_n=momentum of the neutron ejected
m_p=rest-mass of the proton
m_n=rest-mass of the neutron
m_D=rest-mass of the deuteron
E_B=binding energy of the deuteron=$(m_p+m_n-m_D)c^2$
θ_p=angle of proton emission with respect to the direction of the incident photon
θ_n=angle of neutron emission with respect to the direction of the incident photon.

[1] Figures in brackets indicate the references at the end of this paper.
[2] Since this work was completed, it has been brought to the author's attention that a similar derivation, without calculations, was made by J. Blaton. (2)
[3] The laboratory system of reference is that system in which the experiment is performed, whereas the center-of-mass system of reference is that special system in which the total momentum vanishes.

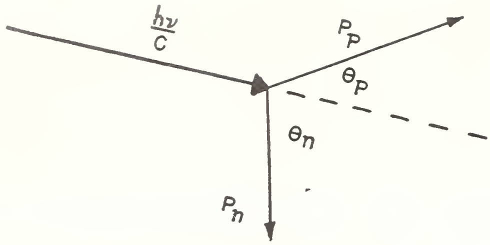


FIGURE 1. *Momentum diagram, in laboratory system, for incident photon, $h\nu$, giving rise to proton with momentum, P_p, and angle of emission, θ_p, and neutron with momentum, P_n, and angle of emission, θ_n.*

The equations expressing the conservation of energy and momentum are (fig. 1):

$$h\nu - E_B = T_p + T_n \tag{1}$$

$$\frac{h\nu}{c} = P_p \cos\theta_p + P_n \cos\theta_n \tag{2}$$

$$0 = P_p \sin\theta_p + P_n \sin\theta_n \tag{3}$$

From eq (2) and (3), eliminating θ_n, one obtains

$$\left(\frac{h\nu}{c} - P_p \cos\theta_p\right)^2 + (P_p^2 \sin^2\theta_p) = P_n^2.$$

Furthermore,

$$P_n^2 = \frac{T_n^2}{c^2} + 2T_n m_n; \quad P_p^2 = \frac{T_p^2}{c^2} + 2T_p m_p.$$

Elimination of T_n from these equations and (1) yields

$$h\nu = \frac{(m_nc^2 + m_pc^2 - E_B)T_p + E_B\left(m_nc^2 - \frac{E_B}{2}\right)}{(m_nc^2 - T_p - E_B) + \sqrt{T_p^2 + 2T_pm_pc^2}\cos\theta_p}. \tag{4}$$

Equation (4) involves no restrictive assumptions. Errors involved in determining $h\nu$ from given values of T_p and θ_p will arise only from errors in the assumed values of the constants m_nc^2, m_pc^2, and E_B.

Equation (4) can also be applied to any simple photodisintegration reaction where the binding energy and the masses of the two products are known.

The following values of masses and their probable errors are derived from Dumond and Cohen [3]: $m_p = 1.007582 \pm 0.000003$ mu; $m_D = 2.014176 \pm 0.000006$ mu; and 1 mass unit $=$ 1 mu $= 931.04 \pm 0.07$ Mev.

The value [4] for the binding energy, E_B, is taken from Bell and Elliott [4]: $E_B = 2.237 \pm 0.005$.

The following value of m_n results from these data: $m_n = 1.008997 \pm 0.000009$ mu.

The corresponding rest energies are $m_nc^2 = 939.41 \pm 0.03$ Mev; $m_pc^2 = 938.10 \pm 0.01$ Mev.

The error in $h\nu$ due to the uncertainties in these constants can then be determined in the usual manner; i. e.,

$$(\Delta h\nu)^2 = \left(\frac{\partial h\nu}{\partial m_D}\right)^2 (\Delta m_D)^2 + \left(\frac{\partial h\nu}{\partial m_p}\right)^2 (\Delta m_p)^2 + \left(\frac{\partial h\nu}{\partial E_B}\right)^2 (\Delta E_B)^2 + \left(\frac{\partial h\nu}{\partial c}\right)^2 (\Delta c)^2,$$

using $c = 299{,}776 \pm 4$ km/sec [3].

From this, it was determined that $(\Delta h\nu)$ max ≤ 0.1 Mev (for the range up to 200 Mev).

Replacing the constants in eq (4) by their numerical values to five significant figures, and expressing T_p and $h\nu$ in Mev, one obtains

$$h\nu = \frac{1875.3T_p + 2099.0}{937.18 - T_p + \sqrt{1876.2T_p + T_p^2}\cos\theta_p}. \tag{5}$$

3. Angular Correlation Between the Laboratory and Center-of-Mass Coordinate Systems

The transformation equations for the angles in the laboratory system to the corresponding angles in the center-of-mass system can be arrived at in the following manner:

In the laboratory system, the proton, with velocity v_p, makes an angle, θ_p, with the direction of the incident photon. If we choose the direction of the x-axis to be that of the incident photon, v_p can be expressed as the sum of two components, v_x, in the direction of the photon, and v_y, perpendicular to this direction. The corresponding quantities in the center-of-mass system are v_p', v_x', v_y', and $\cos\theta_p'$.

The center-of-mass system moves with a velocity, v_c, which is given by the following considerations:

The photon energy in the center-of-mass system is [6]

$$h\nu' = h\nu\sqrt{\frac{1-\beta_c}{1+\beta_c}},$$

where $\beta_c = v_c/c$.

The balance of momentum in the center-of-mass system requires that,

$$\frac{h\nu'}{c} = m_Dc\frac{\beta_c}{\sqrt{1-\beta^2c}}. \tag{6}$$

From this equation, it follows that

$$\beta_c = \frac{h\nu}{h\nu + m_Dc^2}. \tag{7}$$

[4] Since these calculations were made, Bell and Elliott have reported a revised value for E_B of 2.230 ± 0.007 Mev [5]. This revision will introduce an error $(\Delta h\nu)$ max<0.01 Mev, the maximum error occurring in our range of tabulation, at 200 Mev in the backward direction, i. e., $h\nu = 200$ Mev, $\cos\theta_p = -1$.

Similarly, the proton velocity, $v_p=\beta_p c$, can be expressed in the form

$$\beta_p=\frac{\sqrt{2T_p m_p c^2+T_p^2}}{m_p c^2+T_p}. \tag{8}$$

The Einstein velocity addition theorem [7] gives the transformation equations

$$v_x'=\frac{v_x-v_c}{1-v_c v_x/c^2} \qquad v_y'=\frac{v_y\sqrt{1-\beta_c^2}}{1-v_c v_x/c^2}.$$

Therefore,

$$\tan\theta_p'=v_y'/v_x'=v_y\sqrt{1-\beta_c^2}/(v_x-v_c)=\sin\theta_p\sqrt{1-\beta_c^2}/(\cos\theta_p-\beta_c/\beta_p). \tag{9}$$

This relationship is illustrated graphically in the curves of figures 15 and 16.

The inverse relationship is

$$\tan\theta_p=\frac{\sin\theta_p'\sqrt{1-\beta_c^2}}{\cos\theta_p'+\beta_c/\beta_p'} \tag{10}$$

4. Ratio of the Laboratory System Solid Angles to the Center-of-Mass System Solid Angles

In order to transform the density of protons ejected per unit solid angle in the laboratory system to the corresponding density in the center-of-mass system, one must calculate the solid angle ratio,

$$\frac{\sin\theta_p d\theta_p}{\sin\theta_p' d\theta_p'}=\frac{d(\cos\theta_p)}{d(\cos\theta_p')}. \tag{11}$$

The calculation may be performed starting from eq (10) because β_p' is independent of θ_p'. (On the contrary, β_p in eq (9) is a function of θ_p).

$$\cos\theta_p=\frac{1}{\sqrt{[1+\tan^2\theta_p]}}=\frac{1+\frac{\beta_c}{\beta_p'}\sqrt{1+\tan^2\theta_p'}}{\sqrt{\left[1+\frac{\beta_c}{\beta_p'}\sqrt{1+\tan^2\theta_p'}\right]^2+\tan^2\theta_p'(1-\beta_c^2)}}$$

$$=\frac{\cos\theta_p'+\beta_c/\beta_p'}{\sqrt{(\cos\theta_p'+\beta_c/\beta_p')^2+(1-\cos^2\theta_p')(1-\beta^2_c)}}.$$

$$\frac{d(\cos\theta_p)}{d(\cos\theta_p')}=\frac{[\cos\theta_p'+\beta_c/\beta_p']^2+(1-\cos^2\theta_p')(1-\beta_c^2)-(\cos\theta_p'+\beta_c/\beta_p')[(\cos\theta_p'+\beta_c/\beta_p')-\cos\theta_p'(1-\beta_c^2)]}{[(\cos\theta_p'+\beta_c/\beta_p')^2+(1-\cos^2\theta_p')(1-\beta_c^2)]^{3/2}}$$

$$=\frac{\beta_p'^2[\beta_p'+\beta_c\cos\theta_p'](1-\beta_c^2)}{[(1+\beta_p'\beta_c\cos\theta_p')^2-(1-\beta_c^2)(1-\beta_p'^2)]^{3/2}}. \tag{12}$$

Equation (12) gives the relationship in center-of-mass terms. It is convenient to express the solid angle ratio in terms of laboratory system variables.

Using the same transformations as before, namely,

$$\cos\theta_p' = \beta_x'/\beta_p' \qquad \beta_p' = \sqrt{\beta_x'^2 + \beta_y'^2}$$

$$\beta_x' = \frac{\beta_x - \beta_c}{1 - \beta_c\beta_x} \qquad \beta_y' = \frac{\beta_y\sqrt{1-\beta_c^2}}{1-\beta_c\beta_x},$$

one can obtain,

$$\frac{d(\cos\theta_p)}{d(\cos\theta_p')} = \frac{\sqrt{(1-\beta_p\beta_c\cos\theta_p)^2 - (1-\beta_p^2)(1-\beta_c^2)}\,(\beta_p - \beta_c\cos\theta_p)}{\beta_p^2(1-\beta_c^2)}.$$

A family of constant $h\nu$ curves describing this ratio is given in figure 17.

5. References

[1] G. Scharff-Goldhaber, Brookhaven National Laboratory-I-3 (August 1, 1948).
[2] J. Blaton, Matematisk Fysiske Meddelelser **24,** No. 20 (1950).
[3] J. W. Dumond and E. R. Cohen, Rev. Modern Phys. **20,** 82 (1948).
[4] R. E. Bell and L. G. Elliott, Phys. Rev. **74,** 1552 (1948).
[5] R. E. Bell and L. G. Elliott, Phys. Rev. **79,** 392 (1950).
[6] See, for example, G. Joos, Theoretical physics, page 234 (Hafner Publishing Co., New York, N. Y.)
[7] G. Joos, Theoretical physics, p. 232.

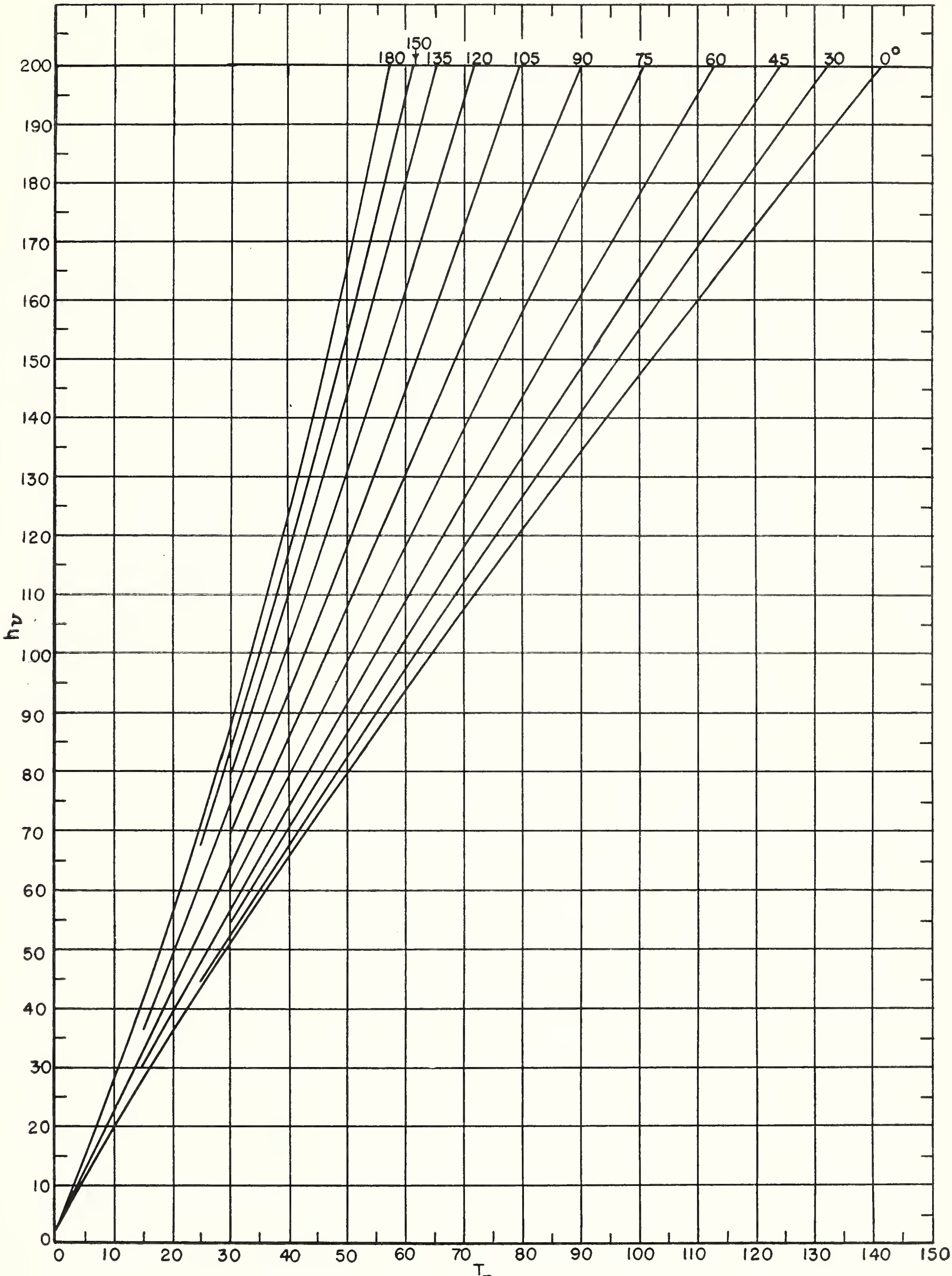
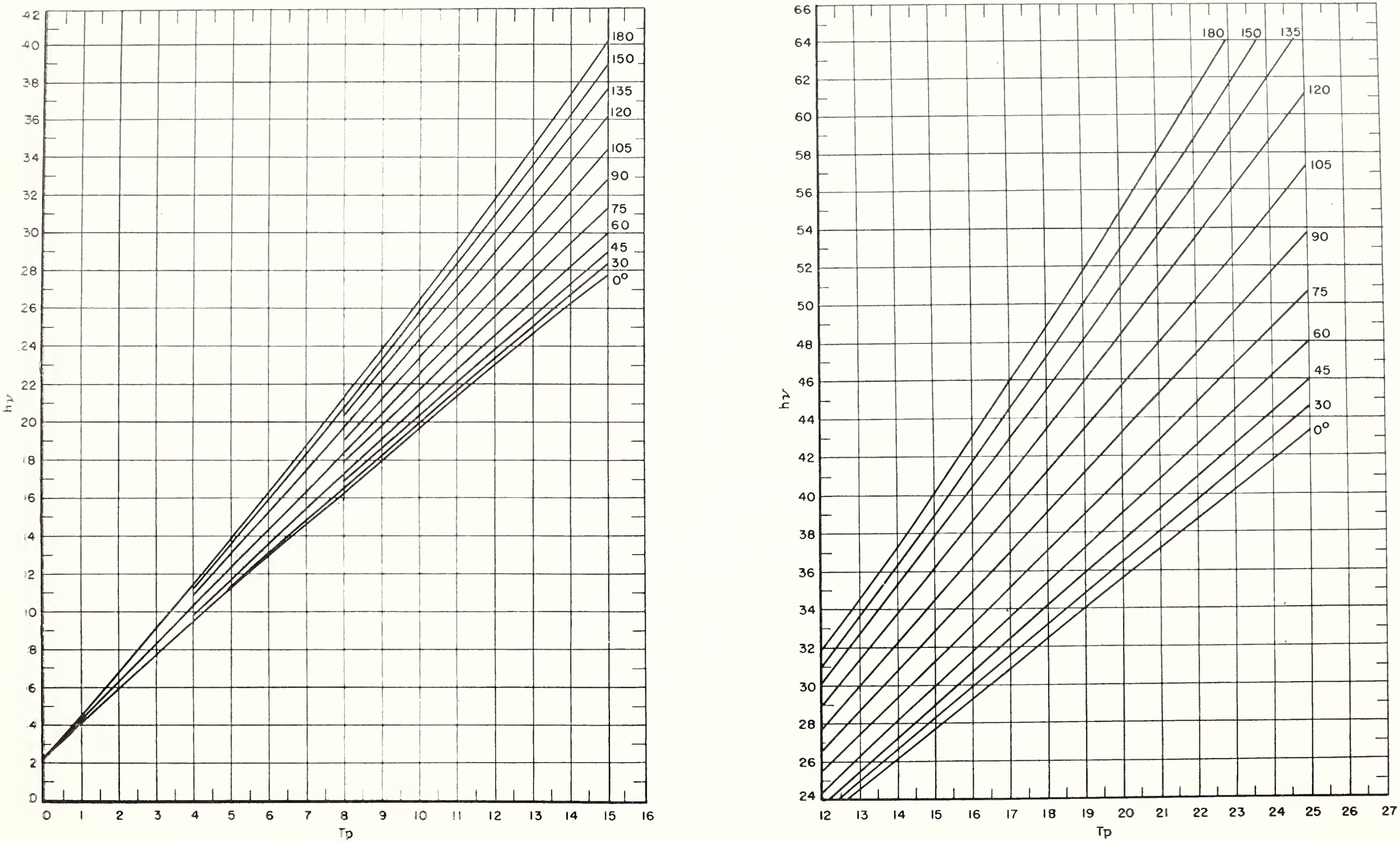
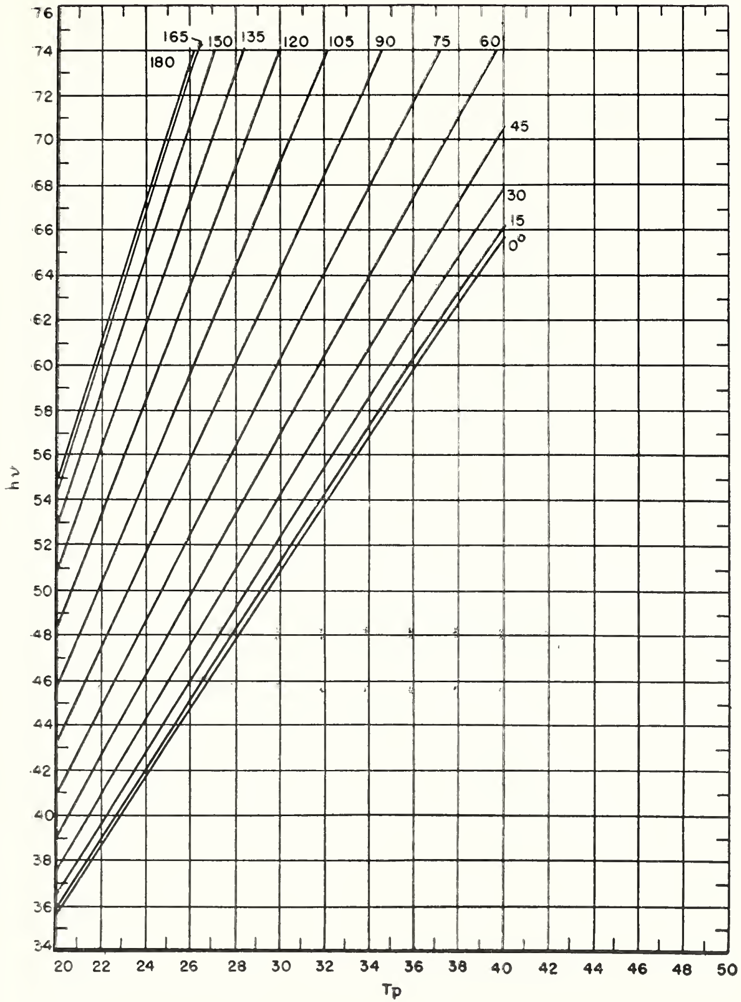
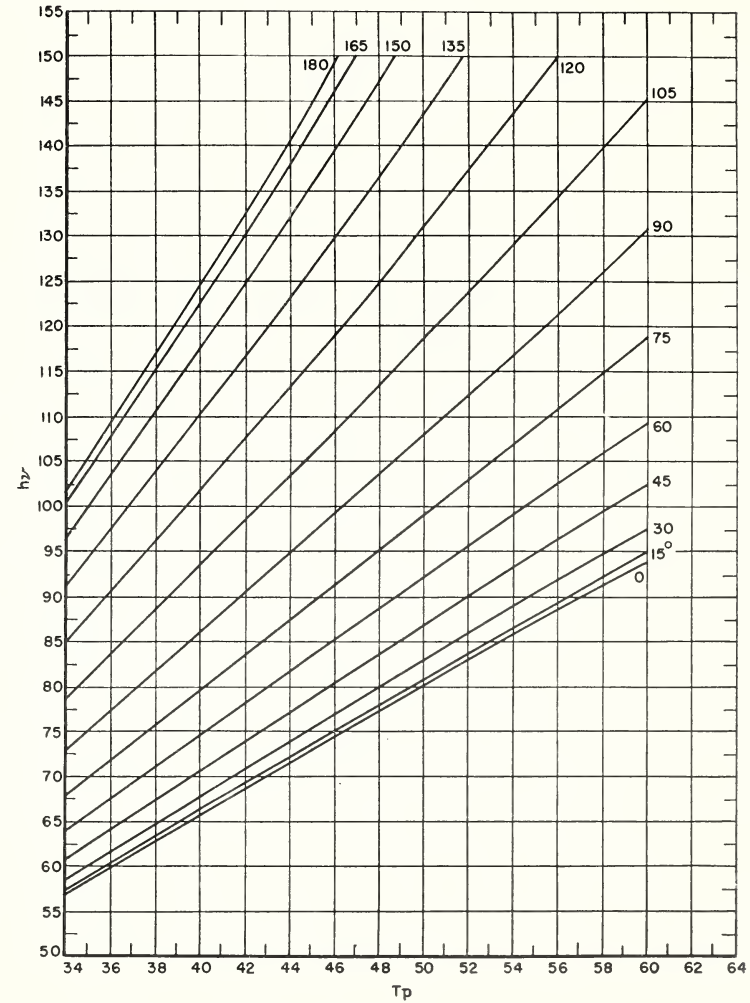


FIGURE 2. *A family of constant θ_p curves for photon energy range 0 to 200 Mev, at 15° intervals.*
Points calculated at proton energies of 2, 3, 4, 5, 8, 10, 15, 17, 20, 22, 25, 30, 35, 40, 45, 50, 60, 70, 75, 80, 90, 100, 110, 120, 125, 130, 140 Mev.

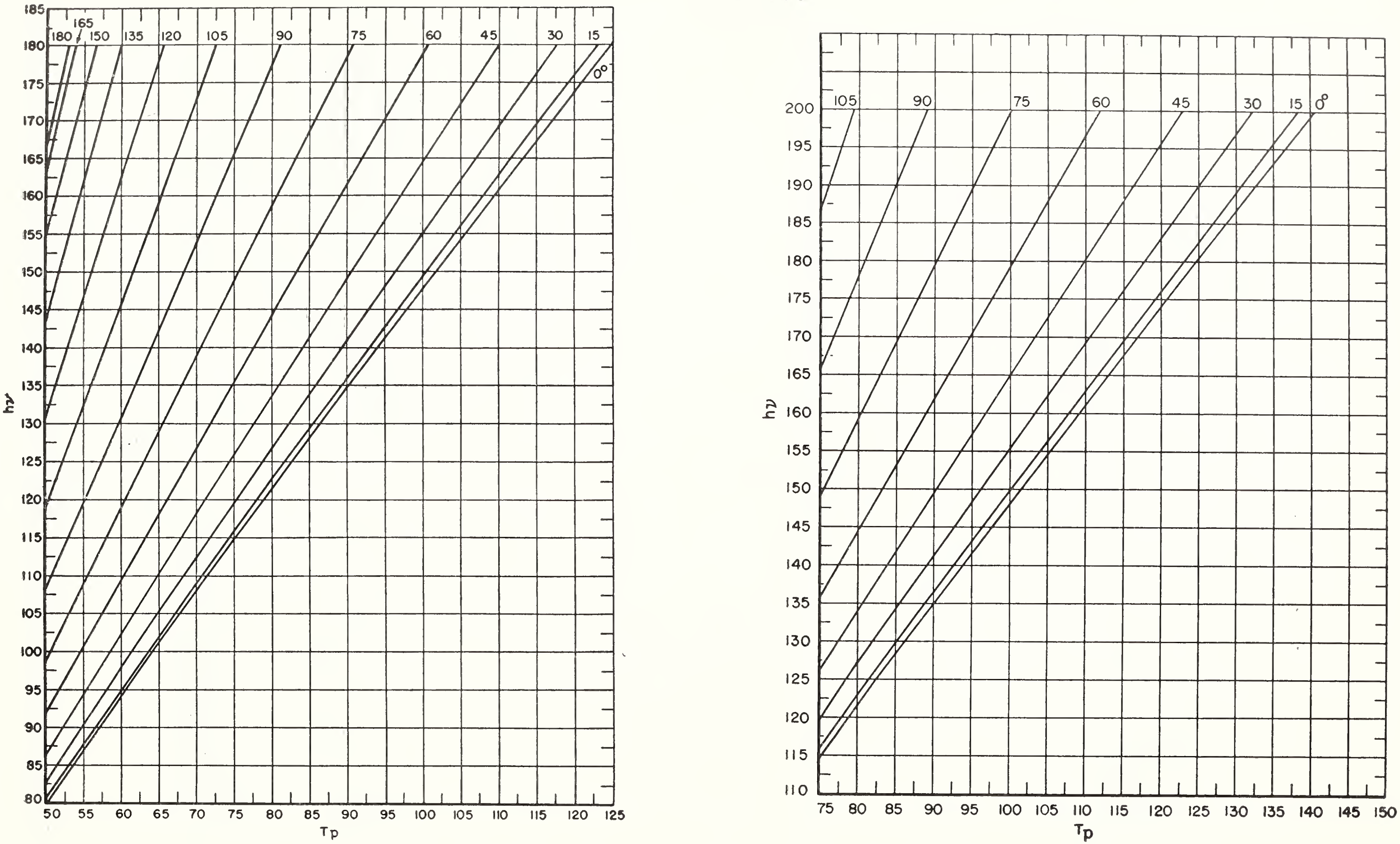


FIGURES 3 AND 4. *Detailed reproductions of figure 2, at 15° intervals for interpolation purposes.*





FIGURES 5 AND 6. *Detailed reproductions of figure 2, at 15° intervals for interpolation purposes.*



FIGURES 7 AND 8. *Detailed reproductions of figure 2, at 15° intervals for interpolation purposes.*

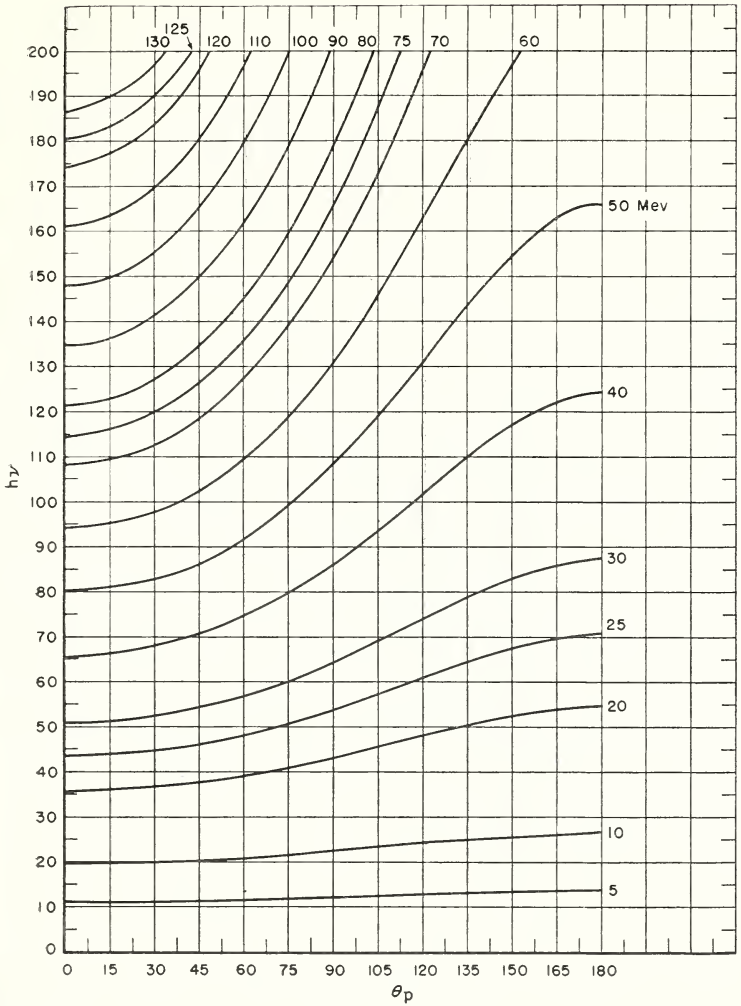


FIGURE 9. *A family of constant T_p curves for the photon energy range 0 to 200 Mev, showing photon energy, $h\nu$, as a function of laboratory angle, θ_p.*

Points calculated at 15° intervals of θ_p.

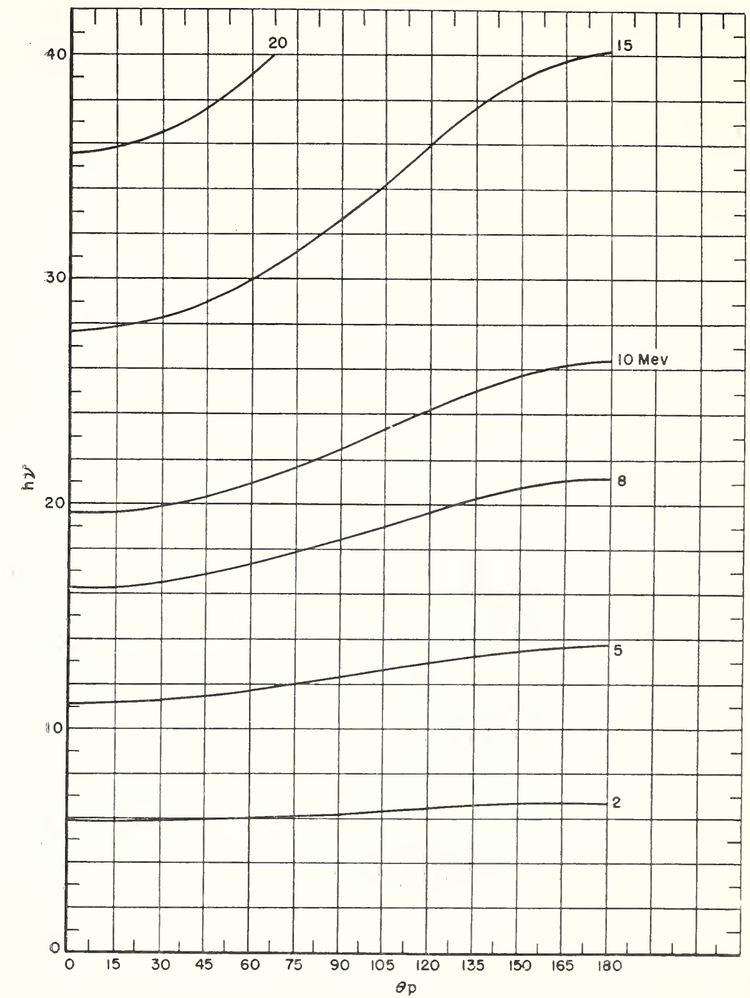
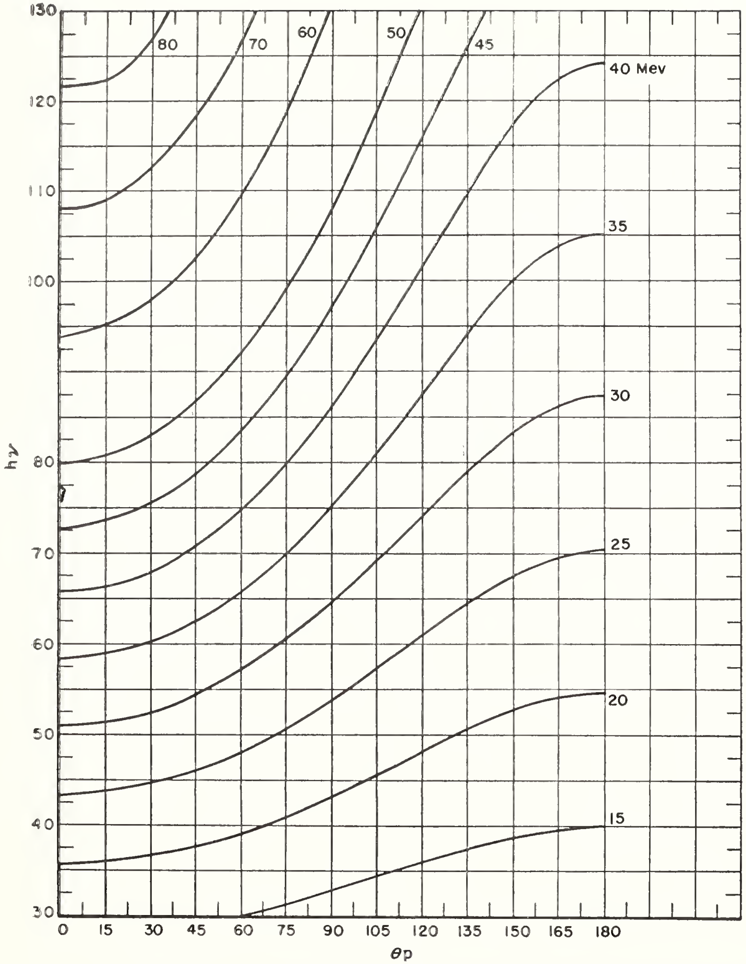
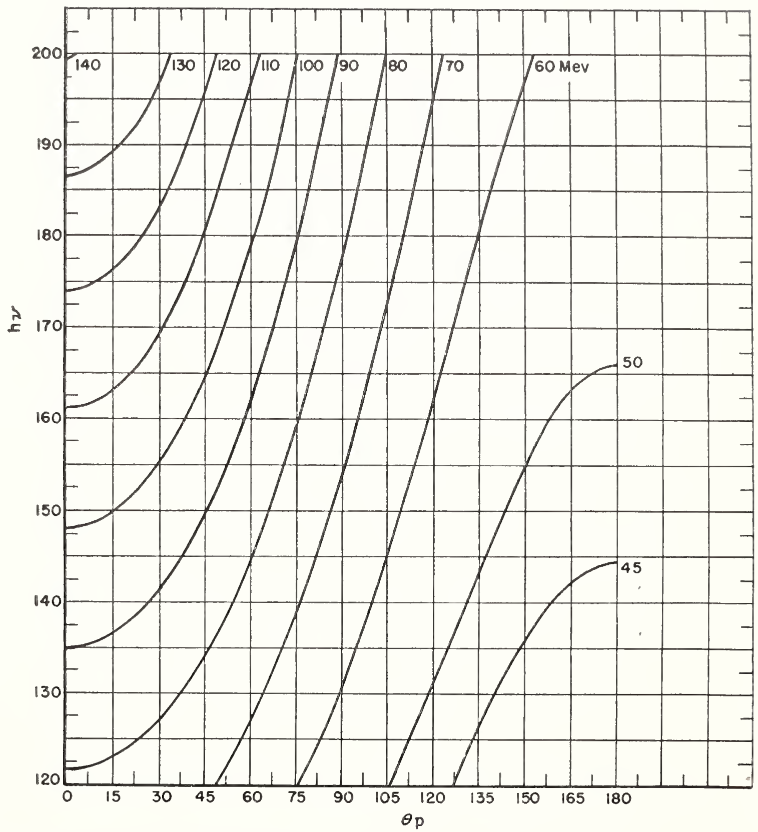
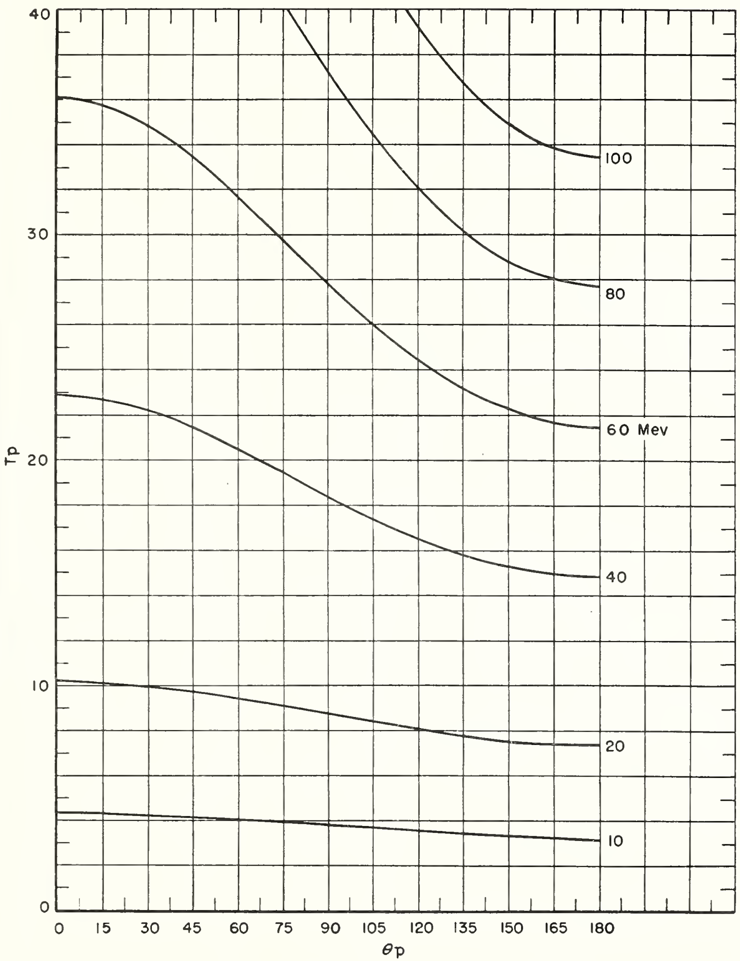


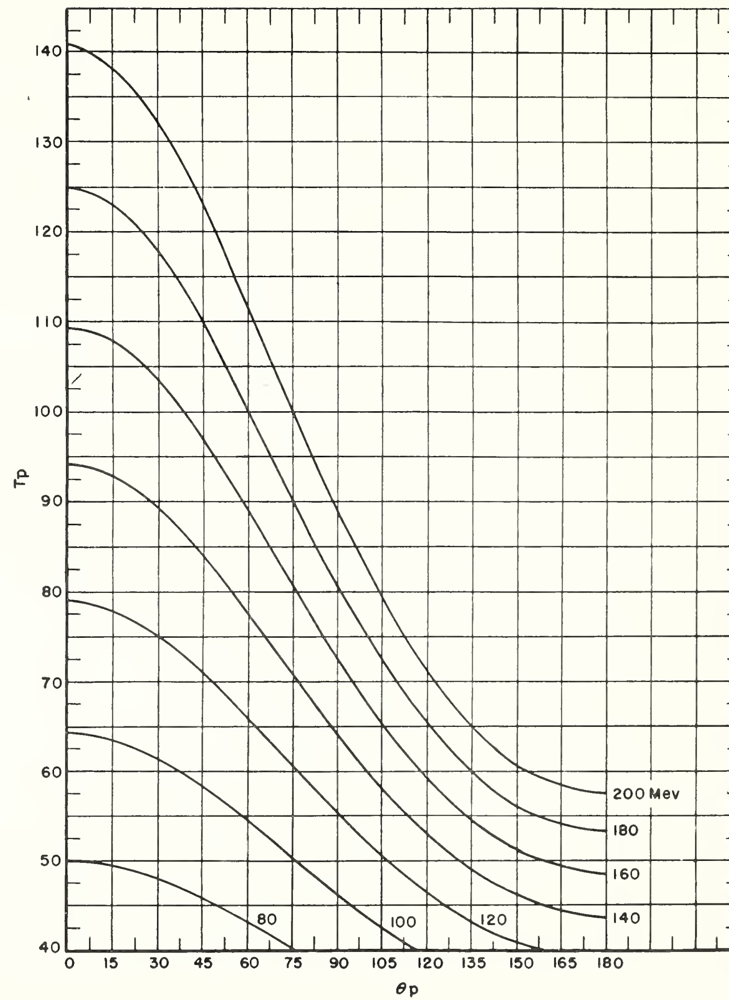
FIGURE 10. *Detailed reproduction of figure 9 for interpolation purposes.*



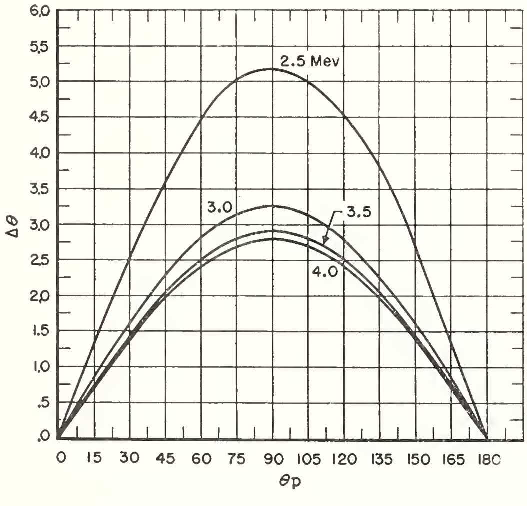


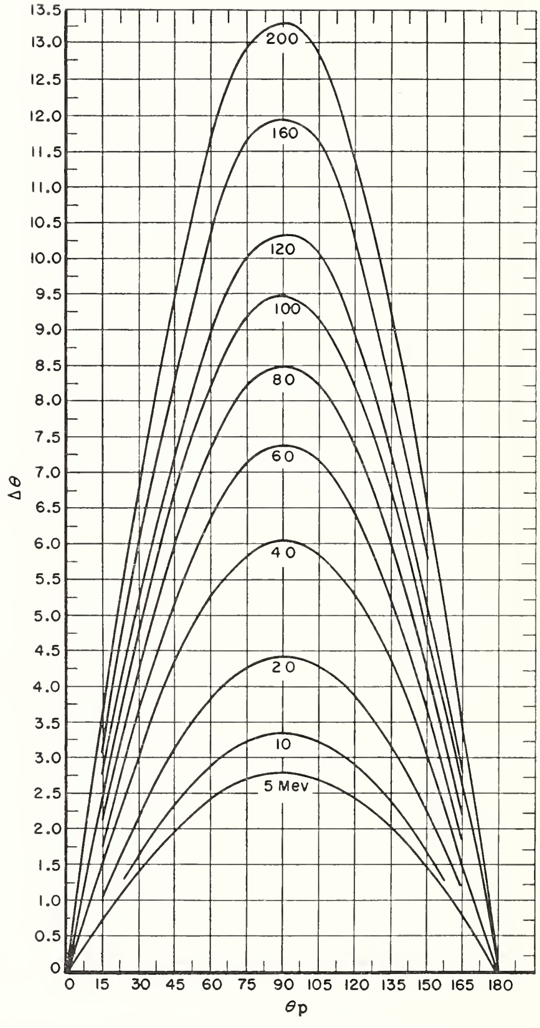
FIGURES 11 AND 12. *Detailed reproductions of figure 9 for interpolation purposes.*





FIGURES 13 AND 14. *Constant hν curves, showing proton kinetic energy, T_p, in Mev, as a function of laboratory angle, θ_p.*





FIGURES 15 AND 16. *A family of constant $h\nu$ curves, showing the angular shift, $\Delta\theta$ as a function of the proton laboratory angle, θ_p.*

These curves were derived from the curves in figures 2 through 12.
$\Delta\theta = \theta_p' - \theta_p$. The points were calculated at 30° intervals. The minimum $\Delta\theta$ occurs at twice the threshold energy.

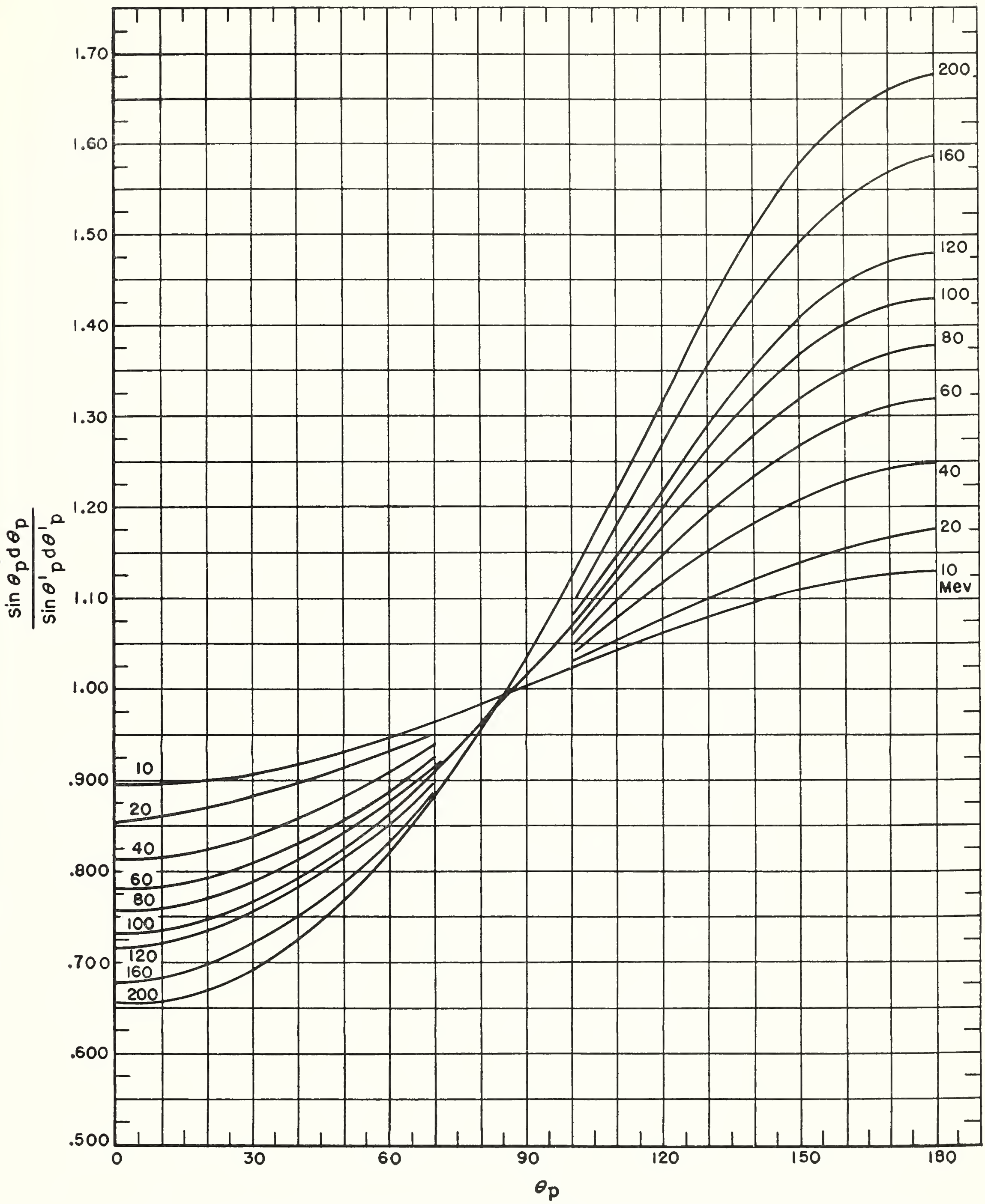


FIGURE 17. *A family of constant $h\nu$ curves, showing the ratio of the solid angle in the laboratory system, $\sin \theta_p d\theta_p$, to the solid angle in the center-of-mass system, $\sin \theta'_p d\theta'_p$, as a function of the proton laboratory angle, θ_p.*

Points calculated at 30° intervals.