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# High-Frequency Voltage Measurement

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NATIONAL BUREAU OF STANDARDS • E. U. Condon, *Director*

# High-Frequency Voltage Measurement

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## Contents

# High-Frequency Voltage Measurements

By Myron C. Selby

The paper presents an up-to-date account of fundamental principles and techniques used for voltage measurements primarily for frequencies in the upper audio- and radio-frequency ranges and including part of the ultrahigh frequency range. Subject matter is limited to principles and methods that have met with some degree of success for both high and moderate precision, emphasizing those developed and applied for primary standard work in this frequency range at the National Bureau of Standards. This paper is intended to give professional workers and graduate students a more comprehensive picture of the methods employed with regard to this subject than is presently available in textbooks and handbooks.

## I. Introduction

Standard procedures of r-f voltage measurements are at the present time based upon direct-voltage calibrations with the voltage of a standard cell as the primary reference. For maximum accuracy, direct substitution is made of d-c for r-f voltage having negligible harmonic components. Reliability is insured by cross-checking results with one or more independent calibration methods based on different principles. In the light of present-day experience at these frequencies, measurements to accuracies of approximately ±1 percent may be considered of high precision, whereas accuracies of the order of ±5 percent are of moderate precision. Direct voltage measurements are readily made to 0.1 percent so that transfer methods, employing measurements made directly in terms of direct voltage without needing corrections for frequency, may be called primary-standard methods. Reproducibility of results, as well as agreement between individual primary-standard methods, is expected to be within ±1 percent or better. In this paper, methods that in terms of primary-standard techniques are only reproducible to ±5 percent will be referred to as "moderate" precision methods of measurement. At this point one should differentiate between accuracy and precision. An accuracy of ±1 percent implies that the value of the quantity measured is within ±1 percent of its true or absolute value. Precision on the other hand refers primarily to sensitivity and repeatability incidental to relative measurements. Thus precise repeatable indications that can be obtained using different instruments may be extremely useful in making circuit adjustments and for experimental purposes, but they need not agree among themselves to the extent of being individually accurate. Accuracy implies ample sensitivity and precision of measurement to establish the extent of agreement with accepted theory or standards.

The most suitable primary-standard r-f voltage measurement methods seem to be those employing:

(a) Power substitution.
(b) Measurement of current through a known resistance.
(c) Deflection of cathode-ray beam.
(d) The electrostatic voltmeter.

Not suitable for d-c calibration are methods employing:

(e) Vacuum tube voltmeters.
(f) Rectifiers (nonthermionic).

## 1. High-Precision Methods Based on D-C Measurements

### (a) Power Substitution

In this method bolometers are made use of with the assumption that the same temperature or heating effect is produced by an equal amount of power whether produced by radio frequency or by direct current, and that the resistance of the bolorimetric element is independent of frequency and is reliably dependent upon its temperature.

Thermistor and Wollaston type bolometers are in general use. A thermistor is a semiconductor, such as uranium oxide ($U_3O_8$), or a mixture of nickel oxide (NiO), having a large negative resistance-temperature coefficient [1, 2]. A Wollaston wire is a platinum wire of the order of 0.001 mm in diameter drawn inside a silver wire; this silver coat is removed over a small section by etching with a solution of nitric acid ($HNO_3$); the exposed platinum core constitutes the active section of the bolometer. Typical thermistors have a resistance versus temperature sensitivity approximately 10 times larger than Wollaston wire, will carry considerably higher overloads, are superior in mechanical ruggedness, have a larger thermal time constant [53], and seem more adaptable for voltage measurements because of their relatively small physical dimensions. Precautions must be observed in mounting the bolometer so that the voltmeter under calibration is connected directly across an r-f current-carrying circuit element having negligible series reactance, since a fundamental assumption of this method is that the r-f resistance of a particular bolometer is equal to its d-c resistance.

Figure 1 shows an elementary circuit diagram of a bridge employing two thermistors. In order to determine the value of a radio-frequency voltage, $E$, the bridge is first balanced using only the direct voltage source, $V_0$, which for a selected value of $R_1$ establishes a definite supply of power to the two thermistors and their corresponding values of resistance, $R_a$ and $R_b$. The r-f voltage $E$, is then applied and the bridge is restored to balance by adjusting $R_1$ to a new value $R_2$. In this manner radio-frequency power is substituted for a portion of the power initially entirely supplied to the thermistors from the battery source. Since $R_a$ and $R_b$ are effectively the same for both balances, the following equations arising from the bridge constants for the two balanced conditions permit an evaluation of $E$. This method is applicable to several hundred and possibly to 1,000 Mc over an approximate range of 0.02 to 1.5 v.

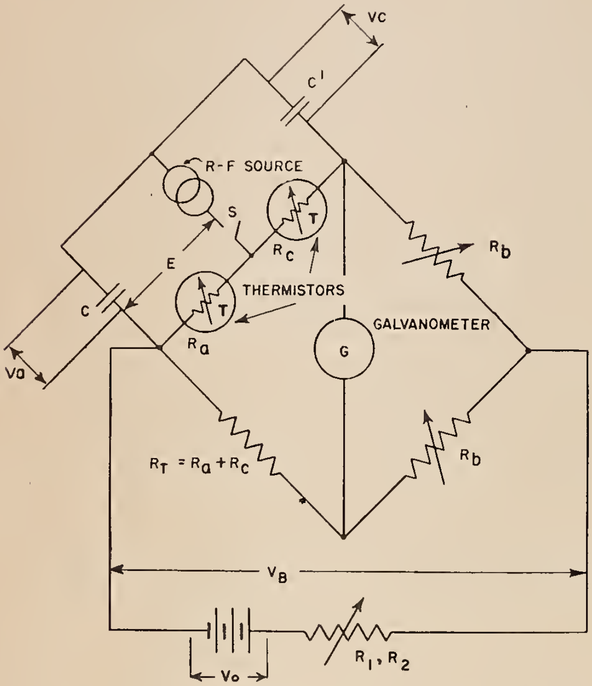


Figure 1. *General bolometer-bridge circuit employing two thermistors.*

The general expression for the r-f voltage is

$$E=\frac{R_T}{(R_T+R_b)(1+\alpha)}\left[\alpha(V_{R_2}-V_{R_1})(2V_0-V_{R_2}-V_{R_1})\right]^{1/2}, \tag{1}$$

and for thermistors matched to have identical resistances and characteristics,

$$E=\frac{R_T}{2(R_T+R_b)}\left[(V_{R_2}-V_{R_1})(2V_0-V_{R_2}-V_{R_1})\right]^{1/2}. \tag{2}$$

$E$=the root-mean-square value of the r-f voltage across the two thermistors in parallel; $V$ designates d-c voltages with the proper subscript. $V_0$, the voltage across the battery, is assumed constant with load variations.

Subscripts 1 and 2=respective values before and after the r-f voltage is applied.

$(R_aR_c)/(R_a+R_c)$=load presented to the r-f source.

$R_a$ and $R_c$=individual thermistor resistances.

$\alpha=R_a/R_c=V_a/V_c$. For matched thermistors $\alpha=1$.

$R_T=R_a+R_c$.

$V_a$ and $V_c$=the d-c individual thermistor voltage drops with switch $S$ (fig 1) closed.

$C$, $C'$=d-c blocking condensers.

When unmatched thermistors are used it is necessary to provide a d-c path through the r-f source (or in shunt with it) to measure $V_a$ and $V_c$ with the r-f voltage applied to the thermistors.

The following special cases are of practical significance:

1. Matched bolometers ($R_a=R_c$), unequal-arm bridge ($R_b\neq R_T$), relatively low $V_0$, ($R_1=0$):

This seems to be the most convenient arrangement for the following reasons: It makes it unnecessary to measure $\alpha$; no decoupling chokes (to keep r-f power out of the bridge) are required; greater accuracy is obtained by measuring $V_{R_2}$ alone (as compared with measuring a small difference between two relatively large voltages), especially when $E$ is low in magnitude.

$$E=\frac{R_T}{2(R_T+R_b)}\left[V_{R_2}(2V_0-V_{R_2})\right]^{1/2}. \tag{3}$$

2. Matched bolometers, unequal-arm bridge, relatively high $V_0$:

It is sometimes advisable to maintain bridge-arm resistances of the same order of magnitude, thereby assuring high bridge sensitivity. $V_0$ is usually not continuously adjustable. $R_1\neq 0$. In this case, eq 2 may be used.

3. Single bolometer ($R_c=0$, $C'=0$); unequal-arm bridge, $R_1=0$:

This arrangement eliminates the necessity of matching bolometers and removes the possibility of only one of the two carrying the entire r-f load. This could occur, for example, when operation takes place over a negative resistance portion of thermistor characteristics. Means are required, however, to keep the r-f power out of the d-c bridge.

$$E=\frac{R_T}{(R_T+R_b)}\left[V_{R_2}(2V_0-V_{R_2})\right]^{1/2}. \qquad (4)$$

4. Single bolometer, equal-arm bridge:

$$E=1/2\left[(V_{R_2}-V_{R_1})(2V_0-V_{R_2}-V_{R_1})\right]^{1/2}. \qquad (5)$$

The major advantage of having $R_b \neq R_T$ arises when a relatively small value of $(V_{R_2}-V_{R_1})$, as compared with $V_{R_2}$, has to be measured, as is the case at very low values of $E$. Other modifications may occasionally be desirable, such as a single-bolometer unequal-arm bridge with high $V_0$, $(R_1 \neq 0)$, or an unmatched two-bolometer bridge when $R_a$ and $R_c$ are known individually and are expected to remain stable in either equal- or unequal-arm bridges. When unmatched thermistors are used, the degree of departure of $R_a$ from $R_c$ may increase considerably with increasing r-f power fed to the two in parallel as a result of their negative resistance vs. temperature coefficient [62].

To obtain maximum accuracy, precision potentiometers and a galvanometer of proper sensitivity are desirable. Among the precautionary requirements of this method, especially in measuring small voltages, are the following: Constant ambient temperature, stable $V_0$, stability of the resistances in the circuit and accurate determination of their values, an r-f voltage source relatively free from harmonics.

### (b) Resistor in series with current measuring device

In employing this method to obtain accuracies of about 1 percent, the d-c calibration of the current indicating device and the value of the series resistor must be known to 0.5 percent. Either changes in current indication or resistance depending on frequency should be negligible, or they should be small and known, so that corrections may be applied. The physical design and arrangement of compenents must insure that all of and only the current entering one end of the resistor passes out the other end and through the current measuring element [62]. Unless the impedance of the current-measuring element is negligible, it must be taken into account and should, in any event, be preferably resistive if the voltage drop across it is part of the total voltage being measured.

Thermoelements (heater with thermocouple for current indication) may be used in series with special resistors [3, 4] up to a frequency of several megacycles to measure voltage ranging from several millivolts to approximately 50 v. Errors at high frequencies are likely to be introduced by stray capacitances associated with either the resistor or thermocouple circuit and by stray fields coupling with thermocouple circuit. Currents induced in the thermocouple circuit whose resistance is low in comparison with that of the r-f current-carrying heater element may introduce spurious heating effects not present during calibration of the thermoelement on direct current [5] and be a large source of error. When the couple is not insulated from the heater, calibration of the thermoelement is usually made on reversed direct current to reduce error caused by the section of the conductor common to both elements.

Instead of using a thermoelement, a heater element alone may be used, depending on other means of correlating the heating effect of r-f current through the heater with a suitable indicating device. An arrangement, applicable to approximately 30 Mc [4], utilizes a photoelectric calibration of the glow from the current-carrying heater element. Under conditions of use the impedance across the heater terminals must be known as a function of voltage and should be essentially resistive.

Skin effect, inductance, and power dissipation limit both the frequency and voltage ranges.

### (c) Cathode-ray beam deflection

This method makes use of the deflection of an electron beam by an electric field. An elementary circuit arrangement is shown in fig. 2.

The deflection is

$$D=E_{\max}\frac{ly}{2V_a d}, \qquad (6)$$

where $V_a$ is the accelerating potential of the beam, $l$ is the effective length of the deflecting field, $y$ is the effective length of the beam between this field and screen, and $d$ is the distance between the deflection plates.

The procedure in measuring an r-f voltage $E$ (fig. 2) is to adjust first the zero spot position by setting $V_1$ with $V=0$ and $E=0$. The r-f voltage is then applied and the position of either end of the deflection trace on the screen is restored to the original position of the spot by applying the proper value of $V$; this value of $V$ is then equal to $E$ peak, provided the transit time and other frequency errors are negligible. For maximum accuracy at low voltage levels it is best to line up the edge of the spot against a fine hair-line with a low-power microscope.

As a result of transit-time effect, the voltage and

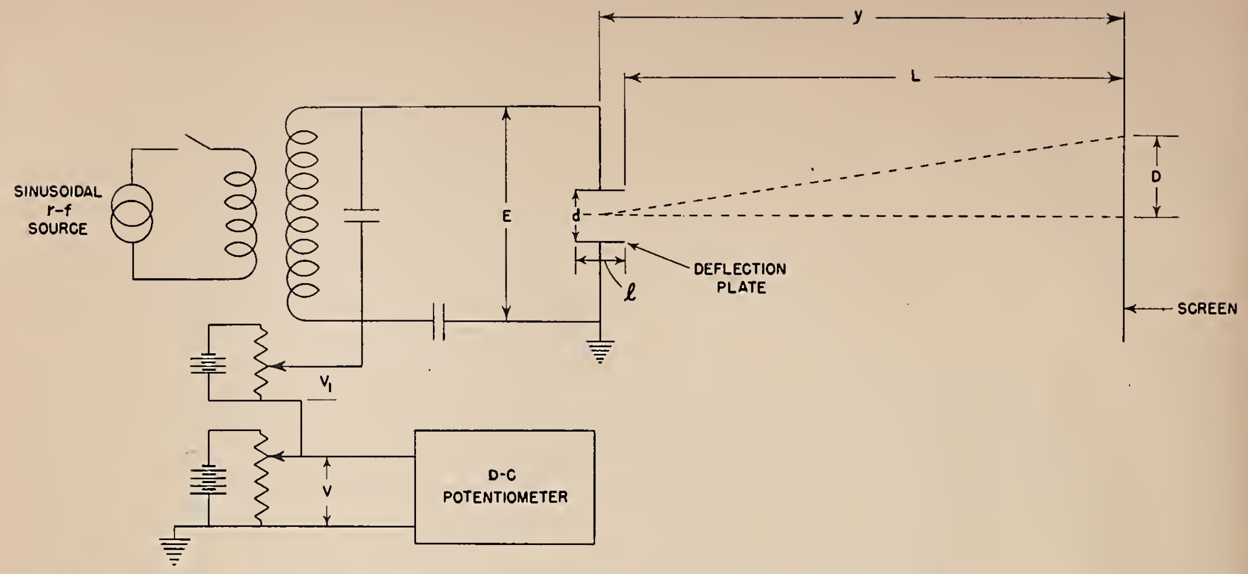


FIGURE 2. *Essential components of r-f voltage-measuring circuit arrangement employing deflection of cathode-ray beam.*

frequency ranges are limited and are interdependent. This effect takes place when the phase of the deflecting voltage begins to reverse before the electrons in the beam have sufficient time to cross the deflecting field. The transit time thus reduces the magnitude of the deflection unless it is smaller than a half-period of the highest frequency to be used. The error caused by this effect is usually expressed as a ratio, $C$, of dynamic to static sensitivity, the sensitivity being the linear deflection per volt. This ratio is expressed as [6, 7, 8]

$$C=\frac{\sqrt{2(1-\cos\phi)+\phi^2-2\phi\sin\phi+2\phi^2\frac{L}{l}\left[1+\frac{L}{l}(1-\cos\phi)\right]}}{\phi^2\left(1/2+\frac{L}{l}\right)}, \qquad (7)$$

where

$$\phi=\frac{\omega l}{v_0} \qquad (8)$$

$$\omega=2\pi f$$

$$v_0=5.97\times10^7\sqrt{V_a}=\text{beam velocity} \qquad (9)$$

$V_a$=d-c voltage accelerating the beam within the deflection field for $V_a \lesssim 10{,}000$v. For higher values of $V_a$, Einstein's correction for the increase in mass of the electron must be applied.

Equation 7 takes into account the beam displacement parallel to the axis. This displacement is usually negligible for large ratios of the $L/l$, where $L$ is the effective distance between the screen and the deflection field as shown in figure 2. In the latter case, for sinusoidal input,

$$C=\frac{\sin\phi/2}{\phi/2}. \qquad (10)$$

Figure 3 shows $C$ as a function of $\phi/2$, given by eq 10 [9, 10].

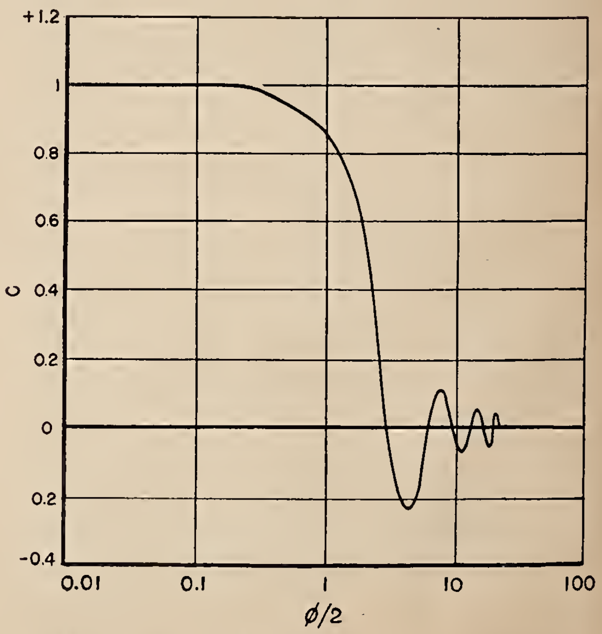


FIGURE 3. *Effect of electron transit time on sensitivity of cathode-ray tube* [9].

Another source of error is the effect on the deflection field of the lead inductance and deflection plate capacity. The voltage between the plates is larger than the voltage applied to the outside terminals of the cathode-ray tube (where the voltmeter under calibration is connected), as given by the following expression [5]:

$$E^1 = E\frac{1}{1-(f/f_r)^2}, \tag{11}$$

where

$E^1$ = deflecting voltage
$E$ = applied voltage
$f$ = operating frequency
$f_r$ = plate and lead series resonance frequency.

The design and construction of present-day commercially available cathode-ray tubes is such that they may be used with negligible frequency error up to 75 Mc for voltages ranging from 5 to several hundred volts. Their suitability and range is dependent upon electrode and lead arrangement, spacing, and insulation.

Deflection sensitivity may be increased by reducing accelerating voltage as is evident from eq 6, permitting satisfactory measurements at less than 5v radio frequency but only at the expense of a reduction of the upper limit of frequency, and an increase in spurious effects from magnetic fields and normally minor voltage fluctuations. Similarly, the upper frequency limit of satisfactory response may be increased by increasing the accelerating voltage (within its rated limits) at the expense of a reduction in deflection sensitivity.

Modified designs of oscilloscope tubes, with proper precautions in circuit connections, may be used for frequencies above 75 Mc. One modification is the replacement of the conventional deflecting plates of an oscillosope by a section of a two-wire transmission line (in a plane normal to the axis of the tube) extending through the tube [11]. The beam passing through the spacing between the conductors is deflected in proportion to the voltage at this point of the line. The voltmeter under calibration is placed either as close as possible to the tube or a distance λ/2 away along the extended transmission line, where the voltage across the line is the same as at the cathode ray. This method is shown in figure 4. The deflection angle $\theta$ is given by

$$\theta = \frac{E_{\max}}{2V_a}\frac{\pi}{\cosh^{-1}(d/2r)}. \tag{12a}$$

For large values of $d/r$

$$\theta \simeq \frac{E}{2V_a}\frac{\pi}{ln(d/r)}, \tag{12b}$$

where $d$ and $r$ are the separation and wire radius, respectively.

The effective deflecting field is largely concentrated within a space between the wires approximately equal in width to the wire separation; the distance responsible for a transit time error is therefore approximately equal to the wire separation.

A line having a 3-mm wire diameter and a 6-mm center-to-center separation results in a sensitivity of 0.3 mm/v for $V_a = 1{,}000$ v at a screen distance of 25 cm; this may be compared with a sensitivity of approximately 0.63 mm under the same conditions with deflecting plates having a transit time distance nearly five times as large.

Experimental work, now in progress at the National Bureau of Standards, with these types

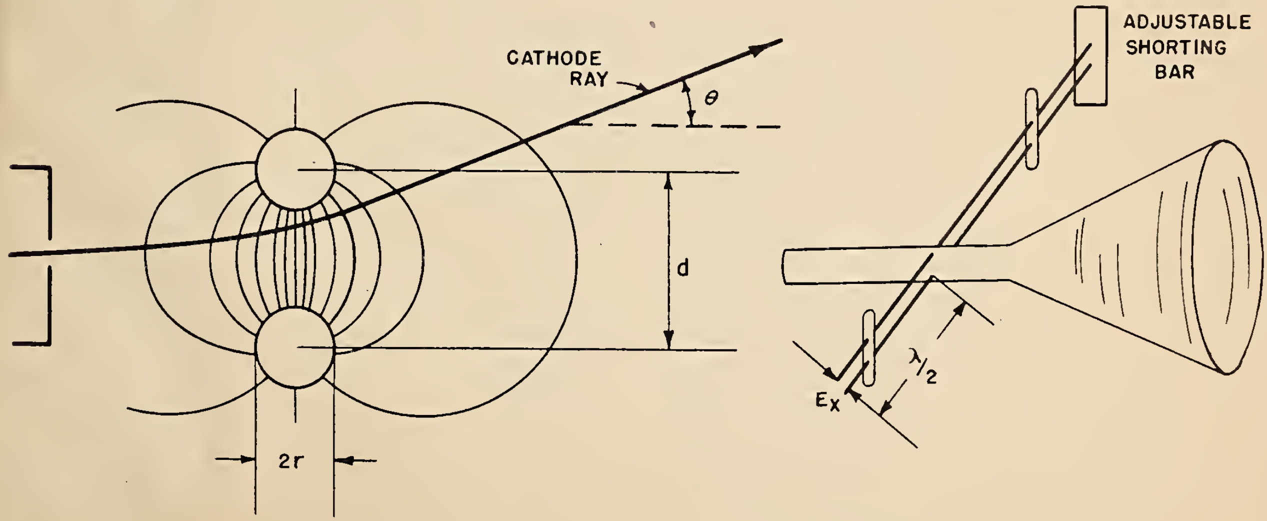


Figure 4. *Tuned transmission-line deflection system* [11].

of tubes resulted in precision measurements of voltages up to 150 Mc.

Another modification consists of a specially constructed cathode-ray tube employing a short focus lens and very small deflecting plates [6, 9]. Figure 5 shows a diagram of this device referred to as a "micro" oscillograph. The initial cross section of the beam is reduced to a degree resulting in a spot size of $10^{-2}$ to $10^{-3}$ mm in diameter. The beam passes through the deflecting plates and into the short-focus electrostatic lens. This oscillograph depends on optical magnification for increased initial deflection. Using a 50× optical magnifier and $V_a=10{,}000$ v, an effective deflection sensitivity of approximately 0.5 mm/v is obtained; the plate length and separation are about 0.2 inch, and the transit time effect is negligible at 1,500 Mc. These two modified applications of the cathode-ray tube seem to indicate that this method can be used for precision wide-range voltage measurements to at least 300 Mc and perhaps to much higher frequencies simply by further reduction in size of cathode-ray tube elements.

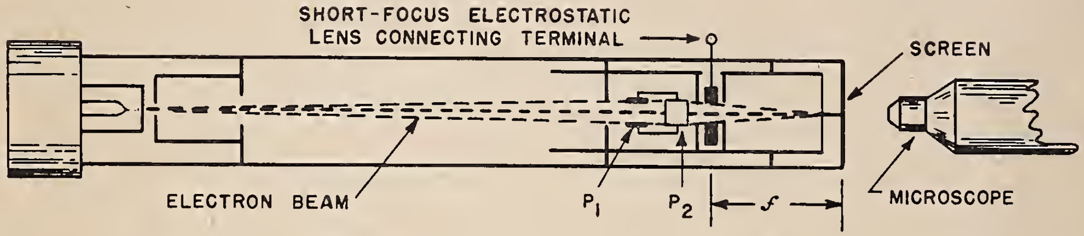


Figure 5. *Diagram of micro oscillograph [6].*

### (d) Electrostatic voltmeter

Electrometers and electrostatic voltmeters make use of the force existing between charged conductors. These instruments are essentially capacitors with the rotor, either fiber suspended or jewel-pivoted. One set of the conductors is usually fixed in position and the other one (the set corresponding to the rotor of a variable capacitor) is fiber-suspended or jewel-pivoted, and so restrained by the suspension or a coiled spring that its displacement is indicative of the voltage applied. Commercial electrostatic voltmeters have a scale (usually nonlinear unless the plates are especially shaped) that is graduated to read rms values of voltage, and have a high input resistance and low power consumption at frequencies up to approximately 5 Mc [12].

The major disadvantages of this type of voltmeter are low sensitivity and high input capacity (a few micromicrofarads to several hundred micromicrofarads), with the capacity a function of the instrument deflection. Thus certain difficulties and limitations are introduced when the instrument is connected across a tuned circuit.

The voltage range of present-day commercial electrostatic voltmeters is approximately 20 to 50,000 v.

## 2. Moderate precision methods

### (a) Vacuum-tube voltmeters

The first vacuum-tube (v-t) voltmeter was patented by R. A. Heising in 1917 [36]. For a number of years prior to that date and up to the present time, v-t voltmeters have been universally used to measure r-f voltages of the order of one millivolt to several kilovolts. Vacuum-tube voltmeters are made in a variety of forms and in various voltage ranges. The useful frequency range in each basic design is limited primarily by the adapted circuit arrangement and characteristics of such components as tubes, amplifiers, voltage dividers, etc. The v-t voltmeter is not satisfactory as a high-precision standard, because it is difficult to determine precisely the law of its voltage-current characteristic as well as to maintain its operation sufficiently constant over a reasonable length of time. In addition, in the case of voltmeters employing the usual form of grid-leak-condenser diode circuits, the analytically derived output contains factors depending upon the nature of the impedance across which the voltage is being measured. Finally the input impedance of the voltmeter is a function of the voltage applied to it [13].

Tubes with more than three electrodes are seldom used for voltages above 0.5 v. Pentodes connected as triodes are desirable for some voltage and frequency ranges as a result of spacing and shielding of the electrodes. Triodes connected as diodes are used for higher voltages.

Vacuum-tube voltmeters are usually calibrated at all frequencies using one of the above standard methods. They may also be calibrated in terms of power or audio-frequency standard instruments, in which case freedom from frequency correction should be verified. For higher accuracy, calibration at the operating frequency is preferred. The major performance desiderata of a v-t voltmeter for freuencies up to several hundred megacycles are:

1. Low-input capacitance.

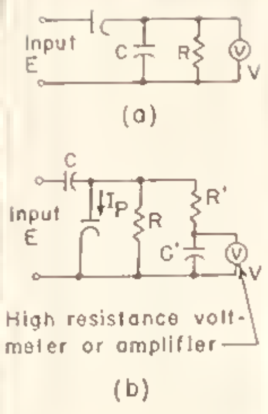
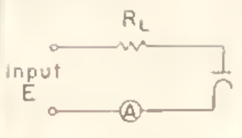
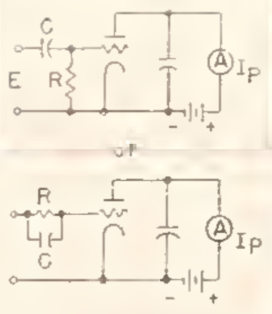
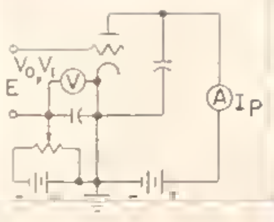
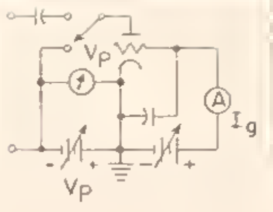
| TYPE | CIRCUITS | PRINCIPLE OF OPERATION | BASIC FORMULAE | APPROXIMATE INPUT IMPEDANCE | OUTPUT OF COMPLEX WAVEFORM | EFFECT OF WAVEFORM | APPROXIMATE VOLTAGE RANGE |
|---|---|---|---|---|---|---|---|
| 1 Diode Detection |  | In circuits (a) and (b) C charges to E peak. R acts to discharge C. 13, 15, 16, 17, 48, 49.<br>In circuit (b), R' and C' act as a filter to keep r-f out of the d-c measuring circuit.<br>NOTE:<br>Numbers refer to list of references. | For linear diode characteristics $V_{dc} = KE$ peak. Equivalent input resistance = $\frac{R}{2\frac{I_P R}{E_{peak}}} \equiv \frac{R}{2\eta}$<br>$\eta$ approximately unity as R and E increase.<br>For square-law type diode $\eta$ is a function of E. 13, 15.<br>NOTE:<br>See list of nomenclature | In general it is a function of R and amplitude of E. There is a value of R at which the input impedance is practically independent of E. The order of magnitude is the equivalent of 1 to 25 megohms resistance shunted with an input capacity of 3-10 μμf.<br>$10^5$ ohms at 300 Mc. is possible. 13 | $E_{peak}$ | The source impedance must be negligible at all harmonics, and the level of harmonics must be low otherwise error may be as large as the percentage harmonics present. 13 | The upper limit depends tube rating. With sensitive d-c voltmeter or with d-c plifier the lower limit is fraction of a volt. Rang depends on frequency; t higher the frequency, t narrower the range; corre may be applied. |
| 2. Diode Rectification |  | $R_P = 0$ by assumption<br>$I_{P_0} = 0$ by assumption<br>$R_L I_{ave}$ = average voltage of the positive half cycle<br>43, 56, 57, 58, 59. | For a sinusoidal input $I_P = \frac{1}{\pi R_L} E_{max}$ | $2R_L$ | $E_{ave}$ | Not subject to turnover. 45 (i.e. no error is caused by reversing input teminals even in the case of unsymetrical wave-form consisting only of fundamental and its harmonics.) | A fraction of a volt t few hundred volts and h er depending on the va of $R_L$ and tube voltage ings. |
| 3. Plate Detection Full wave, square law. |  | Approximate parabolic lower curved portion of $I_P$-$E_C$ characteristic is used. Average plate current is higher than the quiescent plate current. Biased for $i_p > 0$ throughout the cycle. $R_L$ (except of current meter) is usually omitted. 60. | $\Delta I_P \simeq K(E_1^2 + E_2^2 + E_3^2 + \cdots)$ to a first approximation<br>$\Delta I_P = \frac{1}{4\mu_0 g_{m_0}(r_p + R_L)}\left(\frac{\partial g_m}{\partial E_g}\right)(E_1^2 + E_2^2 + E_3^2 + \cdots)$ 40 | Approximately $10^7$ ohms resistive at frequencies up to a few mc. shunted by $(C_{gp} + C_{gc})$. May drop to $10^4$ or $10^3$ at 300 Mc. depending on the tube. 5<br>$R_g = \frac{1}{K g_m f^2 J^2}$ 41, 35, 61. | $(E_1^2 + E_2^2 + E_3^2 + \cdots)$ | In practice $\Delta I_P$ will depend to some degree upon wave form. Theoretically there is no turnover. Phase of harmonic has no effect. | Fraction of a volt to limit within square law of tube (a few volts fo commercial tubes) |
| 4. Plate Detection Half wave, square law. | Same | Same as above except tube is biased to cutoff. For large $R_L$ and C = 0, $i_p$ is nearly proportional to $e_g$ during positive half cycles. | For relatively large values of $R_L$ and E, $I_P \simeq KE_{ave}$. For small r-f voltages and low values of $R_L$, $I_P \simeq K(E_1^2 + E_2^2 + E_3^2 + \cdots)$ | Same as above | $E_{ave}$ if plate current characteristic is linear.<br>$E_{rms}$ if plate current characteristic is parabolic. | Subject to turnover and phase of harmonics. | Fraction of a volt to value of E causing grid c rent flow. |
| 5. Plate Detection Peak | Same | Tube is biased appreciably beyond cutoff. | $I_P \simeq KE_{Peak}$ | At negligible transit time effect $R_g \simeq \frac{K}{f}$ 41, 44<br>$C_g = (C_{gp} + C_{gc})$ | Not recommended. Error might be appreciable. | Subject to turnover and phase of harmonics. | From $E_{max} \simeq V_g$ to va causing flow of grid curr |
| 6. Grid-detection |  | Operation takes place along the lower curved portion of the grid-current grid-voltage characteristic and over the straight or the curved portions of the $I_P$ - $E_g$ characteristic. $X_C \ll R$. | $\Delta I_P = g_m R \Delta I_g$ over the linear portion of the $I_P$-$E_g$ characteristic. | Relatively low. | $E_{rms}$ or $E_{peak}$ depending on input level and operating voltages. | Error may be appreciable. | Fraction of a volt to few volts with receiving tubes. |
| 7. Slide Back |  | D-C bias adjusted to obtain same plate current with r.f. as was obtained without r.f. (i.e. with input terminals shorted) $I_P = I_{P_0}$ = a few microamperes. 34 | The peak of the positive half cycle = $E_{max} = K(V_1 - V_0)$. K approaches unity as E increases and as the sharpness of cutoff increases. It may be as low as 0.2 and lower depending on the tube characteristic and on the magnitude of E.<br>$V_0$ = d-c voltage at E = 0<br>$V_1$ = d-c voltages at other values of E. | $\frac{1}{\omega(C_{gp} + C_{gc})}$<br>Input resistance is approaching leakage resistance across input terminals. 34, 61 | $E_{peak}$ of positive half cycle. | Subject to turnover. | Fraction of a volt to hundred volts. Calibration indispensible especially f voltages below approxima 10 volts; calibration shoul made for a given $I_P$. |
| 8. Inverted Triode |  | $I_g$ is reduced when an r-f voltage is applied to the input terminals. $V_P$ is negative. 38 | $E_{peak} \simeq V_P$ required to produce the same $I_g$<br>Amplification factor $\simeq \frac{1}{\mu}$ | Resistance of the order of 1000 megohms shunted by $C_{gp} + C_{pg}$. 38 | $E_{peak}$ | Subject to turnover. | Large voltages, depen on tube design. |

TABLE 1. *Elementary circuits and characteristics of vacuum-tube voltmeters*

$I_{p0}$ = d-c plate current without applied r-f voltage
$I_p$ = d-c plate current
$I_g$ = d-c grid current
$V$ = d-c voltage
$\Delta V$ = d-c voltage increment
$\Delta I_p$ = increment of d-c plate current
$\Delta I_g$ = increment of d-c grid current
$\mu$ = amplification factor
$g_m$ = mutual conductance
$r_p$ = plate resistance
$R_L$ = load resistance
$R_g$ = resistive component of grid input impedance

$V_g$ = d-c grid bias
$\left.\begin{matrix}\mu_0, g_{m_0}, \\ r_{p0}, \left(\frac{\partial g_m}{\partial e_g}\right)_0\end{matrix}\right\}$ = values determined at a given quiescent point corresponding to $E_{g_0}$ and $I_{p_0}$
$E_1, E_2, E_3$ = amplitudes of harmonic components of a complex periodic wave.
$i_p$ = instantaneous plate current
$e_g$ = instantaneous grid voltage
$e$ = instantaneous voltage
$E_{max}$ = maximum value of r-f voltage
$E$ = rms value
$E_g$ = rms grid voltage
$E_{ave}$ = average voltage over half-cycle of a periodic wave

$f$ =fr
$f_r$ =re
$K$ =a
$d_o$ =di
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2. High-input resistance.
3. Short-input terminals.
4. High series-resonance frequency of input-lead inductance and capacity.
5. Freedom from transit-time correction.
6. Calibration must not be affected by ordinary line-voltage variations, aging and temperature and humidity changes, and must have negligible zero-setting drift. Vacuum-tube voltmeters must not generate disturbing voltages and must give steady indications.
7. Calibration must hold over a reasonable length of time and shall not be affected by tube replacement.
8. Maximum voltage range with minimum auxiliary equipment like amplifiers and voltage dividers.
9. Peak voltage calibration for nonsinusoidal waves; rms for sinusoidal waves.
10. Linear scale or large number of overlapping scales for square-law indications.

Some relative merits of triodes versus diodes for voltmeter applications are listed below.

Triodes are preferable at frequencies below approximately 20 Mc.

1. For high sensitivity to small applied voltages,
2. For lower loading effect on circuit being measured,
3. For greater reliability of calibration at a power frequency.

Their major disadvantages are:

1. Input voltage is usually limited to values low enough to keep the grid from going positive.
2. The d-c plate current has to be stably balanced out to obtain maximum sensitivity.
3. Null point shifts as a result of supply voltage variations, aging and warm-up period required.
4. Triodes may have a shorter life and may require more frequent calibrations as compared with diodes.
5. The input resistance at frequencies of about 100 Mc is lower than that of a diode by a factor of 10 [5].
6. It is difficult to construct a triode having the small interelectrode spacing required to keep transit time and resonance errors to a minimum [5, 15, 23].

A special diode construction was reported employing an indirectly heated cathode having the shape of a cylinder with a sealed-off end. A similar indirectly heated sealed-off cylinder is used as an anode. Both cylinders are lined up end-to-end in such a manner that the two sealed ends face each other, and the gap between them forms the electron transit distance from cathode to anode. The width of this gap, and consequently the transit time, is controlled and is held at a minimum by varying the heating current fed to the anode, thereby causing its expansion or contraction [54]. Diodes were recently developed with an interelectrode spacing and element structure permitting their application without frequency correction for frequencies well above several hundred megacycles.

Table 1 lists fundamental detecting circuit elements of vacuum-tube voltmeters and their major functional characteristics. Associated circuits including regular and feedback amplifiers, current balancing circuits, voltage dividers, voltage stabilizing elements, etc., are equally important in determining sensitivity, linearity, stability, and range of the meter [14, 16, 17, 18, 19]. Voltage dividers, specially constructed to fit given mechanical and electrical requirements, may be used to measure high voltages at high frequencies. One arrangement is shown in figure 6 [20].

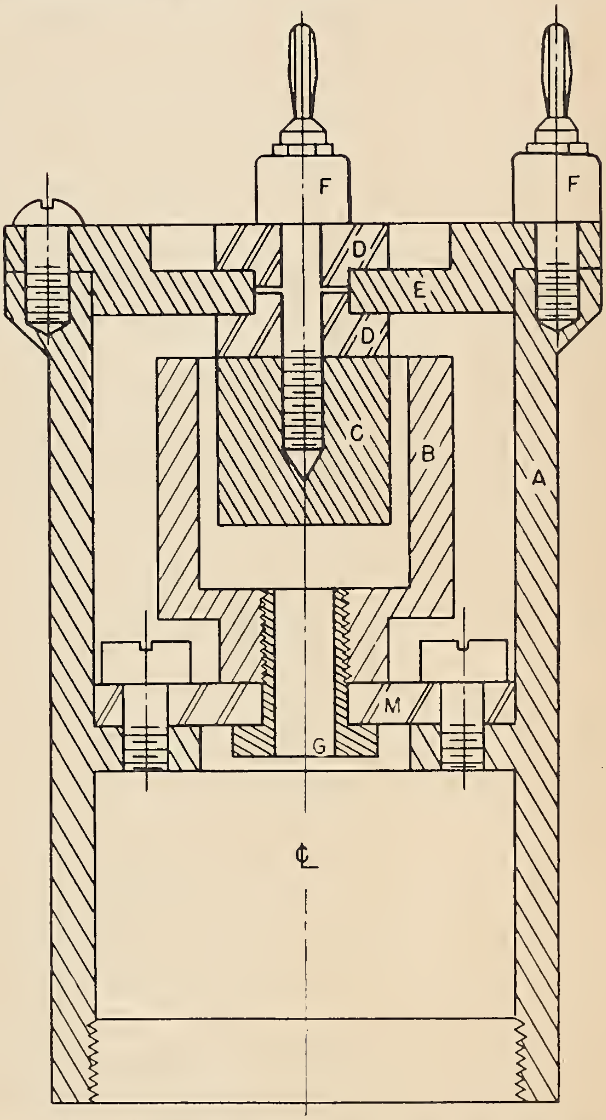

FIGURE 6. *Voltage divider for high r-f voltage measurements*

*G* is the terminal connected to a calibrated low voltage meter. The high voltage is coupled through the capacity between parts B and C. Various sizes of part *C* are provided to obtain different voltage ratios.

### (b) Nonthermionic rectifiers

In addition to thermionic diodes, other rectifiers are used as r-f voltmeter diode elements. These may be broadly subdivided into two classes, copper-oxide or selenium rectifiers and crystal diodes.

Copper-oxide and selenium rectifiers have good overload characteristics and ruggedness. They are, however, affected by temperature and aging, have a relatively large shunt capacitance and high forward resistance. The approximate equivalent circuit is given in figure 7 [21, 22]. The capacitance is approximately 0.02 $\mu$f/cm$^2$ of contact surface. $R_1 \simeq 2$ ohms for 1 cm,$^2$ and $R_2$ with polarity connections for maximum resistance (that is, backward resistance) is approximately 11,000 ohms for 1 cm$^2$ at applied voltages of $-0.25$ to $-3$: this holds over a frequency range of 50 kc to 5 Mc. The forward resistance is several ohms per square centimeter at 26° C and decreases slightly with increasing temperature. Backward resistance is considerably affected by temperature changes. Figure 8 shows direct-current characteristics of some of these rectifiers [22]. The rectified current depends on temperature, load resistance, frequency, and current density. This type of rectifier is manufactured in all sizes down to a small fraction of an inch in cross-sectional area for currents of a few milliamperes.

Copper-oxide rectifiers are preferable to selenium types for instrument application because of their lower resistance. Selenium types may be operated up to about 10 v per element as against about 2 v for copper oxide.

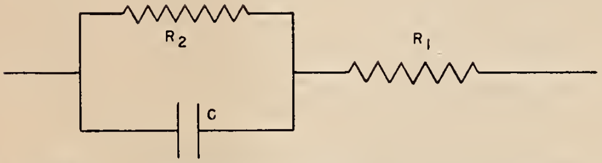

FIGURE 7. *Approximate equivalent circuit of copper-oxide rectifier in the 50 kc to 5 Mc range.*

$R_1$ represents the resistance of the body of the oxide. $R_2$ represents the resistance of the oxide-copper interface and is a function of applied voltage.

Copper-oxide-rectifier-type voltmeters are available for frequencies to approximately 30 kc and can be designed up to 1 Mc [22]. The major frequency-limiting element is the shunt capacitance, $C$. The effect of the wave form of the applied voltage is appreciable; the error in the indicated output, calibrated in terms of voltage free of harmonics, may approach in magnitude the percentage of harmonic content of the voltage measured. These rectifiers are used in series with resistors in 1,000-ohm/v instruments. They are very much higher in sensitivity and draw considerably less current from the source than the iron-vane or thermocouple type instruments. When the power of the voltage source exceeds 1 w and a high impedance instrument is not necessary it is preferable to employ an instrument having a higher inherent order of accuracy than that of the rectifier-type voltmeter [22].

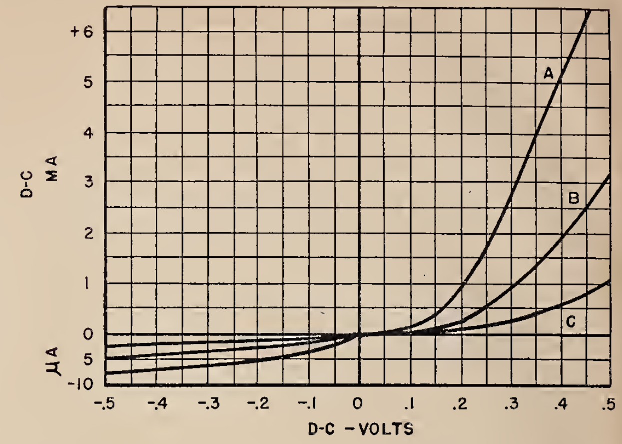

FIGURE 8. *Direct-current characteristics of copper-oxide rectifiers.*

A, 0.200 disk; B, 0.130 disk; C, 0.008 disk.

Whereas copper-oxide rectifiers are applicable as voltage indicators only at the lowest frequencies considered here, modern crystal rectifiers are useful up to 300 Mc and higher.

The crystals most commonly used at present are silicon and germanium.

Table 2 shows the chemical constituents of some of these crystals [26]. Other useful crystals are galena, iron-pyrites and carborundum. In measurement applications involving present-day dry rectifiers, one should recognize the following difference in design: the contact area, and thus power-handling capacity, of crystal diodes is much smaller than for copper-oxide rectifiers.

TABLE 2. *Composition of crystal rectifiers*

| Bulk material | Impurities added | | |
|---|---|---|---|
| | High-frequency mixer crystals | High-back voltage crystals | Low-frequency rectifiers |
| Silicon | Aluminum<br>Boron | Germanium<br>Also<br>Ni<br>Sn<br>Bi<br>Ca | Aluminum<br>Boron<br>Germanium<br>Also<br>Mo Ta<br>Zr Co<br>W Re<br>Be Fe |
| Germanium | Antimony<br>Also<br>P<br>Fe | Tin<br>Also<br>Ca<br>Ni<br>Sr<br>Bi<br>N | Antimony<br>Tin |

Figure 9 shows the mechanical construction of a modern crystal diode; figure 10 shows typical static characteristics for three germanium diodes, and figure 11 shows the rectification-efficiency characteristic of a germanium diode for different loads and frequencies; figure 12 shows the rectification efficiency of an iron-pyrites rectifier [24, 25, 26, 27]

Figure 13 shows the equivalent circuit of a crystal diode where $R_e$ and $C_b$ are the nonlinear resistance and shunt capacity of the barrier layer, and $R_s$ is the resistance of the body of the semiconductor [27, 28].

Relative merits of crystal versus thermionic diodes for r-f voltage measurements are listed as follows.

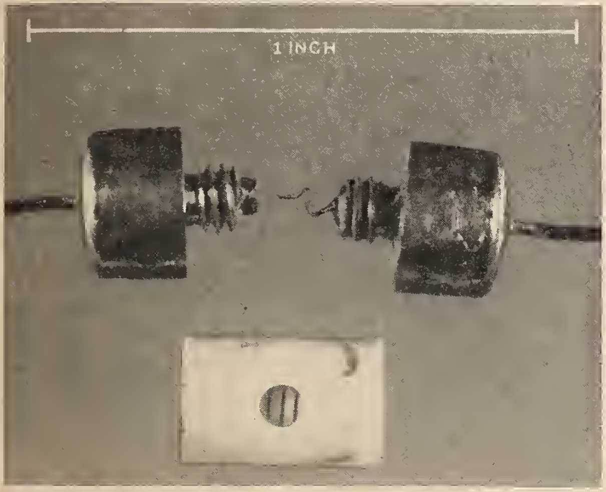

Figure 9. *Components of a 1N34 germanium crystal.*
The wire "whisker" points toward the small germanium block mounted on the lower threaded terminal.

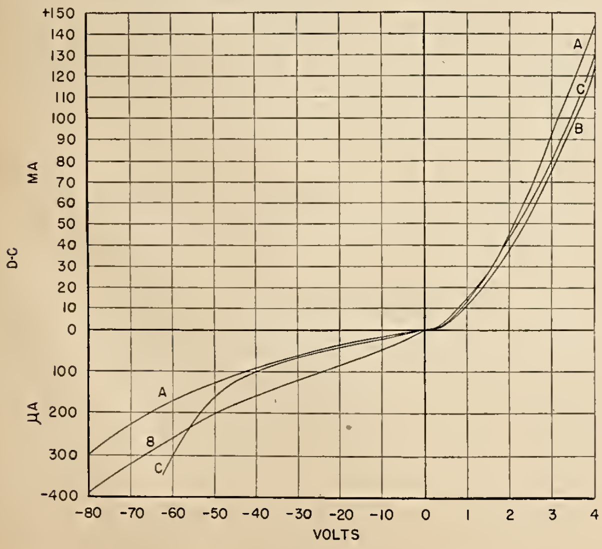

Figure 10. *Individual static characteristics of three small germanium diodes A, B, and C.*

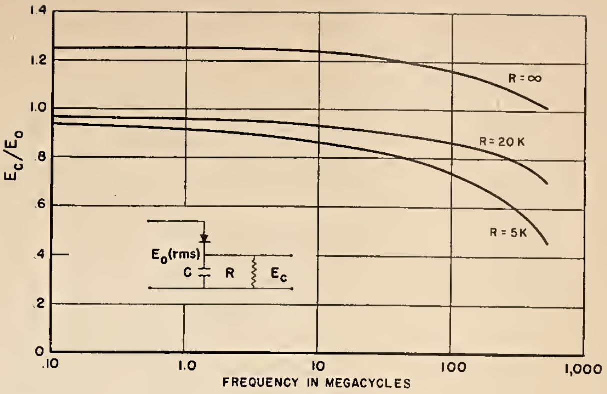

Figure 11. *Rectification-efficiency characteristics of a germanium crystal.*

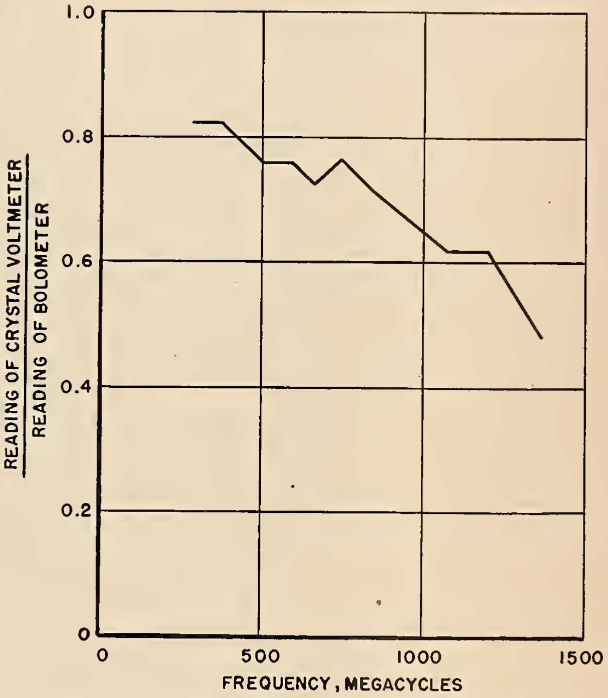

Figure 12. *Rectification efficiency of an iron-pyrites rectifier.*

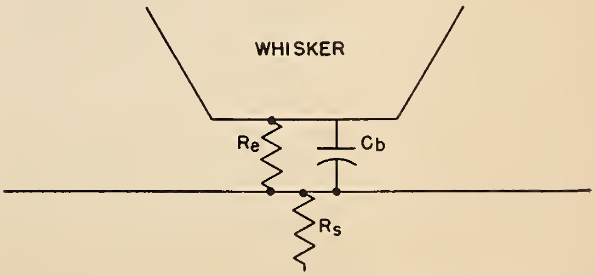

Figure 13. *Equivalent circuit of a crystal rectifier.*

Relative advantages of crystal diodes:

1. Transit-time effect is negligible. For comparison the rectification efficiency in acorn-type thermionic diodes as a result of transit-time effect begins to drop off at 30 Mc for voltage levels of 0.5 v. This reduction is 30 percent at 500 Mc.

2. The crystal has smaller physical dimensions and therefore a higher input resonant frequency. This is approximately 3,500 Mc as against 1,500 Mc for the smallest commercial thermionic diode and 2,800 Mc claimed for the latest special type diode (Eimac type 2-01C).

3. The cathode does not have to be maintained at constant temperature, an added requirement in the case of thermionic tubes to insure constant emission.

4. Crystals are useful in rectifying lower voltages than vaccum diodes.

Relative disadvantages of crystals:

1. They have poorer stability, are less rugged, and show greater variation in characteristics for units of the same type.

2. They are frequency-sensitive partly for the following reasons: the capacity $C_b$ (fig. 13) ($C_b$ ranged from 0.2 to 0.6 $\mu\mu$f for good commercial units) shunts the "reverse" resistance of the barrier; this produces a drop in rectification efficiency. The effective resistance and capacity of the barrier layer are functions of the voltage applied across this barrier [25, 28]; the magnitude of this voltage is in turn a function of $C_b$, $R_e$, and $R_s$ acting as a voltage divider; $R_s$ varies between 5 to 100 ohms for different types of crystals. This effect is negligible, however, for some units at frequencies below 500 Mc in circuits having relatively high crystal-load resistance. Figures 11 and 12 show typical frequency characteristics of two commercial types of crystals.

3. Reverse rectification at the contact between the crystal and its supporting electrode and the relatively large shunting capacity at this contact introduces another error. In the particular case of an iron-pyrites crystal this error amounts to a 50-percent increase in the total output at 10 Mc as compared with the output at 1 Mc [27]. This takes place because the r-f voltage across these contacts is a function of the ratio of the contact resistance to the capacitive reactance shunting it; at 10 Mc the r–f voltage rectified in the reverse direction is therefore lower than that of 1 Mc; consequently the total rectified output at 10 Mc is higher. Plating or fusing the crystal in place largely eliminates this effect.

4. The input impedance of crystal and probe is comparable with that of a thermionic diode and its probe at ultrahigh frequencies. At lower frequencies the v-t diode has a higher input resistance than the crystal.

5. The voltage range of commercially available crystal units designed for high back voltage and for frequencies up to 100 Mc is limited to a maximum of approximately 30 v rms. Those recommended for higher frequencies have a maximum rating of approximately 1 v rms [25, 26]. Overloading causes a change in characteristics or permanent damage to the contacts.

6. Resistance and sensitivity vary with temperature as shown in figure 14 [26].

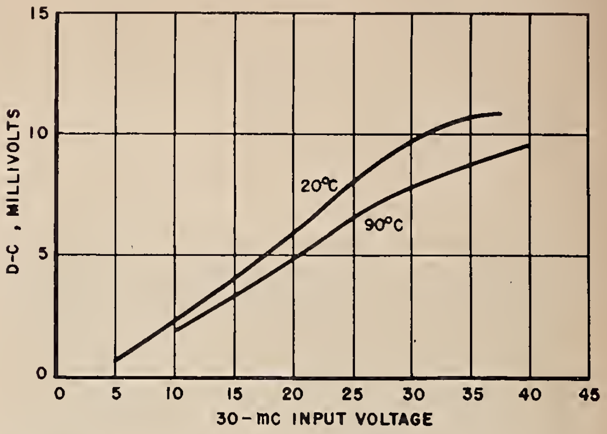

FIGURE 14. *Rectified current through a 1,000-ohm load on a high-back-voltage silicon-crystal rectifier as a function of input voltage at 20° and 90° C.*

A crystal-type voltage indicator was recently placed on the market [28]. It has a range of 0.1 to 1 v with a ±5 percent claimed accuracy from 10 to 300 Mc. The "forward" resistance of the crystal is of the order of a few hundred ohms; in the "reverse" direction it is 15,000 to 100,000 ohms. The crystal is used in a peak-reading circuit. The input resistance of the meter is approximately one-third of the "reverse" resistance. An improved construction of a germanium crystal was announced having an optically polished face of specially processed germanium and a platinum "whisker" point welded to that plane; stability and constancy of performance superior to that employing pressure-type contact is claimed [29].

## 3. Pulse-peak voltage measurement

### (a) Cathode-ray beam deflection

The most accurate method of measuring peaks of voltage-pulses employs a cathode-ray oscilloscope. Proper synchronization of the sweep circuit of an oscilloscope with the pulse source facilitates detailed measurement of the pulse shape including the evaluation of the crest voltage of the pulse. Deflections can be measured directly on the screen, or a d-c voltage slide-back circuit arrangement similar to the one shown in figure 2 may be used. Resistance or capacitance dividers are frequently used with oscilloscopes in making high peak measurements. The design of the divider should be such that in reducing the relative magnitude of the voltage pulse it does not alter the shape of the pulse applied to the oscilloscope.

### (b) Diode peak voltmeters

Diode peak voltmeters are generally used as convenient moderate-precision indicators [30, 31]. However, the discrepancy between the voltmeter reading and true peak value may be very large. Figure 15 shows the response of a diode-type vacuum-tube voltmeter as a function of pulse-repetition frequency for a rectangular pulse shape and pulse duration of 3.5 microseconds. An approximate expression is given by Easton [30] for the rectangular pulse peak in terms of the d-c diode output as follows:

$$e_0 \cong E_{dc}\left[1+\frac{T}{t_1}\frac{R_1}{R_2}\right], \qquad (13)$$

where

$e_0$=the peak voltage of a rectangular pulse
$E_{dc}$=the d-c voltage across $R_2$
$T$=the duration of the discharging interval
$t_1$=the duration of the charging interval
$R_1$=the total resistance during charging
$R_2$=the total resistance during discharging.

One of the curves of figure 15 shows values computed on the basis of this expression. The effective input impedance of this type of voltmeter for pulse voltages may be very low. The direct-voltage output (measured across $R_2$) decreases with increasing rate of pulse repetition, with increasing values of $R_2$ and with decreasing values of $R_1$. $R_1$ is a function of the combined source and diode resistances at the particular operating conditions. Improved performance may be obtained by means of auxiliary circuits like cathode followers [30] and automatic slideback arrangements [31].

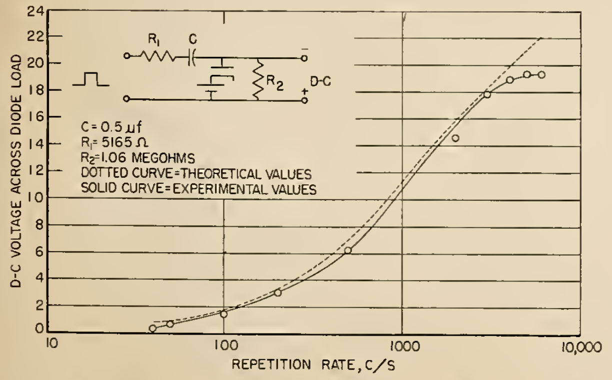

Figure 15. *Response of a diode-type VTVM as a function of pulse-repetition frequency for a charging-interval duration of 3.5 microseconds and a peak of 26 v.*

## 4. Miscellaneous Methods

The following voltage-measuring methods are of interest as relatively independent and useful for certain applications.

### (a) Heterodyne Method of Extending the Voltage Range [32]

This principle is illustrated in figure 16. A linear diode frequency changer, $D$, mixes voltages, $E_0$, supplied from an auxiliary source, with $E_{x1}$, a fraction of the unknown voltage $E_x$ used to calibrate the voltmeter. The intermediate-frequency output voltage is directly proportional to $E_x$ for large ratios of $E_0/E_{x1}$. This proportionality holds for values of $E_{x1}$ not exceeding the order of magnitude of 1 volt. The capacitor, $C$, is therefore indicated as an element of a linear voltage dividing network used to maintain $E_{x1}$ considerably below $E_x$. The interelectrode capacity of the diode, shunted by other circuit capacities (not shown in the figure), constitutes the second element of the voltage divider. One can thus use a single calibration point of the same voltmeter determined at a low voltage (for example, 1 volt obtained by means of the bolometer or any of the other standard methods listed above) and proceed with calibrating it at higher voltages. The advantage of this method is that it can be used to calibrate high voltage levels at high frequencies in terms of a standard attenuator used at a relatively low intermediate frequency. Let, for example, $C$ be chosen at such a value that $E_{\text{i-f}}$, corresponding to the calibrated $E_{x1}$ (of, let us say, 1 volt), is some small value (say $X_1$ mv). $E_x$ is then increased until a new value of $E_{\text{i-f}}$ (say $X_2$ mv) is obtained; the new higher value of $E_x$ is then equal the previous times the ratio of these i-f voltages (that is, $E_{x2}=(X_2/X_1)E_{x1}$ volts). This latter ratio can be accurately determined with a standard attenuator placed in series with the i-f output terminals. The use of a low transit-time diode may eliminate the transit time error at frequencies up to several hundred megacycles.

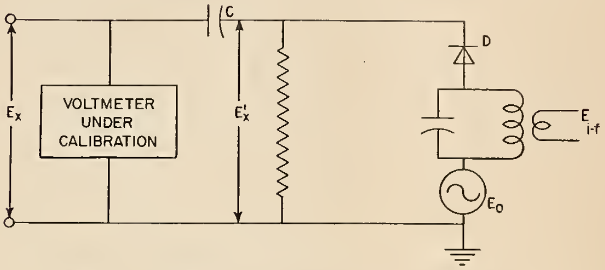

Figure 16. *Heterodyne principle of calibrating r-f voltmeters.*

### (b) Spark-gap method

Spark gaps may be used to measure peak voltages of the order of 1 to 30 kv at all frequencies up to about 100 kc [33]. The sphere spark gap is preferred to other electrodes because the breakdown voltage changes little up to about 25 kc. For a symmetrical sphere-gap voltmeter the peak voltage is given approximately by

$$E=\sqrt{2}\,\epsilon\frac{l}{m}, \tag{14}$$

where

$\epsilon=19.3\rho[1+0.76/\sqrt{pD}]$kv/cm
$\rho=3.92p/T=$relative air density
$D=$sphere diameter in centimeters
$l=$maximum gap-length in centimeters at which spark-over may take place
$p=$atmospheric pressure in centimeters of a mercury column
$T=(273+N°\mathrm{C})=$absolute temperature in deg K, and
$m=0.25[2l/D+1+\sqrt{(2l/D+1)^2+8}]$.

Actual voltages at 100 kc are 10 percent lower than corresponding power frequency voltages normally used to calibrate the gap voltmeter.

### (c) Glow-discharge voltmeter

A method applicable for peak voltages up to about 15 kv and frequencies up to 1 Mc makes use of a glow tube as shown in figure 17 [33]. The value of $C$ is continuously decreased until the peak value of the voltage across it is just equal to a predetermined critical value $E_c$ that is causing the tube to glow. The value of $E_x$ may then be computed from known values, of $C$, $C_1$, and $C_2$, where $C_2$ is the tube and distributed input lead capacity.

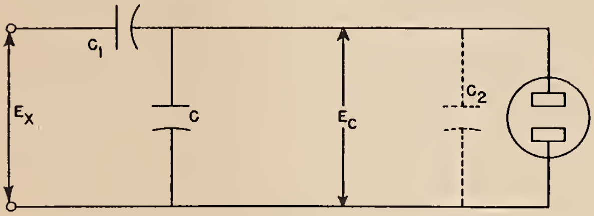

FIGURE 17. *Glow-discharge tube voltmeter.*

### (d) Electrometer employing miniature open-wire line

An electrometer consisting of a short platinum open parallel-wire line is reported applicable for measurement accuracies of 1.5 percent over a frequency range of 30 to 100 Mc and voltage range of 10 to 70 volts effective [42]. The wire diameter is 0.01 mm, line separation 1 mm, and line length approximately 5 cm. One of the lines is tightly mounted whereas the other is kept under relatively low tension. The deflection of the latter is observed under a microscope and calibrated at 100 kc and at direct current, the two calibrations yielding identical results. Varying the tension of the wire under observation provides a control of the voltage range that could be increased to 300-v effective value. A major advantage of this method is claimed to be the high input impedance, the capacity of the electrometer being less than 0.5 $\mu\mu$f. The maximum reduction in line separation (with a consequent effect on the characteristic impedance and voltage distribution) is 10 percent; this contributes an error of less than 0.1 percent to the voltage measurements at all frequencies up to 100 Mc.

### (e) Electrometer employing suspended wire

An electrostatic voltmeter employing a suspended wire described by Peterson [55] may be used for frequencies up to 1,000 Mc or higher. It consists of a 3.5-cm-long, 0.0013-cm-diameter platinum wire suspended inside a 2.5 by 1.1-cm opening of a brass block. The wire is spaced 0.16 cm from the 1.1-cm side and the deflection of its free end is measured when the r-f voltage is applied between the insulated wire-suspension terminal and the brass block. A deflection of about 2 cm may be obtained for 10v when the shadow of the wire is projected optically on a screen with an effective deflection magnification of 1,000. The deflection is increased by about 3 percent at 300 Mc and 33 percent at 1,000 Mc as compared with that of direct current. The top value of the voltage range is limited by the corresponding charging current that might cause the fine platinum wire to fuse.

The listed difficulties encountered with this voltmeter were:

1. A darkened room may be required.
2. The electrometer is very sensitive to motions of the building and should preferably be used in the dead of night.
3. The heat from the projector lamp causes a drift of the wire position.
4. It cannot be used at low frequencies where the low inertia of the wire is insufficient to prevent wire vibration.
5. The input capacity is a function of the voltage applied, which may sometimes be objectionable.

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The author welcomes an opportunity to express his appreciation of the large amount of careful work covered by the references cited in this paper and for the kind permission of some of those authors to quote and include data and reproduce some of their curves and drawings and acknowleges helpful suggestions of some of his colleagues at the National Bureau of Standards during the course of preparation of this paper.

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