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Colorimetry

by Deane B. Judd

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Preface

The judgment of color permeates our industrial life. The farmer finds his vegetables and fruits often graded and priced according to color. Raw wool, cotton, and silk are graded and priced according to color. Only in this way can the textile manufacturer produce goods of uniform color at a price low enough to sell. Oils and resins used for paint have their colors measured with extreme care, and pigments and dyes likewise. Poor quality edible oils (cottonseed, cocoanut, olive, peanut) can often be detected by color; and light-colored cooking oils bring a premium price apart from quality because bakers know they can sell the resulting light-colored cakes and pies without any trouble.

Color measurement is especially vital in mass production. The assembly of parts from different sources to make a complicated machine like a refrigerator often requires a color specification for each of several parts. The white plastic door to the freezing unit must be related in color to the interior walls of the refrigerator by color tolerances no less important to the final sale than are the length and width tolerances that insure the fit of the door. There is scarcely a segment of industry untouched by color considerations.

Precise judgment of color is made possible by color measurement, or colorimetry. During the past two decades the National Bureau of Standards has played a leading part in establishing practical working standards of color and in setting up and administering color tolerances. These standards and methods have been described in some scores of scientific and technical publications. It is the purpose of this Circular to bring together this information in concise form, and by giving the basis for each technique to show more clearly how they supplement each other than has been possible in the several separate descriptions of them now available. It is hoped that this information will still further facilitate the purchase and sale of commodities and manufactured goods on a fair basis. Conformity to a color requirement can now be determined with the same assurance as size, shape, or strength.

E. U. Condon, *Director.*

Contents

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The definition of color as a characteristic of light is given, and the basic principles of its measurement are outlined. Reduction of spectrophotometric data is outlined, and three methods are illustrated by examples. The various methods of direct colorimetry are treated briefly, and a more detailed treatment is given of visual and photoelectric methods of colorimetry by difference from a material color standard. The most useful collections of material color standards (Lovibond, Arny, Maerz & Paul, TCCA, Ostwald, Munsell) are described, and the most widely used one-dimensional color scales (color temperature, Union, Saybolt) are explained. Five widely used specifications of color having general applicability are summarized.

I. Introduction

Traditionally, two techniques are designated by the name colorimetry. In the first, the color of a transparent medium is used as an indication of the amount of some constituent in it, as in so-called chemical colorimetry. The development of this technique has shown that the appearance of the medium serves as an unnecessarily complicated indication of its constitution, but that the absorption spectrum may give a comparatively direct and precise index. For each wavelength the logarithm of the reciprocal of the internal transmittance (log $1/T$) is often directly proportional to the concentration of the solution. The variation of log $1/T$ with wavelength often identifies the constituents in the solution. The second technique relates to the measurement of objects, both self-luminous and nonself-luminous, so that the aspect of their appearance known as color can be expressed numerically. By *colorimetry* in this work we mean only this second technique.

1. Definition and Nature of Color

It is common practice to regard color as a property of objects, and in a limited sense this is true. However, light sources as well as objects have color. The flame of a bunsen burner can be changed from bluish purple to orange by the introduction of sodium. And since even objects lose their colors and become invisible unless they reflect, scatter, or transmit radiant energy, or form a part of an illuminated scene, a broader view is that color is a property of light, and of light alone.

Since it is possible to measure by a spectrophotometer the spectral-energy distribution of any light beam, and since the color of a light correlates closely with its spectral composition, some of the more physically minded have contended that color is a physical entity measurable by the techniques of physics; but this is not the most useful view. The color change of the bunsen flame from bluish purple to orange can be shown by a purely physical measurement to be caused by a change in the spectral composition of the emitted energy; but it takes more than physics to decide whether this flame has the same orange color as the light reflected from the peel of a given citrus fruit in daylight. Application of the spectrophotometer to the orange peel will show that the spectral composition of the light reflected from it under daylight illumination is radically different from that emitted by the sodium flame. The former includes energy from every portion of the visible spectrum (about 380 to 770 mμ) except for a number of narrow regions (Fraunhofer lines). The latter is confined almost entirely to two narrow spectral regions (589.0 and 589.6 mμ) and is indeed comprised of energy almost completely missing from the light reflected from the orange peel illuminated by daylight. Physically, therefore, the two lights are almost as different as it is possible to obtain, but they have closely the same color. The two lights must therefore be identical in some respect other than physical identity; this identity consists in some aspect of the response made by a normal observer to the sodium flame being the same as the corresponding aspect of the response to the peel of the citrus fruit. The broader view of color must, therefore, include not only the spectral composition of the radiant energy reaching the eye of the observer, but the properties of the observer as well. These properties have been evaluated by finding equivalent stimuli, somewhat like the energy of the sodium flame and that reflected from a daylight-illuminated orange peel, that have different spectral compositions, but still manage to have exactly the same appearance to the normal observer. Such equivalent stimuli are called *metamers* and are said to form a *metameric pair*. The properties of the observer are completely defined by stating which pairs of stimuli are found to be metameric by him. Modern colorimetry is based upon spectrophotometry interpreted according to the properties of an observer.

The most widely accepted technical definition of color is that given by the Committee on Colorimetry of the Optical Society of America [107]:[1] *"Color*

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[1] Figures in brackets indicate the literature references at the end of this Circular.

consists of the characteristics of light other than spatial and temporal inhomogeneities; light being that aspect of radiant energy of which a human observer is aware through the visual sensations which arise from the stimulation of the retina of the eye." Colorimetry is here taken to be the measurement of color. It will be noted that this definition relates color and light to radiant energy only insofar as the energy produces a visual effect within an observer. On this account, color and light are said to be psychophysical entities, and in their evaluation it is ordinarily not necessary to pay attention to energy of wavelength less than 380 mμ nor greater than 770 mμ, because the eye is relatively insensitive to such energy.

2. Basic Principle of Measurement

If a normal observer attempts to adjust one controllable element of his central visual field so that it matches a neighboring element, he will ultimately discover that three independent adjustments have to be at his disposal. If he is using the red, yellow, and blue paints frequently found in primary grade schools, only by chance will he obtain a match from a mixture of two of them. Even a brown color requires blue in addition to red and yellow. Within the color gamut of the three paints, an exact match for any given color is easily possible, but three primary paints are the irreducible minimum. The same considerations apply to the magenta, yellow, and cyan colorants used in color photography, lithography, and color printing. Similarly, if he is trying to color-match one spot of light by shining several spotlights of different color onto the same neighboring spot of a screen, he finds, in general, that either three lights of fixed spectral composition are required, or, if two lights be added together, not only the amounts of both but also the spectral composition of at least one has to be adjustable. The same rule applies to rotary mixture on a sector disk, four sectors, giving three independent adjustments, are necessary and sufficient. Normal color vision is tridimensional.

Since the color vision of a normal observer is tridimensional, it follows that a color specification is expressible by three numbers. For normal observers three numbers are necessary, for partially colorblind observers only two numbers are necessary, and for totally colorblind observers, only one.

In the examples given (paints, spotlights, sector disks) the observer by adjustment of three variables obtains a color match, that is, he has set up a second stimulus equivalent to the first. Except by accident, however, the ternary or binary mixture does not match the unknown in spectral composition. In the usual case the mixture is equivalent to the unknown in color but not in spectral composition, and the unknown and the mixture therefore form a metameric pair. There are, however, degrees of difference in spectral composition. If one painted panel be matched by a mixture of red, yellow, and blue paints, the degree of metamerism is likely to be only moderate; but if the paint panel illuminated by daylight be matched by shining on a white card three spotlights, each one of which contains energy restricted to a narrow wavelength band (such as spectrum red, green, and blue), the degree of metamerism will ordinarily be extreme. Normal observers will agree closely on matches that are but slightly metameric; but they may disagree widely on highly metameric matches.

(a) Ocular Pigmentation

Studies of extremely metameric pairs in which mixtures of two parts of the spectrum are set up to color match other two-part spectrum mixtures have yielded our most valuable knowledge regarding the properties of the average normal eye [1, 2, 42, 70, 83, 140]. An outstanding fact derived from observation of such metamers is that the center of the retina (fovea centralis) has somewhat different properties from that immediately surrounding it; that is, a color match set up for the central 2° or 3° of the retina becomes an easily detectable mismatch if the eye be turned so as to allow the stimuli to affect a portion of the retina, say, 6° from the fovea. Furthermore, if the metamers are compared in large patches (6° or more), this mismatch causes a central spot to appear temporarily on a field that is physically uniform. This dependence of metamerism on the portion of the retina used arises chiefly from the existence of a spot of brownish or yellowish pigment irregularly covering and interpenetrating the central 3° or 4° of the normal retina; it is called the macula lutea, or sometimes the yellow spot. The pigment acts as a selective filter interposed between the vitreous humor and receptor layer at the retina. Metamers set up for one normal observer usually fail to hold strictly for anyone else. This failure is ascribable to variation in amount of pigmentation of the eye media, the macular pigment being one of the chief variables. Table 1 and figure 1 show the spectral transmittance of the macu-

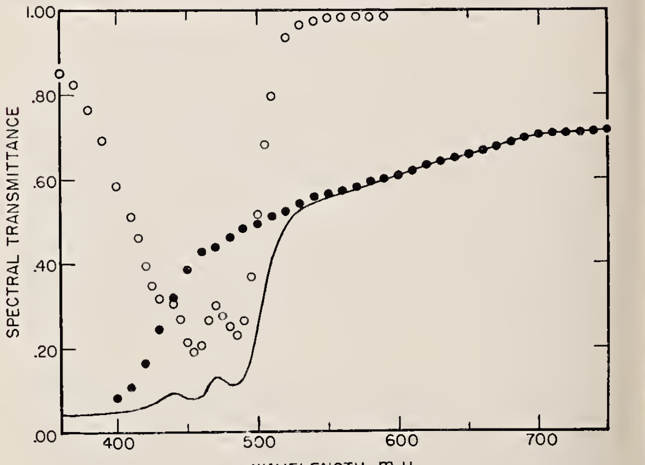


FIGURE 1. *Spectral transmittance of the ocular media, Ludvigh-McCarthy [75], Wald [135].*

The macula, or yellow spot, absorbs chiefly in the blue region of the spectrum (see circles); all other ocular media (chiefly the crystalline lens) absorb more strongly in the ultraviolet and violet regions (see dots). The center of the eye is thus protected against over-stimulation by short-wave energy by the combination of these two absorptions (see curve).

lar pigment according to Wald [135] and the spectral transmittance of all other ocular media (aqueous, vitreous, lens, and cornea) according to Ludvigh and McCarthy [75]. The product of these two (also shown) is an estimate of the average transmittance of the ocular media, but there is a large individual-observer difference. The properties of the normal eye derived from a study of extreme metamers therefore refer only to the central 2° of the retina, and they refer to an hypothetical average eye. Probably nobody has an eye that is sufficiently close to the average to be a substitute for it in all applications.

Table 1. *Spectral transmittance of ocular media*

Wave-length	Spectral transmittance of cornea, lens, and aqueous and vitreous humors (Ludvigh-McCarthy)	Spectral internal transmittance of macula lutea (Wald)	Spectral transmittance of ocular media, including the macula lutea
mμ			
360	[1]0.052	0.859	0.045
370	[1].056	.826	.046
380	[1].062	.762	.047
390	[1].069	.695	.048
400	.086	.577	.050
410	.106	.506	.054
420	.160	.396	.063
430	.248	.316	.078
440	.318	.305	.097
450	.388	.212	.082
460	.426	.206	.088
470	.438	.299	.131
480	.458	.250	.115
490	.481	.263	.126
500	.495	.516	.256
510	.510	.798	.407
520	.525	.935	.491
530	.543	.968	.526
540	.559	.977	.546
550	.566	.985	.557
560	.572	.989	.566
570	.583	.989	.577
580	.594	.989	.587
590	.602	.989	.595
600	.610	[1]1.000	.610
610	.619	[1]1.000	.619
620	.631	[1]1.000	.631
630	.641	[1]1.000	.641
640	.649	[1]1.000	.649
650	.657	[1]1.000	.657
660	.664	[1]1.000	.664
670	.676	[1]1.000	.676
680	.690	[1]1.000	.690
690	.698	[1]1.000	.698
700	.705	[1]1.000	.705
710	.707	[1]1.000	.707
720	.708	[1]1.000	.708
730	.710	[1]1.000	.710
740	.711	[1]1.000	.711
750	.713	[1]1.000	.713

[1] Extrapolated.

(b) Grassman's Law

From a knowledge of spectral metamers it has been possible to summarize concisely the properties of a reasonably average normal eye. This summary is made in accord with the principle known as Grassman's law [39] foreshadowed by Newton's laws of color-mixture [98]. If a light composed of known amounts of three components (called primaries) is equivalent in color to an unknown light, the three known amounts may be used as a color specification for this light. These amounts are called the *tristimulus values* of the color. Grassman's law states that when equivalent lights are added to equivalent lights, the sums are equivalent. Thus, if an unknown spot of color were matched by shining on the same spot of a white screen two component spotlights of tristimulus values, X_1, Y_1, Z_1, and X_2, Y_2, Z_2, respectively, by Grassman's law, the tristimulus values, $X,Y,Z,$ of the unknown spot of color would be simply:

$$\left.\begin{aligned} X&=X_1+X_2,\\ Y&=Y_1+Y_2,\\ Z&=Z_1+Z_2.\end{aligned}\right\} \tag{1}$$

Any beam of light, whether it originates from a self-luminous body, or comes by transmission, scattering, or reflection, from a nonself-luminous object, may be considered as made up of a large number of portions of the spectrum. The amounts of these various portions may be determined by spectrophotometry. The tristimulus specifications, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, of each of these portions have been determined for a number of normal observers, and average values are given in table 2 in arbitrary units for a spectrum of unit spectral irradiance. The amounts, $X,Y,Z,$ of the primaries required to produce a stimulus equivalent to any light beam may therefore, by Grassman's law, be found simply by adding together the tristimulus values of the various parts of its spectrum. Modern colorimetry is based upon this application of Grassman's law.

The principle expressed in Grassman's law has been established by repeated experiment over a wide middle range of retinal illuminance. It breaks down for very high retinal illuminance [136, 141, 142] that begins to approach those sufficient to do the eye permanent harm; and it breaks down if the illuminance of the whole retina continues for 10 min. or more to be so slight that vision by the retinal rods (twilight vision) intrudes significantly [71]. Between these two extremes, however, Grassman's law holds independent of the adaptive state of the eye. Thus, if two stimuli of different wavelength distributions of energy be found that are once responded to alike by the eye, they will be seen alike even after exposure of the eye to another stimulus sufficient to change considerably the appearance of the two equivalent stimuli. For example, if a portion of the spectrum near 640 mμ (red) be superposed on a portion near 550 mμ (yellowish green), it will be found possible to obtain the color of this combination from an intermediate portion of the spectrum, say 590 mμ (orange). If the retina of the eye be highly illuminated by light of wavelength near 640 mμ, and its sensitivity to radiant flux of this wavelength region considerably reduced in this way, it is found that, although neither of the equivalent stimuli any longer appears orange, they still give identical colors, for example, they may yield identical yellows or identical greenish yellows. The eye thus cannot be trusted to yield the same color perception from a given stimulus; simultaneous and successive contrast affect it profoundly. But it is still a satisfactory null instrument and obeys Grassman's law.

By Grassman's law it is possible to test whether any two beams of light of differing spectral composition form a metameric pair for any observer defined by $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$. The condition for metamerism of two beams of light of spectral irradiance, $(E_1)_\lambda$ and $(E_2)_\lambda$, is that simultaneously:

$$\left.\begin{aligned}\sum_0^\infty (E_1)_\lambda \bar{x}_\lambda \Delta\lambda &= \sum_0^\infty (E_2)_\lambda \bar{x}_\lambda \Delta\lambda,\\ \sum_0^\infty (E_1)_\lambda \bar{y}_\lambda \Delta\lambda &= \sum_0^\infty (E_2)_\lambda \bar{y}_\lambda \Delta\lambda,\\ \sum_0^\infty (E_1)_\lambda \bar{z}_\lambda \Delta\lambda &= \sum_0^\infty (E_2)_\lambda \bar{z}_\lambda \Delta\lambda.\end{aligned}\right\} \qquad (2)$$

The wavelength interval, $\Delta\lambda$, to be used in these summations depends upon the irregularity of the curve of spectral irradiance with wavelength; intervals of 20 mμ are sometimes sufficient to yield a significant result; intervals of 10 mμ often do; and intervals of 5 mμ are usually sufficient except for discontinuous spectral distributions such as those characterizing gaseous discharge lamps. The wavelength functions, $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, characterize the observer; they indicate the amounts of the three primaries required by him to match the various parts of the spectrum at unit irradiance. If these functions be divided by the spectral transmittance of the ocular media (such as estimated in table 1), the ratios so obtained characterize the foveal receptors of the observer.

Any three lights may be used as primaries in a system of tristimulus color specifications, provided only that no one of them is equivalent to a combination of the other two. Tristimulus specifications, X,Y,Z, expressed relative to one set of primaries may be transformed into specifications, R,G,B, relative to any other set of primaries by transformation equations of the form:

$$\left.\begin{aligned}R&=K_1X+K_2Y+K_3Z\\ G&=K_4X+K_5Y+K_6Z\\ B&=K_7X+K_8Y+K_9Z\end{aligned}\right\}. \qquad (3)$$

The constants, K_1 to K_9, may take on any arbitrary values, positive, negative, or zero, provided they are not such as to make one of the new primaries identical to a combination of the other two; that is, provided that:

$$\begin{vmatrix} K_1 & K_2 & K_3 \\ K_4 & K_5 & K_6 \\ K_7 & K_8 & K_9 \end{vmatrix} \neq 0. \qquad (3a)$$

Since the exceptions that cause the determinant of the system to vanish are trivial, the choice of coordinate system is very wide. The primaries do not even have to correspond to physically realizable lights. Imaginary lights defined by spectral compositions having negative values for some parts of the spectrum are admissible, and indeed are preferred for routine colorimetry, because by their use the computation of tristimulus values from spectrophotometric data is somewhat simplified.

It is generally presumed that the initial responses of the retinal cones are photochemical in nature, and that there are three of them, each independent of the other two. If we knew what colors (imaginary or real) correspond to each of these responses, we could evaluate the constants K_1 to K_9, and by finding R,G,B, by setting $\bar{x}$, $\bar{y}$, $\bar{z}$, for X, Y, and Z, in formula 3 and by dividing by the spectral transmittance of the ocular media (such as estimated in table 1), we could find the spectral sensitivities of the presumed photosensitive substances required for the observer. It will be noted, however, that lack of this theoretical information is no hindrance to the use of the functions $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$, for colorimetric purposes. It makes no difference in the end result whether these functions or any analogous functions computed from formula 3 be used. Grassman's law is a well-established experimental fact, and is not dependent upon any theory of vision.

II. The Standard Observer, Illuminants and Coordinate System

In 1931 the International Commission on Illumination recommended that all subsequent color data be expressed in terms of the same tristimulus system, so that the results would be immediately comparable. The standard observer and coordinate system recommended [20, 44, 56, 126] are defined by the tristimulus values of the spectrum colors given in table 2 and plotted in figure 2. It will be noted that the primaries chosen are such that none of these tristimulus values is less than zero. It is further true that the green primary chosen, whose amounts are designated by Y, is such as to carry all of the luminosity, the other two primaries (red, blue) whose amounts are designated by X and Z, respectively, being unassociated with luminosity. Therefore the values of $\bar{y}_\lambda$ for the spectrum correspond to the standard luminosity function, and it is convenient and customary to express the Y value of a luminous area as its *luminance* (photometric brightness) in terms of some recognized unit (such as candles per square meter, millilambert, or foot lambert), and the Y value of a transmitting specimen is customarily put in terms of *luminous transmittance* (ratio of transmitted to incident luminous flux). However, the more complicated concept, *luminous directional reflectance,* which refers only to that flux reflected in the direction of the observer, is more closely related to the appearance of the specimen, and is therefore the more commonly used as an evaluation of Y for opaque specimens. *Luminous directional reflectance* is the reflectance that a perfectly diffusing surface illuminated and viewed like the specimen would need to have in order to attain the same luminance as the specimen. Directional reflectance was called apparent reflectance by

McNicholas, originator of the concept [84, 107]. The Y value of an opaque specimen may also be conveniently expressed as its *luminous reflectance* (ratio of reflected to incident luminous flux).

If, as is usual, light combinations or mixtures are not the chief interest, it is convenient to substitute for the tristimulus values, X,Y,Z, the two ratios $X/(X+Y+Z)$ and $Y/(X+Y+Z)$, combined with the luminous value, Y. The two ratios are known as chromaticity coordinates, x,y, because they serve to specify the chromatic aspect of the light. The analogous ratio, $Z/(X+Y+Z)$, is also known as a chromaticity coordinate, z, but only two of the three coordinates, x,y,z, give independent information since by definition the sum of all three is unity. Table 2 also gives the chromaticity coordinates, x,y,z, of the spectrum colors; and figure 3 shows the points representing the spectrum colors in the (x,y)-chromaticity diagram. This diagram is also known as a *Maxwell triangle* because of Maxwell's first use of such a diagram.

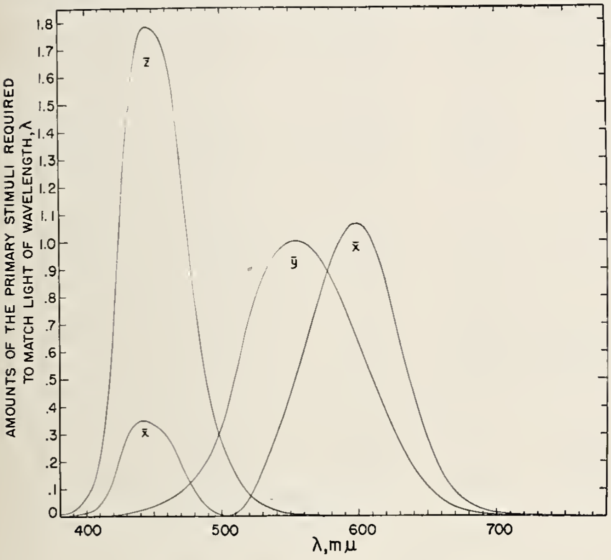


FIGURE 2. *Tristimulus values of spectrum stimuli of unit irradiance according to the standard observer and coordinate system recommended by the International Commission on Illumination in 1931 (the ICI or CIE system).*

The $\bar{y}$-function is the relative luminosity function for daylight vision. By reading the three ordinates at any wavelength the amounts of the three primaries required in an additive combination to color match the spectrum at that wavelength may be found. For example, equal amounts of the X- and Y-primaries combine to produce for the standard observer a close match for the color of the spectrum at 578 mμ.

Furthermore, it has aptly been called a *mixture diagram* because it indicates in a very simple way the chromaticity of the color resulting from the additive combination of any two lights. The point representing this chromaticity is found on the straight line connecting the points representing the two lights. The primary lights are represented by points at the corners of a triangle, and every point within the triangle represents the chromaticity of a mixture of the primary lights whose proportions are indicated by the chromaticity coordinates, x,y,z. The spectrum colors are shown by dots connected by a smooth curve known as the spectrum locus. They are identified by wavelength in millimicrons. It will be noted from figure 3 that the spectrum locus is substantially straight from 540 mμ to the longwave extreme. This means that the standard observer would find binary mixtures of, say, 540 mμ with 640 mμ, closely equivalent to some intermediate portion of the spectrum. But the spectrum locus from 540 mμ to the shortwave extreme is convex. This means that for the standard observer a binary mixture of 540 mμ with, say, 440 mμ would differ importantly in chromaticity from the intermediate parts of the spectrum. By drawing straight lines through any central point (such as $x = y = 1/3$, representing the stimulus whose spectrum has constant radiance per unit wavelength, the so-called equal-energy stimulus) and extending them until they cut the spectrum locus, we may find the spectral complementaries relative to a stimulus represented by that point, that is, we may find the two parts of the spectrum that when combined in proper proportions will for the standard observer be equivalent to the central stimulus.

The straight line in figure 3 joining the extremes of the spectrum locus represents the chromaticities of the mixtures of the two extremes of the visible spectrum. The area bounded by the closed curve made up of the spectrum locus and this straight line is the locus of all physically realizable chromaticities. Note that the points representing the primaries of the ICI coordinate system. the apices of the triangle ($x=1$, $y=z=0$; $y=1$, $x=z=0$; $z=1$, $x=y=0$), all fall outside this area; that is, the primaries are imaginary. Note also that both the X and Z primaries fall on the line $y=0$, which is unassociated with luminosity and is known as the *alychne* or *lightless line;* this means that although it has the power to elicit in the standard observer a considerable X and Z response, resulting in a vivid bluish purple color, radiant flux of wavelength 380 to 420 mμ is only slightly luminous.

The areas in figure 3 corresponding to common color designations for lights are those proposed by Kelly [66] on a basis to be discussed later.

Figure 3a illustrates the use of the (x,y)-chromaticity diagram for the specification of railway signal colors [4a].

At the time of setting up the standard observer and coordinate system, the International Commission on Illumination [20] recommended three standard illuminants for colorimetry, and standard angular conditions and a reflectance standard for the colorimetry of opaque specimens. The standard illuminants are: illuminant A, representative of gas-filled incandescent lamps; illuminant B, representative of noon sunlight; and illuminant C representative of average daylight such as that from a completely overcast sky. Illuminant A is an incandescent lamp operated at a color temperature of 2,854°K on the international temperature scale (C_2=14,380), or at a color temperature of 2,850°K if the value of C_2=14,360 recommended by Wensel [137] be used in Planck's formula for spectral emittance. Illuminant B is obtained by using

TABLE 2. *The 1931 ICI standard observer*

Chromaticity coordinates			Wave-length mμ	Tristimulus values of equal-energy spectrum			Chromaticity coordinates			Wave-length mμ	Tristimulus values of equal-energy spectrum		
x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$	x	y	z		$\bar{x}$	$\bar{y}$	$\bar{z}$
0.1741	0.0050	0.8209	380	0.0014	0.0000	0.0065	0.5125	0.4866	0.0009	580	0.9163	0.8700	0.0017
.1740	.0050	.8210	385	.0022	.0001	.0105	.5448	.4544	.0008	585	.9786	.8163	.0014
.1738	.0049	.8213	390	.0042	.0001	.0201	.5752	.4242	.0006	590	1.0263	.7570	.0011
.1736	.0049	.8215	395	.0076	.0002	.0362	.6029	.3965	.0006	595	1.0567	.6949	.0010
.1733	.0048	.8219	400	.0143	.0004	.0679	.6270	.3725	.0005	600	1.0622	.6310	.0008
.1730	.0048	.8222	405	.0232	.0006	.1102	.6482	.3514	.0004	605	1.0456	.5668	.0006
.1726	.0048	.8226	410	.0435	.0012	.2074	.6658	.3340	.0002	610	1.0026	.5030	.0003
.1721	.0048	.8231	415	.0776	.0022	.3713	.6801	.3197	.0002	615	0.9384	.4412	.0002
.1714	.0051	.8235	420	.1344	.0040	.6456	.6915	.3083	.0002	620	.8544	.3810	.0002
.1703	.0058	.8239	425	.2148	.0073	1.0391	.7006	.2993	.0001	625	.7514	.3210	.0001
.1689	.0069	.8242	430	.2839	.0116	1.3856	.7079	.2920	.0001	630	.6424	.2650	.0000
.1669	.0086	.8245	435	.3285	.0168	1.6230	.7140	.2859	.0001	635	.5419	.2170	.0000
.1644	.0109	.8247	440	.3483	.0230	1.7471	.7190	.2809	.0001	640	.4479	.1750	.0000
.1611	.0138	.8251	445	.3481	.0298	1.7826	.7230	.2770	.0000	645	.3608	.1382	.0000
.1566	.0177	.8257	450	.3362	.0380	1.7721	.7260	.2740	.0000	650	.2835	.1070	.0000
.1510	.0227	.8263	455	.3187	.0480	1.7441	.7283	.2717	.0000	655	.2187	.0816	.0000
.1440	.0297	.8263	460	.2908	.0600	1.6692	.7300	.2700	.0000	660	.1649	.0610	.0000
.1355	.0399	.8246	465	.2511	.0739	1.5281	.7311	.2689	.0000	665	.1212	.0446	.0000
.1241	.0578	.8181	470	.1954	.0910	1.2876	.7320	.2680	.0000	670	.0874	.0320	.0000
.1096	.0868	.8036	475	.1421	.1126	1.0419	.7327	.2673	.0000	675	.0636	.0232	.0000
.0913	.1327	.7760	480	.0956	.1390	0.8130	.7334	.2666	.0000	680	.0468	.0170	.0000
.0687	.2007	.7306	485	.0580	.1693	.6162	.7340	.2660	.0000	685	.0329	.0119	.0000
.0454	.2950	.6596	490	.0320	.2080	.4652	.7344	.2656	.0000	690	.0227	.0082	.0000
.0235	.4127	.5638	495	.0147	.2586	.3533	.7346	.2654	.0000	695	.0158	.0057	.0000
.0082	.5384	.4534	500	.0049	.3230	.2720	.7347	.2653	.0000	700	.0114	.0041	.0000
.0039	.6548	.3413	505	.0024	.4073	.2123	.7347	.2653	.0000	705	.0081	.0029	.0000
.0139	.7502	.2359	510	.0093	.5030	.1582	.7347	.2653	.0000	710	.0058	.0021	.0000
.0389	.8120	.1491	515	.0291	.6082	.1117	.7347	.2653	.0000	715	.0041	.0015	.0000
.0743	.8338	.0919	520	.0633	.7100	.0782	.7347	.2653	.0000	720	.0029	.0010	.0000
.1142	.8262	.0596	525	.1096	.7932	.0573	.7347	.2653	.0000	725	.0020	.0007	.0000
.1547	.8059	.0394	530	.1655	.8620	.0422	.7347	.2653	.0000	730	.0014	.0005	.0000
.1929	.7816	.0255	535	.2257	.9149	.0298	.7347	.2653	.0000	735	.0010	.0004	.0000
.2296	.7543	.0161	540	.2904	.9540	.0203	.7347	.2653	.0000	740	.0007	.0003	.0000
.2658	.7243	.0099	545	.3597	.9803	.0134	.7347	.2653	.0000	745	.0005	.0002	.0000
.3016	.6923	.0061	550	.4334	.9950	.0087	.7347	.2653	.0000	750	.0003	.0001	.0000
.3373	.6589	.0038	555	.5121	1.0002	.0057	.7347	.2653	.0000	755	.0002	.0001	.0000
.3731	.6245	.0024	560	.5945	0.9950	.0039	.7347	.2653	.0000	760	.0002	.0001	.0000
.4087	.5896	.0017	565	.6784	.9786	.0027	.7347	.2653	.0000	765	.0001	.0000	.0000
.4441	.5547	.0012	570	.7621	.9520	.0021	.7347	.2653	.0000	770	.0001	.0000	.0000
.4788	.5202	.0010	575	.8425	.9154	.0018	.7347	.2653	.0000	775	.0000	.0000	.0000
.5125	.4866	.0009	580	.9163	.8700	.0017	.7347	.2653	.0000	780	.0000	.0000	.0000
										Totals	21.3713	21.3714	21.3715

this same lamp in combination with a two-cell Davis-Gibson liquid filter giving a nearest color temperature of about 5,000°*K*. Illuminant *C* is obtained similarly and results in a source of nearest color temperature about 6,800°*K*. These illuminants are recommended for general use, or whenever there is no special reason for using some other source. Detailed directions for making the filters are given by Davis and Gibson [22], and the composition of the solutions is specified in many accessible publications [44, 56, 108]. Table 19 gives the relative spectral irradiance of illuminants *A*, *B*, and *C*.

The angular conditions recommended for the colorimetry of opaque specimens are that the light shall strike the specimen at 45° and that the specimen shall be viewed along the perpendicular to its surface. Since, as in the inspection of the glossy and semiglossy materials making up the bulk of commerce, these conditions avoid by a wide margin the inclusion of the main beam of specularly reflected light, they should be used particularly when appearance is the chief concern, and in the absence of circumstances requiring another choice of angular conditions. However, one nonstandard set of angular conditions deserves special mention because of its wide use; this is to illuminate the specimen along or near the perpendicular and to collect by means of a white-lined sphere all of the reflected light regardless of direction, except for a small solid angle centering along the average direction of the illuminating beam. The particular merit of this choice of angular conditions is that it does not waste any of the reflected flux; and so it is well suited to photoelectric devices with a small margin of sensitivity. The results obtained by using these normal-diffuse conditions correlate with the appearance of the specimen viewed under a completely uniform sky. But since inspectors are usually not concerned with the appearance of specimens so illuminated, it is customary to apply to results so obtained a correction adjusting them at least approximately to the standard angular conditions.

The reflectance standard recommended for the colorimetry of opaque specimens is a white surface prepared by collecting on a metallic or other suitable surface an opaque layer formed by the smoke from magnesium ribbon or shavings burning in air. Details for the preparation of a magnesium-oxide reflectance standard are given in NBS Letter Circular LC547 [113]. It is now nearly universal practice

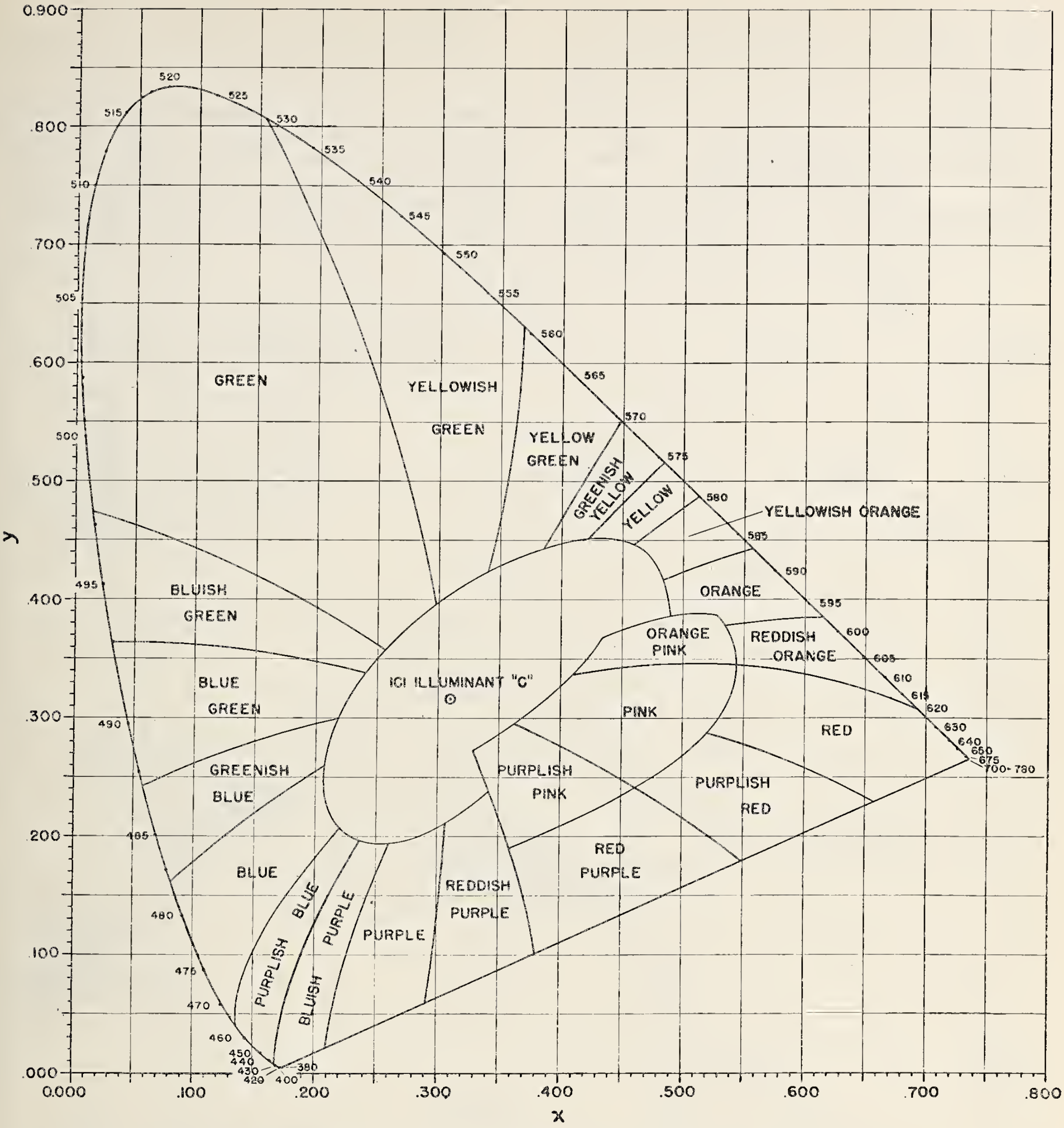


FIGURE 3. *The (x,y)-chromaticity diagram of the ICI system.*

The abscissa, x, is the ratio of the tristimulus value, X, to the sum of all three $(X+Y+Z)$. The ordinate, y, is the ratio of Y to this sum. The parts of the spectrum locus are identified by wavelength in millimicrons. The region bounded by this locus and the straightline (purple border) joining its extremes represents all chromaticities producible by actual stimuli. The division of this chromaticity diagram into a number of regions corresponding to various hues surrounding a central region to which no hue name is given is from Kelly's work [66] on color designations for lights. It refers to observation of self-luminous areas against a dark background.

to carry out the spectrophotometry of color standards relative to this magnesium-oxide standard; that is, the ratio of the directional reflectance of the unknown specimen to that of the magnesium-oxide standard, similarly illuminated and viewed, is found rather than the absolute value of directional reflectance itself. For the standard (45°-normal) angular conditions, it has been found [114, 117] that a freshly prepared magnesium-oxide reflectance standard has the very convenient directional reflectance of 1.00 with variations throughout the visible spectrum of less than 0.01. Because a magnesium-oxide surface gradually becomes yellow with age, it is often more feasible to use a secondary standard of more permanent nature, such as a vitreous plaque. These secondary standards are, however, calibrated by comparison with freshly prepared magnesium oxide, and the results obtained by using them are corrected to correspond with what would have been obtained had the magnesium-oxide standard, itself, been used.

1. Reduction of Spectrophotometric Data to Colorimetric Terms

The fundamental nature of the tristimulus specification of color permits it to be used as a common denominator by means of which colorimeters (involving color standards of glass, plastic, or solutions) or systems of material color standards, transparent and opaque, may be intercompared. To demonstrate how the ICI standard observer and coordinate system may be used for this purpose, the steps in the evaluation of two of the reference samples issued by us, Kitchen Green (SKC-15) [94] and Orchid (SBC-20) [93] are reproduced in detail. Figure 4 shows spectral reflectances of the master standards for these two kinds of vitreous enamel samples.

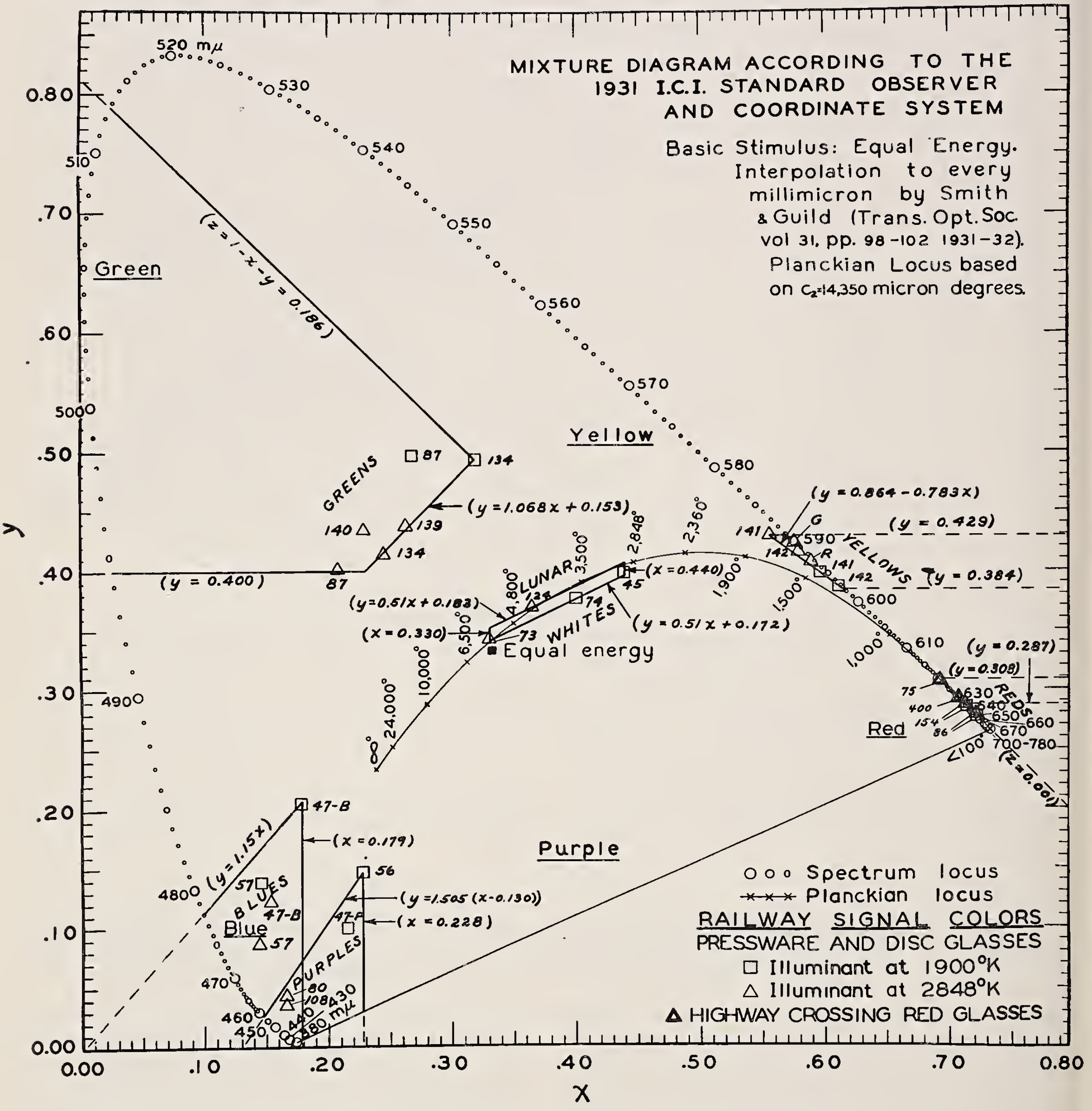


Figure 3a. *Illustration of the use of the (x,y)-chromaticity diagram to specify the railway signal colors.*

Note that areas corresponding to permitted colors for red, yellow, green, blue, purple, and lunar white signals are shown (AAR, [4a]).

TABLE 3(a)

ICI Coordinates. Illuminant *A* (2,854°K)

Sample.................................... Source of Trans Data....................................

Submitted by ....

λ	$(\bar{x}E)_\lambda$	$(\bar{y}E)_\lambda$	$(\bar{z}E)_\lambda$	T_λ	$(\bar{x}ET)_\lambda$	$(\bar{y}ET)_\lambda$	$(\bar{z}ET)_\lambda$
380	1		6	0.			
90	5		23	.			
400	19	1	93	.			
10	71	2	340	.			
20	262	8	1256	.			
30	649	27	3167	.			
40	926	61	4647	.			
450	1031	117	5435	.			
60	1019	210	5851	.			
70	776	362	5116	.			
80	428	622	3636	.			
90	160	1039	2324	.			
500	27	1792	1509	.			
10	57	3080	969	.			
20	425	4771	525	.			
30	1214	6322	309	.			
40	2313	7600	162	.			
550	3732	8568	75	.			
60	5510	9222	36	.			
70	7571	9457	21	.			
80	9719	9228	18	.			
90	11579	8540	12	.			
600	12704	7547	10	.			
10	12669	6356	4	.			
20	11373	5071	3	.			
30	8980	3704		.			
40	6558	2562		.			
650	4336	1637		.			
60	2628	972		.			
70	1448	530		.			
80	804	292		.			
90	404	146		.			
700	209	75		.			
10	110	40		.			
20	57	19		.			
30	28	10		.			
40	14	6		.			
750	6	2		.			
60	4	2		.			
70	2			.			
Sums ⟶	109828	100000	35547	Sums ⟶	$X=$	$Y=$	$Z=$
x_w, y_w, z_w	0.4476	0.4075	0.1449	$S=X+Y+Z$			
x, y, and z ⟶ $(x=X/S,\ y=Y/S,\ z=Z/S)$							

Planck 2,854°K, $C_2=14{,}380$

Planck 2,848°K, $C_2=14{,}350$ Computed by........................ Checked by........................

TABLE 3(b)

ICI Coordinates. Illuminant *B*

Sample.. Source of Trans Data..

Submitted by ....

λ	$(\bar{x}E)\lambda$	$(\bar{y}E)\lambda$	$(\bar{z}E)\lambda$	$T\lambda$	$(\bar{x}ET)\lambda$	$(\bar{y}ET)\lambda$	$(\bar{z}ET)\lambda$
380	3		14	0.			
90	13		60	.			
400	56	2	268	.			
10	217	6	1033	.			
20	812	24	3899	.			
30	1983	81	9678	.			
40	2689	178	13489	.			
450	2744	310	14462	.			
60	2454	506	14085	.			
70	1718	800	11319	.			
80	870	1265	7396	.			
90	295	1918	4290	.			
500	44	2908	2449	.			
10	81	4360	1371	.			
20	541	6072	669	.			
30	1458	7594	372	.			
40	2689	8834	188	.			
550	4183	9603	84	.			
60	5840	9774	38	.			
70	7472	9334	21	.			
80	8843	8396	16	.			
90	9728	7176	10	.			
600	9948	5909	7	.			
10	9436	4734	3	.			
20	8140	3630	2	.			
30	6200	2558		.			
40	4374	1709		.			
650	2815	1062		.			
60	1655	612		.			
70	876	321		.			
80	465	169		.			
90	220	80		.			
700	108	39		.			
10	53	19		.			
20	26	9		.			
30	12	4		.			
40	6	2		.			
750	2	1		.			
60	2	1		.			
70	1			.			
Sums	99072	100000	85223	$S=X+Y+Z$			
x_w, y_w, z_w	0.34848	0.35175	0.29977				
x, y, and z ⟶ ($x=X/S$, $y=Y/S$, $z=Z/S$)							

Computed by........................ Checked by........................

TABLE 3(c)

ICI Coordinates. Illuminant *C*

Sample.. Source of Trans Data..

Submitted by ....

λ	$(\bar{x}E)\lambda$	$(\bar{y}E)\lambda$	$(\bar{z}E)\lambda$	$T\lambda$	$(\bar{x}ET)\lambda$	$(\bar{y}ET)\lambda$	$(\bar{z}ET)\lambda$
380	4		20	0.			
90	19		89	.			
400	85	2	404	.			
10	329	9	1570	.			
20	1238	37	5949	.			
30	2997	122	14628	.			
40	3975	262	19938	.			
450	3915	443	20638	.			
60	3362	694	19299	.			
70	2272	1058	14972	.			
80	1112	1618	9461	.			
90	363	2358	5274	.			
500	52	3401	2864	.			
10	89	4833	1520	.			
20	576	6462	712	.			
30	1523	7934	388	.			
40	2785	9149	195	.			
550	4282	9832	86	.			
60	5880	9841	39	.			
70	7322	9147	20	.			
80	8417	7992	16	.			
90	8984	6627	10	.			
600	8949	5316	7	.			
10	8325	4176	2	.			
20	7070	3153	2	.			
30	5309	2190		.			
40	3693	1443		.			
650	2349	886		.			
60	1361	504		.			
70	708	259		.			
80	369	134		.			
90	171	62		.			
700	82	29		.			
10	39	14		.			
20	19	6		.			
30	8	3		.			
40	4	2		.			
750	2	1		.			
60	1	1		.			
70	1			.			
Sums	98041	100000	118103	Sums ⟶	$X=$	$Y=$	$Z=$
x_w, y_w, z_w	.31012	.31631	.37357	$S=X+Y+Z$			
x, y, and z ⟶ $(x=X/S, y=Y/S, z=Z/S)$							

Computed by........................ Checked by........................

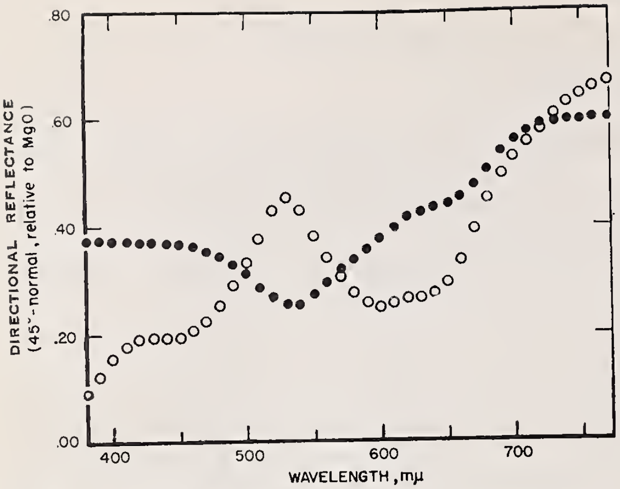


FIGURE 4. *Spectral directional reflectance (45°- normal relative to magnesium oxide) of two of the master standards of colors for kitchen and bathroom accessories (Orchid, see solid circles; Kitchen Green, see open circles).*

It is often hard to tell from inspection of spectrophotometric curves such as these what color will be perceived. Note, for example, that the Kitchen Green standard reflects more strongly in the long-wave (red) extreme of the spectrum than the Orchid standard does.

(a) Summation

Tables 3(a), (b) and (c) give computation forms for evaluation of the colors of nonself-luminous specimens that transmit, scatter, or reflect incident light. Table 3(a) refers to illuminant A; table 3(b) to illuminant B; and table 3(c), to illuminant C. Table 4 gives the spectral reflectances of the two master standards illustrated in figure 4, and it also gives the products provided for in the computation form shown in table 3(*c*). These products evaluate the spectral irradiance of the observer's eye by energy reflected perpendicularly from the master standards illuminated by standard illuminant C (representative of average daylight), relative to a magnesium-oxide reflectance standard similarly illuminated and viewed. This evaluation is in terms of the amount of each of the primaries $\bar{x}_\lambda$, $\bar{y}_\lambda$, $\bar{z}_\lambda$ required by the standard observer to form a stimulus equivalent to each 10-mμ band of the spectrum of the reflected energy. The sums of these products gives by Grassman's law a similar evaluation of the entire beam reflected from the specimen toward the eye of the observer; these sums are called the *tristimulus values, X,Y,Z.* The luminous directional reflectance is found as $Y/100{,}000$; and the chromaticity coordinates, x,y,z, are found by dividing X, Y, and Z, respectively, by the corresponding sum, $X+Y+Z$. Table 5 lists these results for the two master standards and they are engraved on each duplicate standard issued by the National Bureau of Standards. Figure 5 is the (x,y)-chromaticity diagram showing the location of the points representing the chromaticities of these standards in accord with their chromaticity coordinates (x,y). Comparison of figure 5 with figure 3 indicates that the light reflected from these two standards, if seen as from a self-luminous source with dark surroundings, would differ too little from daylight to deserve a special hue name. The orchid specimen illuminated by daylight would reflect light almost distinctive enough to take the designation, purplish pink; that reflected from the kitchen green specimen would almost take the designation, yellowish green. It is thus important to note that color perception is not related in a simple way to the chromaticity diagram, but depends importantly on whether the object is seen as a self-luminous source in a dark surrounding field, or as a part of an illuminated scene in an object-color environment.

TABLE 4. *Spectral directional reflectances of the Orchid and Kitchen Green standards together with details of computation of their tristimulus values X, Y, Z*

Wave-length	Re-flect-ance	Kitchen Green tristimulus values			Re-flect-ance	Orchid tristimulus values		
		X	*Y*	*Z*		*X*	*Y*	*Z*
mμ								
380	0.091	0	......	2	0.372	1	......	7
390	.122	2	......	11	.372	7	......	33
400	.153	13	0	62	.372	32	1	150
410	.179	59	2	281	.371	122	3	582
420	.191	236	7	1.136	.371	459	14	2,207
430	.193	578	24	2,823	.371	1,112	45	5,427
440	.193	767	51	3.848	.370	1,471	97	7,377
450	.196	767	87	4,045	.367	1,437	163	7,574
460	.208	699	144	4,014	.362	1,217	251	6,985
470	.224	509	237	3,354	.355	807	376	5,315
480	.251	279	406	2,375	.344	383	557	3,255
490	.291	106	686	1,535	.330	120	778	1,740
500	.331	17	1,126	948	.312	16	1,061	894
510	.376	33	1,817	572	.289	26	1,397	439
520	.428	247	2,766	305	.267	154	1,725	190
530	.451	687	3.578	175	.253	385	2,007	98
540	.429	1,195	3,925	84	.255	710	2,333	50
550	.380	1,627	3,736	33	.271	1,160	2,664	23
560	.341	2,005	3,356	13	.294	1,729	2,893	11
570	.308	2,255	2,817	6	.318	2,328	2,909	6
580	.277	2,332	3,214	4	.338	2,845	2,701	5
590	.260	2,336	1,723	3	.355	3,189	2,353	4
600	.249	2,228	1,324	2	.378	3,383	2,009	3
610	.256	2,131	1,069	1	.399	3,322	1,666	1
620	.263	1,859	829	1	.416	2,941	1,312	1
630	.267	1,418	585	0	.426	2,262	933	.....
640	.276	1,019	398	......	.433	1,599	625	.....
650	.295	693	261	......	.441	1,036	391	.....
660	.335	456	169	......	.454	618	229	.....
670	.394	279	102	......	.476	337	123	.....
680	.450	166	60	......	.504	186	68	.....
690	.495	85	31	......	.534	91	33	.....
700	.529	43	15	......	.559	46	16	.....
710	.554	22	8	......	.575	22	8	.....
720	.580	11	3	......	.587	11	4	.....
730	.607	5	2	......	.592	5	2	.....
740	.628	3	1	......	.596	2	1	.....
750	.644	1	1	......	.598	1	1	.....
760	.657	1	1	......	.600	1	1	.....
770	.668	1	0	......	.602	1	......	.....
Total	.....	27,170	33,561	25,633	.....	35,574	31,750	42,378

TABLE 5. *Luminous directional reflectance, Y, and chromaticity coordinates, x,y, of the Kitchen Green and Orchid master standards for standard illuminant C*

Color standard	Luminous directional reflectance, *Y*	Chromaticity coordinates	
		x	*y*
Kitchen Green	0.34	0.315	0.389
Orchid	.32	.324	.289

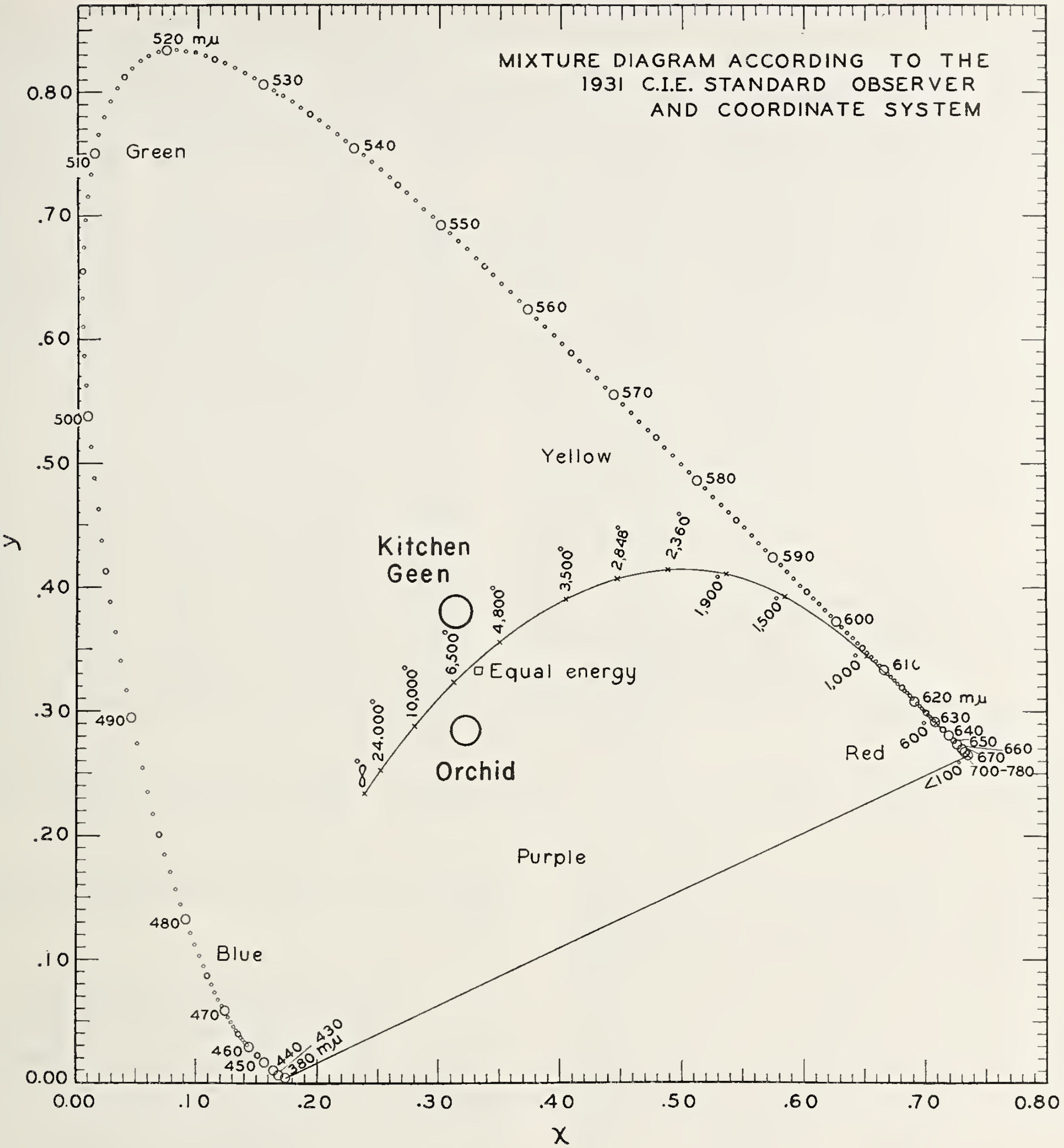


FIGURE 5. *Chromaticities of Kitchen Green and Orchid on the (x,y)-chromaticity diagram.*

Note that both of these colors are represented by points within the area found by Kelly (see fig. 3) to be too little different from daylight to deserve a hue name. This serves to emphasize that color perception is not related to position on the chromaticity diagram in any simple way. It depends importantly upon whether the object is self-luminous and viewed against a dark background or is nonself-luminous and viewed in the object-color environments that we are accustomed to.

(b) Selected Ordinate Method

The labor of computing X,Y,Z or Y,x,y, corresponding to pairs of spectrophotometric curves in accord with tables 3(a), 3(b), or 3(c) to see how the colors of the corresponding specimens compare is considerable. Often the degree of metamerism exhibited by the pair is sufficiently small that the comparison can be made directly from the curves, themselves. There is still frequent need, however, in the establishment of color standards and tests for conformity to those standards to compute the tristimulus values, X,Y,Z, and a short-cut method yielding nearly the accuracy of the summation method has come into considerable use. In this method the spectrophotometric curve, instead of being read at every 5 or 10 mμ, is read at a series of selected ordinates different for each illuminant. Instead of multiplying the reflectance or transmittance of the specimen by the tristimulus values of the spectrum of the illuminant, $E\bar{x}_\lambda$, $E\bar{y}_\lambda$, $E\bar{z}_\lambda$, the selected ordinates are spaced proportionately closer in the wavelength regions where the tristimulus values are higher, and the corresponding readings of spectral reflectance are simply added. Table 6 gives [44] selected ordinates

TABLE 6. *Selected ordinates (in millimicrons) for computing tristimulus values, X, Y, Z, for specimens under standard illuminants*

Number	X	Y	Z	X	Y	Z	X	Y	Z
	STANDARD ILLUMINANT A			STANDARD ILLUMINANT B			STANDARD ILLUMINANT C		
1	444.0	487.8	416.4	428.1	472.3	414.8	424.4	465.9	414.1
2*	516.9	507.7	424.9	442.1	494.5	422.9	435.5	489.4	422.2
3	544.0	517.3	429.4	454.1	505.7	427.1	443.9	500.4	426.3
4	554.2	524.1	432.9	468.1	513.5	430.3	452.1	508.7	429.4
5*	561.4	529.8	436.0	527.8	519.6	433.0	461.2	515.1	432.0
6	567.1	534.8	438.7	543.3	524.8	435.4	474.0	520.6	434.3
7	572.0	539.4	441.3	551.9	529.4	437.7	531.2	525.4	436.5
8*	576.3	543.7	443.7	558.5	533.7	439.9	544.3	529.8	438.6
9	580.2	547.8	446.0	564.0	537.7	442.0	552.4	533.9	440.6
10	583.9	551.7	448.3	568.8	541.5	444.0	558.7	537.7	442.5
11*	587.2	555.4	450.5	573.1	545.1	446.0	564.1	541.4	444.4
12	590.5	559.1	452.6	577.1	548.7	448.0	568.9	544.9	446.3
13	593.5	562.7	454.7	580.9	552.1	450.0	573.2	548.4	448.2
14*	596.5	566.3	456.8	584.5	555.5	451.9	577.3	551.8	450.1
15	599.4	569.8	458.8	588.0	559.0	453.9	581.3	555.1	452.1
16	602.3	573.3	460.8	591.4	562.4	455.8	585.0	558.5	454.0
17*	605.2	576.9	462.9	594.7	565.8	457.8	588.7	561.9	455.9
18	608.0	580.5	464.9	598.1	569.3	459.8	592.4	565.3	457.9
19	610.9	584.1	467.0	601.4	572.9	461.8	596.0	568.9	459.9
20*	613.8	587.9	469.2	604.7	576.7	463.9	599.6	572.5	462.0
21	616.9	591.8	471.6	608.1	580.6	466.1	603.3	576.4	464.1
22	620.0	595.9	474.1	611.6	584.7	468.4	607.0	580.5	466.3
23*	623.3	600.1	476.8	615.3	589.1	470.8	610.9	584.8	468.7
24	626.9	604.7	479.9	619.1	593.9	473.6	615.0	589.6	471.4
25	630.8	609.7	483.4	623.3	599.1	476.6	619.4	594.8	474.3
26*	635.3	615.2	487.5	628.0	605.0	480.2	624.2	600.8	477.7
27	640.5	621.5	492.7	633.4	611.8	484.5	629.8	607.7	481.8
28	646.9	629.2	499.3	640.1	619.9	490.2	636.6	616.1	487.2
29*	655.9	639.7	508.4	649.2	630.9	498.6	645.9	627.3	495.2
30	673.5	659.0	526.7	666.3	650.7	515.2	663.0	647.4	511.2
Multiplying factors:									
30 ordinates	0.03661	0.03333	0.01185	0.03303	0.03333	0.02842	0.03268	0.03333	0.03938
10 ordinates	.10984	.10000	.03555	.09909	.10000	.08526	.09804	.10000	.11812

for illuminant A (incandescent lamp light), illuminant B (noon sunlight), and illuminant C (average daylight). Table 7 gives the spectral directional reflectances of the Kitchen Green standard read by interpolation from a large-scale copy of figure 4 for the selected ordinates for illuminant C, together with the sums of these reflectances both for 10 ordinates and for 30. It will be noted that after applying the multiplying factors listed in table 6 under standard illuminant C, the tristimulus values, *X,Y,Z,* for the Kitchen Green standard are found again to a close approximation (compare 0.271, 0.335, and 0.256 from table 7 with 0.272, 0.336, and 0.256, respectively, found from table 4 by dividing the totals by 100,000).

Ten selected ordinates sometimes give significant information; note in table 7 that they have yielded results differing by not more than 3 in the third decimal from those (table 4) obtained by 10-mμ summation. Thirty selected ordinates often are sufficient; and one hundred selected ordinates are sufficient for all but a few very irregular spectral distributions (such as produced by gaseous discharge tubes) to which 5 mμ-summation is also inapplicable. These wavelengths are available for many illuminants in other publications [16, 108]. Nickerson [100] has published a study of the reliability of the selected-ordinate method of computation and of 10-mμ summation, both relative to 5-mμ summation taken as correct.

(c) Semigraphical Centroid Method

A still more rapid method has been developed by Robinson [121] for the reduction of spectrophotometric curves that can be satisfactorily approximated by a series of straight lines. The curve is divided into a number of segments, each segment covering a wavelength interval for which the wavelength centroid has already been computed, and a straight-line approximation is drawn in by visual estimation for each segment. The straight-line approximating one segment of the curve need not, and usually does not, join that approximating the next segment. A few spectrophotometric curves will be encountered that are sufficiently simple to be reduced by means of two segments or even by a single straight line; four to eight segments are most often used; segments as small as 20 mμ are occasionally required. For curves so irregular as to require segments smaller than 20 mμ this method is scarcely shorter than 10-mμ summation.

The method is based upon two principles. First, the spectral distribution of each primary may be considered as a variable mass distribution along the spectrophotometric curve. If the curve is a straight line for a certain wavelength interval, the center of gravity for that interval must fall somewhere on the line, and the wavelength of this center of gravity can be computed from the spectral distribution of the primary itself. Table 8 gives the wavelength centroids for X, Y, and Z in the spectra of illuminants A, B, and C from 400 to 700 mμ taken in 1, 2, 4, 8, and 15 intervals.

Table 7. *Spectral directional reflectances of Kitchen Green standard (see fig. 5) read for the selected ordinates for standard illuminant C (see table 6)*

Number	X		Y		Z	
1	0.192	.....	0.216	.....	0.187	.....
2*	.193	0.193	.285	0.285	.192	0.192
3	.194	.....	.334	.....	.193	.....
4	.197	.209	.371	.....	.193	.....
5*	.209	.209	.401	.401	.193	.193
6	.232	.....	.430	.....	.193	.....
7	.450	.....	.445	.....	.193	.....
8*	.410	.410	.451	.451	.193	.193
9	.370	.....	.447	.....	.193	.....
10	.346	.....	.439	.....	.193	.....
11*	.328	.328	.423	.423	.194	.194
12	.311	.....	.406	.....	.194	.....
13	.297	.....	.388	.....	.195	.....
14*	.284	.284	.372	.372	.196	.196
15	.275	.....	.359	.....	.199	.....
16	.268	.....	.346	.....	.201	.....
17*	.262	.262	.334	.334	.203	.203
18	.256	.....	.325	.....	.205	.....
19	.250	.....	.311	.....	.208	.....
20*	.248	.248	.299	.299	.211	.211
21	.251	.....	.287	.....	.214	.....
22	.254	.....	.276	.....	.217	.....
23*	.258	.258	.269	.269	.221	.221
24	.261	.....	.261	.....	.226	.....
25	.263	.....	.251	.....	.233	.....
26*	.266	.266	.250	.250	.242	.242
27	.267	.....	.254	.....	.255	.....
28	.271	.....	.262	.....	.276	.....
29*	.285	.285	.266	.266	.294	.294
30	.354	.....	.289	.....	.382	.....
Totals	8.302	2.743	10.047	3.350	6.489	2.139
Times factor ..	0.271	0.269	0.335	0.335	0.256	0.253

The second principle is that the center of gravity of a reflectance curve over any interval lies somewhere on the straight line connecting the centers of gravity of any two subdivisions of that interval. The method is therefore simply to locate the point on the straight-line approximations for each segment that corresponds to the wavelength in table 8; then connect these points in pairs by straight lines, locate new points at the wavelengths given in table 8 for the double intervals, and so on until one straight line for the entire spectrum is found. The tristimulus value is then found by reading on the scale of spectral reflectance the ordinate on this line corresponding to the wavelength given in the final columns of table 8 for X, Y, or Z, as the case may be, and applying the indicated multiplying factor.

Figure 6 shows the application of this method to the curve of spectral-directional reflectance for the Kitchen Green (SKC-15) standard to obtain its luminous directional reflectance for illuminant C. Eight segments have been used, 400 to 440, 440 to 480, 480 to 520, 520 to 560, 560 to 600, 600 to 640, 640 to 680, and 680 to 700 mμ. The straight-line approximations to these segments are shown by solid lines. The points on these straight lines for the wavelength centroids given in table 8 under Y_2 for illuminant C (431.7, 466.0, 505.0, 541.8, 577.4, 616.4, 653.3, and 688.0 mμ) are connected in pairs by four dashed lines. The points on these lines for the wavelength centroids given under Y_3 (463.1, 531.0, 588.6, 655.2 mμ) are connected in pairs by two dashed lines. The points on these lines for the wavelength centroids given under Y_4 (526.6, 592.2 mμ) are connected by the final dashed line, and the scale of directional reflectance read for the wavelength centroid given under Y_5 (557.0 mμ) is found to be 0.334 (point indicated by small square). This value multiplied by the factor 1.0000 given in table 8 is the desired evaluation of luminous directional reflectance, $Y=0.334$, which is seen to be in good agreement with the value (0.336) found by 10-mμ summation in table 4.

2. Direct Colorimetry

The scales of the standard coordinate system [20] are defined independent of the properties of the standard observer, and it is possible at least in theory to obtain the tristimulus values, X,Y,Z, of any color by comparison of it by a normal observer with a mixture of a limited number of fixed or variable stimuli.

(a) Tristimulus Colorimeters

Tristimulus values, X,Y,Z, may be obtained by direct comparison of the unknown light with an optical mixture of three primary lights in a divided

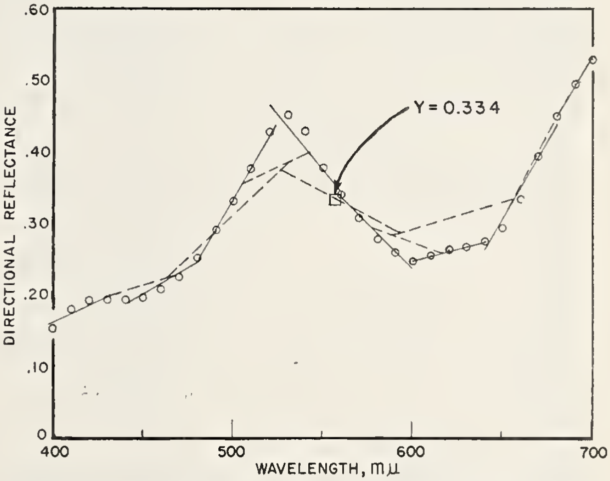


Figure 6. *Application of the Robinson [121] centroid method to find the reflectance of the Kitchen Green standard for standard illuminant C (representative of average daylight).*

The solid lines are the approximate representation of the spectrophotometric curve, segment by segment. Four dashed lines connect the centroids of these segments (see table 9, Y-scale for illuminant C). Two dashed lines in turn connect the centroids of these four lines; and the value of daylight reflectance (Y=0.334) is read from the centroid (about 557 mμ) of the line connecting the centroids of the semifinal lines.

TABLE 8.

X scale for *A*—illuminant 400 to 700 mμ

λ	X_1	X_2	X_3	X_4	X_5
400 420	414.33	429.83	449.96	514.83	595.22
421 440	432.25				
441 460	450.64	458.44			
461 480	469.06				
481 500	486.79	499.02	544.50		
501 520	515.90				
521 540	532.95	547.10			
541 560	551.89			608.94	
561 580	571.42	583.12	601.63		
581 600	590.93				
601 620	610.30	618.33			
621 640	629.58				
641 660	649.00	654.03	656.35		
661 680	668.56				
681 700	688.29	688.29			

Multiplying factor — 1.0983

Y scale for *A*—illuminant 400 to 700 mμ

λ	Y_1	Y_2	Y_3	Y_4	Y_5
400 420	414.44	431.85	465.26	532.52	572.07
421 440	433.47				
441 460	452.47	467.42			
461 480	472.27				
481 500	492.24	506.95	534.45		
501 520	512.08				
521 540	531.24	542.48			
541 560	550.81				
561 580	570.49	579.81	593.17	598.07	
581 600	590.16				
601 620	609.84	617.06			
621 640	629.37				
641 660	648.92	653.83	656.09		
661 680	668.53				
681 700	688.28	688.28			

Multiplying factor — 1.0000

Z scale for *A*—illuminant 400 to 700 mμ

λ	Z_1	Z_2	Z_3	Z_4	Z_5
400 420	414.34	429.95	452.08	461.83	462.11
421 440	432.32				
441 460	450.87	459.84			
461 480	469.66				
481 500	489.04	494.74	498.73		
501 520	508.77				
521 540	528.64	532.64			
541 560	548.05				
561 580	569.23	576.74	582.30	582.46	
581 600	589.55				
601 620	608.39	612.31			
621 640	627.70				
641 660	644.17	644.17	644.17		
661 680	0				
681 700	0	0			

Multiplying factor — 0.3555

X scale for *B*—illuminant 400 to 700 mμ

λ	X_1	X_2	X_3	X_4	X_5
400 420	414.40	429.71	447.98	494.44	576.20
421 440	432.14				
441 460	450.33	457.40			
461 480	468.81				
481 500	486.55	496.34	542.78		
501 520	515.70				
521 540	532.84	546.69			
541 560	551.73				
561 580	571.17	582.14	599.04	605.19	
581 600	590.68				
601 620	610.16	617.85			
621 640	629.46				
641 660	648.91	653.68	655.71		
661 680	668.43				
681 700	688.12	688.12			

Multiplying factor — 0.9907

Y scale for *B*—illuminant 400 to 700 mμ

λ	Y_1	Y_2	Y_3	Y_4	Y_5
400 420	414.50	431.72	463.74	528.45	561.45
421 440	433.37				
441 460	452.17	466.46			
461 480	472.00				
481 500	491.87	505.55	531.95		
501 520	511.70				
521 540	531.10	541.97			
541 560	550.66				
561 580	570.24	578.80	590.42	594.28	
581 600	589.91				
601 620	609.70	616.59			
621 640	629.25				
641 660	648.83	653.49	655.46		
661 680	668.40				
681 700	688.11	688.11			

Multiplying factor — 1.0000

Z scale for *B*—illuminant 400 to 700 mμ

λ	Z_1	Z_2	Z_3	Z_4	Z_5
400 420	414.42	429.83	449.99	456.78	456.89
421 440	432.21				
441 460	450.55	458.75			
461 480	469.40				
481 500	488.68	493.43	496.33		
501 520	508.39				
521 540	528.51	532.27			
541 560	547.92				
561 580	568.98	575.76	580.37	580.49	
581 600	589.32				
601 620	608.28	611.95			
621 640	627.65				
641 660	644.07	644.07	644.07		
661 680	0				
681 700	0	0			

Multiplying factor — 0.8522

Table 8—Continued

X scale for C—illuminant 400 to 700 mμ					
λ	X_1	X_2	X_3	X_4	X_5
400 420	414.42	429.64	447.17	486.21	565.28
421 440	432.09				
441 460	450.20	456.98			
461 480	468.71				
481 500	486.46	495.35	542.00		
501 520	515.63				
521 540	532.81	546.57			
541 560	551.69				
561 580	571.07	581.78	598.03	603.74	
581 600	590.59				
601 620	610.10	617.67			
621 640	629.42				
641 660	648.87	653.53	655.46		
661 680	668.37				
681 700	688.05	688.05			
Multiplying factor — 0.9804					

Y scale for C—illuminant 400 to 700 mμ					
λ	Y_1	Y_2	Y_3	Y_4	Y_5
400 420	414.52	431.66	463.06	526.57	556.95
421 440	433.32				
441 460	452.05	466.05			
461 480	471.90				
481 500	491.72	504.97	530.95		
501 520	511.55				
521 540	531.06	541.82			
541 560	550.61				
561 580	568.27	577.35	588.62	592.16	
581 600	589.82				
601 620	609.64	616.40			
621 640	629.21				
641 660	648.79	653.34	655.21		
661 680	668.35				
681 700	688.03	688.03			
Multiplying factor — 1.0000					

Z scale for C—illuminant 400 to 700 mμ					
λ	Z_1	Z_2	Z_3	Z_4	Z_5
400 420	414.43	429.76	449.12	454.99	455.06
421 440	432.16				
441 460	450.42	458.30			
461 480	469.29				
481 500	488.54	492.94	495.51		
501 520	508.24				
521 540	528.47	532.17			
541 560	547.88				
561 580	568.88	575.39	579.68	579.78	
581 600	589.22				
601 620	608.20	611.82			
621 640	627.66				
641 648	643.83	643.83	643.83		
Multiplying factor — 1.1810					

photometric field. Since the primaries of the ICI standard colorimetric coordinate system are imaginary, such a tristimulus colorimeter cannot be made directly. It must be calibrated by measurements of four known stimuli, and then may yield tristimulus values, X,Y,Z, by a transformation the reverse of that indicated in eq. 3. Since, except by accident, the color matches set up in a tristimulus colorimeter designed to cover any substantial part of the color totality with a single set of primaries exhibit serious metamerism, the field has to be relatively small, such as to subtend about two degrees at the observer's eye. Even with this small field, the fixation point has to be kept near the center of the field in order to prevent the observer from becoming confused by the appearance of the parafoveal mismatch (see fig. 1, circles). This small angular size of field severely limits the precision of setting compared to what is possible by direct comparison of large specimens in daylight. Furthermore, the metamerism also prevents one normal observer from getting the same reading as another except by accident. To mitigate the effects of these individual observer differences, it is customary to adjust the scales of the instrument separately for each observer so that for some standard stimulus (such as illuminant B or C), the tristimulus values will agree with those for the standard observer. Even with this correction, if a reasonable approximation to the standard values of X,Y,Z, is to be assured, either the readings of a group of five or ten observers must be averaged, or a color standard yielding a spectral composition similar to that of the unknown specimen must be used.

It will be seen that tristimulus colorimeters give but poor information regarding the unknown specimen. Their application to product-control problems is negligible. But because of the ease of calibration and simplicity of the theory, they are very useful visual research tools. Tristimulus colorimeters have been built and described by Allen [4], Guild [40], Wright [139], Verbeek [134], Donaldson [25], and Newhall [95]. The Wright instruments have spectrum primaries; the other five have primaries formed by combining a light source with glass filters. To the Guild and Wright instruments we owe our accurate information regarding the properties of the normal visual system, which have been expressed in terms of the standard observer. Only the Donaldson [25] colorimeter in the six-primary form is applicable to product control.

The foregoing instruments make up the comparison-field mixture by optical combination of light beams from different sources either so that a sum of the separate effects is obtained by continuous exposure, or so that a time-weighted average is obtained by successive stimulation at a frequency too high to produce flicker. A very simple and widely used tristimulus colorimeter based on this latter principle is obtained by taking four disks that have been cut along a radius, by interlocking them so as to expose a sector of each, and by causing them to rotate on the spindle of a motor so rapidly that neither the separate sectors nor even flicker is perceived. Such an arrangement for combining colors by rotary mixture is called a Maxwell disk. The four disks provide the necessary three degrees of freedom in the adjustment for a match, and if the tristimulus values of the component disks be known (X_1,Y_1,Z_1; X_2,Y_2,Z_2; X_3,Y_3,Z_3; X_4,Y_4,Z_4) the tristimulus values (X,Y,Z) of the mixture can be computed from the fractions, f_1,f_2,f_3,f_4, of the total area occupied by the respective sectors:

$$\left.\begin{aligned} X&=f_1X_1+f_2X_2+f_3X_3+f_4X_4, \\ Y&=f_1Y_1+f_2Y_2+f_3Y_3+f_4Y_4, \\ Z&=f_1Z_1+f_2Z_2+f_3Z_3+f_4Z_4 \end{aligned}\right\}, \qquad (4)$$

where, of course, $f_1+f_2+f_3+f_4=1$.

If the disks are chosen anew for each kind of unknown color to be measured so as to be all fairly similar in color to the unknown, the spectral composition of the mixture color is usually sufficiently nonmetameric that no restriction to the central 2° of the retina is required. Furthermore, any two normal observers with some experience at making the adjustment can check each other closely. The chief drawback of this simple arrangement for product-control work in color is the time lost in adjustment of the sector-disk areas. The motor must be turned off, brought to a stop, the disks loosened and readjusted, the motor turned on and allowed to resume speed several times to obtain a final setting of reasonably good precision.

Nickerson has described a disk colorimeter [99] that avoids the difficulties of the elementary Maxwell disk. Light reflected from the unknown specimen fills one half of a photometric field, and that from a stationary sector disk fills the other. By having the observer look at the sector disk through a rapidly rotating glass wedge, each sector is presented to view in sufficiently quick succession that no flicker is produced; and at the same time the sectors, since they are stationary may be continuously adjusted until a color match is obtained. A further advantage is obtained by extending the rotary scanning to the unknown specimen. In this way the average color of a notably nonuniform specimen such as that made up of coarse salt crystals may be obtained. The disk colorimeter is extensively used by the U. S. Department of Agriculture [102] for the color-grading of food products, and is well adapted to product-control colorimetry of many kinds. Frequent checks of the working standards of color are required for accurate work lest fading or soiling of them render calibration by formula 4 inapplicable.

(b) Dominant Wavelength and Purity

Another form of identifying a combination of lights to specify a color, alternate to the tristimulus form, is to determine the luminance (photometric brightness) of one spot of light of fixed spectral composition (such as average daylight) and the luminance of a spot of light of continuously variable spectral composition separately identified (as by wavelength in the spectrum). In this way the requisite three degrees of freedom in adjustment to a color match are supplied. This form of identification leads naturally into a specification in which the luminance (or luminous transmittance, or luminous reflectance) of the unknown is given and the chromaticity is specified by two variables in polar coordinates. One of these variables is an angle, the other a radius, and both can be computed from the chromaticity coordinates of the fixed spot of light, the variable spot of light, and the unknown, these coordinates serving to locate the respective positions in the Maxwell triangle. If the fixed light is nearly achromatic, the angle often correlates well with the hue of the color perception, and the radius fairly well with its saturation.

The most fundamental way to specify the direction on the Maxwell triangle from the point representing the fixed light to the point representing the unknown light is by wavelength of the part of the spectrum required to make the match. If the unknown color can be matched by adding some part of the spectrum to the fixed light, it is said to have a spectral color, and the required wavelength is called its *dominant wavelength*. But if a color match is produced for the fixed light by adding some part of the spectrum to the unknown color, the unknown is said to be nonspectral, and the required wavelength is called the *complementary wavelength*. Either dominant wavelength or complementary wavelength may be obtained for the standard observer by drawing on the standard Maxwell triangle a straight line through the point representing the fixed light and that representing the unknown color, and then reading the wavelength corresponding to the point at which this line extended intersects the locus of spectrum colors. If the unknown color is plotted between the fixed light and the spectrum, the intersection gives the dominant wavelength; but if the fixed light is represented by a point intermediate to the unknown and the intersection of the straight line with the spectrum locus, the intersection indicates the complementary wavelength. Tables of slopes (or reciprocals of slopes) of lines connecting points representing various fixed lights (illuminants A, B and C, and the "equal energy" point) with various parts of the spectrum locus on the (x,y)-diagram have been published by Judd [56] as an aid in finding dominant wavelength from given chromaticity coordinates, x,y.

The degree of approach of the unknown color to the spectrum color is commonly indicated by the ratio of the amount of the spectrum color to the total amount of the two-part combination; this ratio is called purity, and if the amounts are specified in luminance units, the ratio is called *colorimetric purity*.

Colorimetric purity for nonspectral colors has been defined in two ways [53, 55, 131], both awkward and little used in actual practice. By far the most common convention is to express the amounts in units of the excitation sum, $X+Y+Z$; the resulting ratio is called *excitation purity* and corresponds simply to distance ratios on the Maxwell triangle of the standard colorimetric coordinate system. It is also conventional to define excitation purity for nonspectral colors by assigning to colors that plot on the purple border a value of excitation purity equal to unity. Excitation purity, p_e, can therefore be defined simply in terms of the chromaticity coordinates, x_b, y_b, of the border (be it spectrum locus or purple boundary) at the intersection of the straight line connecting the point, x,y, representing the unknown color with the point, x_w, y_w, representing the fixed light, thus:

$$\left.\begin{aligned} p_e &= (x-x_w)/(x_b-x_w), \\ \text{or} \quad &= (y-y_w)/(y_b-y_w), \end{aligned}\right\} \qquad (5)$$

whichever form is determinable with the least rejection error.

Formulas for colorimetric purity and for converting colorimetric purity of spectral colors to excitation purity and the reverse have been derived by Judd [56], Hardy [44], and MacAdam [78, 108].

$$\left.\begin{aligned} p_c &= (y_b/y)(x-x_w)/(x_b-x_w), \\ \text{or} \quad &= (y_b/y)(y-y_w)/(y_b-y_w), \end{aligned}\right\} \qquad (6)$$

whichever is determinable with the least rejection error.

$$p_e = \frac{y_w p_c}{y_b + (y_w - y_b) p_c}. \qquad (7)$$

$$p_c = \frac{y_b p_e}{y_w + (y_b - y_w) p_e}. \qquad (8)$$

Similar formulas may be derived for nonspectral colors, depending upon which definition of colorimetric purity is followed. MacAdam [78, 108] gives tables facilitating these conversions.

Figure 7 indicates how dominant wavelength and excitation purity of the two standards would be found from their chromaticity coordinates, x,y, relative to illuminant C taken as the fixed light. Table 9 gives the dominant (or complementary) wavelength found as in figure 7 by the intersections of the straight lines with the spectrum locus. Table 9 also gives the excitation purities found by dividing the distance from the fixed point (illuminant C) to the

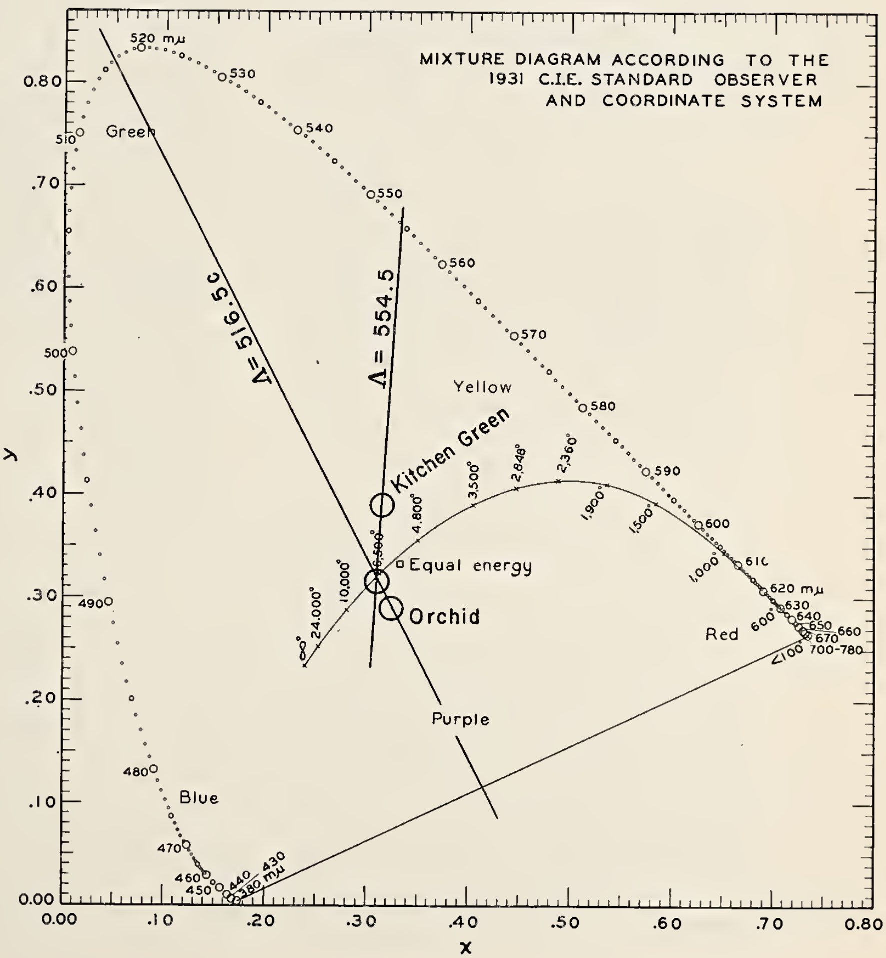


FIGURE 7. *Graphical determination of the dominant (or complementary) wavelength and excitation purity of the Kitchen Green and Orchid standards illuminated by standard source C.*

The wavelength is found by drawing a straight line between the point representing the chromaticity of the specimen and the point representing the chromaticity of the illuminant and reading the wavelength of the part of the spectrum locus intersected by an extension of this straight line. Excitation purity is the ratio of the distance between the specimen point and the illuminant point to the distance between the illuminant point and the border of the diagram (either spectrum locus or purple border).

specimen point by the total distance from the fixed point to the boundary (spectrum locus plus straight line connecting its extremes), or by eq. 5. Large-scale charts for reading dominant wavelength and purity relative to illuminant C are provided in the Hardy Handbook of Colorimetry [44].

Table 9. *Dominant wavelengths and excitation purities of the Kitchen Green and Orchid master standards*

Color standard	Chromaticity coordinates (from table 5)		Dominant wavelength	Excitation purity
	x	y		
			mμ	*Percent*
Kitchen Green..	0.315	0.389	554.5	21.1
Orchid	.324	.289	516.5 C	13.4

Apparatus for the direct measurement of dominant wavelength and colorimetric purity has been designed by Nutting [105] and by Priest [115]. The degree of metamerism ordinarily obtained with such apparatus leaves it open to the same objections as have prevented tristimulus colorimeters with single sets of primaries from being useful for production control. There is a further disadvantage in the direct measurement of colorimetric purity that the luminance of the spectrum component has to be determined relative to the luminance of the mixture by separate photometry. Since there is usually a large chromatic difference between these two fields, simple equality-of-brightness settings are not reliable, and an auxiliary flicker photometer, as in Priest's apparatus [115], must be used. This method has been found to exaggerate individual-observer differences; oftentimes observers will differ only slightly in the mixtures of spectrum light and fixed light that they find to be equivalent to an unknown color, but they will disagree importantly in their photometry of the components.

(c) Guild Vector Method

By using two known fixed lights and one variable (spectrum) light, Guild [41] was able to determine chromaticity coordinates, $x,y,$ without resort to auxiliary photometric devices. A match is first set up for the unknown by a mixture of one fixed light with some portion of the spectrum, the amounts of both components being adjustable by the observer. A straight line is drawn on the chromaticity diagram connecting the points representing the fixed light and the spectrum light. Then the process is repeated with a second fixed light, and a second straight line is drawn on the chromaticity diagram. The intersection of the two straight lines gives the chromaticity coordinates of the unknown. This method was used by Guild chiefly to supplement the tristimulus colorimeter by measuring unknowns whose chromaticities plot outside of the triangle formed by plotting the points representing the primaries of the tristimulus colorimeter. Such unknowns may be evaluated on the tristimulus colorimeter only by the inconvenient expedient of mixing one (or two) of the primaries with the unknown. Since the matches obtained by the vector method are in general highly metameric, a group of observers is required to obtain a representative result.

III. Small-Difference Colorimetry

In the fundamental colorimetry of lights and objects, a single standard is used for each class of specimen. Opaque surfaces are referred either to the ideal perfect diffuser or to a sufficiently thick layer of magnesium oxide deposited from the smoke of magnesium turnings or ribbon burning in air [113]. Transparent objects, such as gelatine films, crystal, or glass plates, are referred to the same thickness of air; transparent solutions, to the same thickness of distilled water or solvent. Self-luminous objects, such as fluorescent lamps, cathode-ray tubes, television screens, and incandescent lamps, are measured relative to one of the standard illuminants, usually illuminant A [20], or they are measured relative to illuminant A viewed through a filter of known spectral transmittance. The colors of specimens closely resembling the respective standards can be evaluated by direct colorimetry quite precisely and accurately; those differing radically in spectral composition, only with relative uncertainty. That is, near-white specimens, nearly clear glass plates, and incandescent lamps nearly equivalent to illuminant A present the simplest colorimetric problem; highly selective absorbers and emitters, like the rare-earth glasses and gaseous discharge tubes, present difficult measurement problems. In general, the greater the deviation in spectral composition between the unknown specimen and the standard, the greater the uncertainty of the result obtained by a visual or a photoelectric colorimeter.

Modern photoelectric spectrophotometers have greatly extended the application of both visual and photoelectric colorimetry. They have supplied a rapid, practical way to calibrate working standards of color. If a fairly large group of specimens is at hand to be measured, say 20 or more, all of similar spectral composition, the most satisfactory way to measure them in the present state of colorimetric science is to evaluate one or two of them carefully by means of the spectrophotometer to serve as working standards, then obtain the color specifications of the remainder by visual or photoelectric determination of the difference between specimen and standard.

Another important application of small-difference colorimetry is the solution of such problems as the color changes produced by various deteriorative agents, or those produced by admixture of a colorant, or toner. In these problems, the color standard is the original unexposed or untoned specimen; and the chief concern is to evaluate character and amount of the color differences, the absolute evaluation of the colors being of secondary interest, or required only approximately.

1. Martens Photometer

One of the most useful visual devices for determining relative luminance of surfaces of nearly identical spectral selectivity is the Martens photometer [82, 106]. Figure 8 shows the Martens photometer combined with a diffuse illuminator to form the Priest-Lange reflectometer [116]. This reflectometer is intended for the measurement of luminous reflectance of opaque specimens relative to reflecting standards of similar spectral reflectance. It is also adaptable to the measurement of luminous transmittance of transparent plates relative to transmitting standards similar in spectral transmittance to the unknown. Finally the Martens photometer removed from the mounting, shown in figure 8, may be used for the determination of the luminance of an unknown self-luminous surface relative to a spectrally similar standard of known luminance. The superior usefulness of the Martens photometer arises from the convenience of the adjustment for equality of brightness betwen the two halves of the photometer field and from the fact that in a well-made instrument the dividing line between the half-fields is exceptionally narrow so that it is often invisible when a brightness match has been set. The addition of the simple lens in front of the photometer permits the object viewed to be thrown into focus and extends the use of the device to the reflectometry of small specimens.

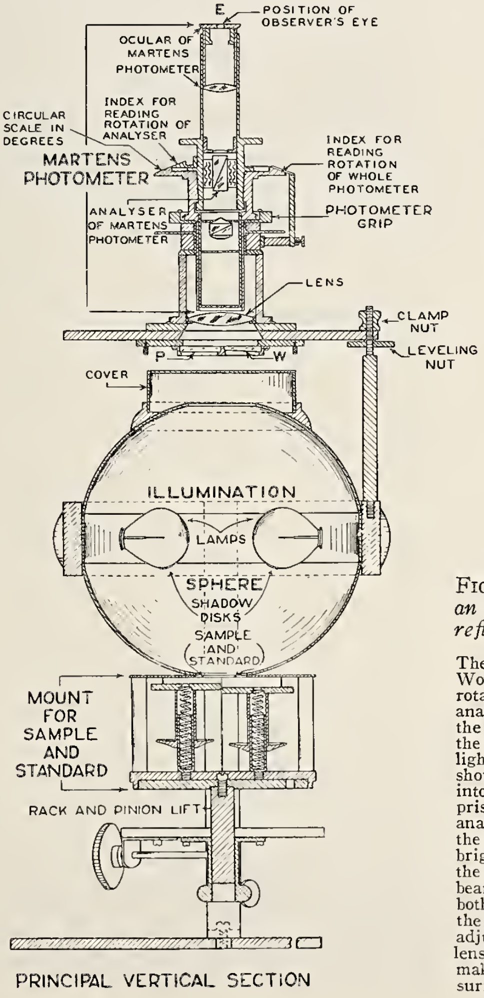
The Martens photometer is customarily used in one of two ways. In the interchange method the specimen is compared directly with the standard and the angle Θ_1, required for a brightness match is read from the instrument scale set so that the zero of the scale corresponds to maximum obscuration of the standard side of the field. Then specimen and standard are interchanged, and angle Θ_2 read. The luminance ratio, B/B_s, is found as

$$B/B_s = (\tan \Theta_1)(\cot \Theta_2). \tag{9}$$

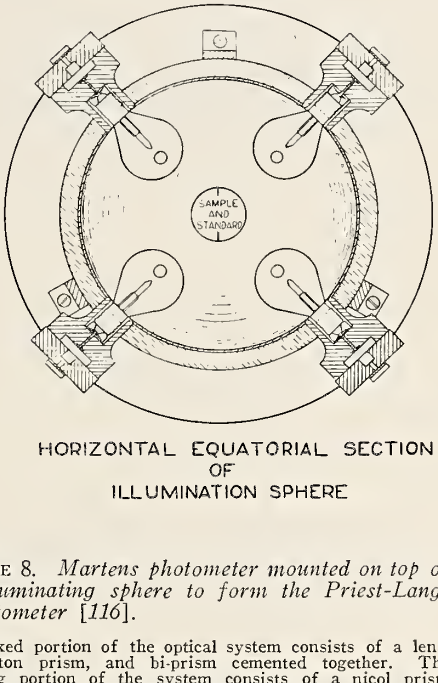
In the substitution method the standard is first compared with a comparison surface and the angle Θ_1 required for a brightness match is read. Then the specimen is substituted for the standard and the angle Θ_2 found. The luminance ratio is found as

$$B/B_s = (\tan^2\Theta_2)/(\tan^2\Theta_1). \tag{10}$$

If specimen and standard are of considerably different chromaticities, the setting for equality-of-brightness is not adequately reproducible. If the specimen and standard are of considerably different



PRINCIPAL VERTICAL SECTION



HORIZONTAL EQUATORIAL SECTION
OF
ILLUMINATION SPHERE

FIGURE 8. *Martens photometer mounted on top of an illuminating sphere to form the Priest-Lange reflectometer [116].*

The fixed portion of the optical system consists of a lens, Wollaston prism, and bi-prism cemented together. The rotating portion of the system consists of a nicol prism analyser and an eye-piece permitting the observer to bring the edge of the bi-prism into sharp focus. This edge forms the dividing line of the photometric field. The two beams of light to be compared enter two small circular openings (not shown) at the bottom of the photometer. Each beam is split into two mutually perpendicular beams by the Wollaston prism, only one of which falls on the aperture of the analyser. Adjustment of the position of the analyser causes the two halves of the photometric field to become equally bright. The angle at which this condition exists indicates the relative luminance of the two surfaces from which the beams originated. The lens beneath the photometer serves both to bring light from the sample and standard positions of the reflectometer into the field of the photometer and also by adjustment of the distance between the photometer and the lens to bring the sample plane into focus if desired, thus making possible the measurement of reflectance of small surfaces.

spectral compositions but of nearly the same chromaticities, a projection of the macular pigment of the observer appears on the field of the instrument (usually 6° diameter) and renders a reproducible setting very difficult. These limitations apply to all visual photometers.

2. Chromaticity-difference colorimeter

The determination of chromaticity coordinates, x,y, by comparison of the unknown specimen with a working standard of similar spectral reflectance can be carried out visually with high precision by means of a differential colorimeter described by Judd [61]. The adjustment of the chromaticity of the comparison field to match the standard field is by two double wedges, one of greenish and the other of yellowish glass. Since the light from the comparison field must pass through both the yellow and the green wedge, some of the radiant energy being subtracted by each, it is sometimes called a subtractive colorimeter; see figure 9, which gives a schematic diagram. The standard and comparison fields are brought into juxtaposition by means of a Lummer-Brodhun cube having a double-trapezoid pattern subtending 9x13° at the observer's eye. The adjustment to near equality of brightness to facilitate detection of chromaticity differences is by movement of the projection lamp that illuminates both standard and comparison surfaces. The comparison is made either by incandescent-lamp light, or by artificial daylight obtained by insertion of Corning Aklo and Daylite glass found to yield a satisfactory heat-free duplication of illuminant C for this purpose.

A substitution method is usually employed with this colorimeter, a match first being set up between the standard and comparison surfaces by adjustment of the wedges. Then the unknown specimen is substituted for the standard and the wedges readjusted to restore the match. Auxiliary yellow and green filters may be inserted either in the beam reflected from the specimen or in the comparison beam, if the specimen differs sufficiently from the standard to require it. The differences in wedge settings can be calibrated in terms of differences in the chromaticity coordinates, x,y, from the known spectral transmittances of the wedges by computation as in table 3(c), provided the spectral directional reflectances of the comparison surface are known approximately. The calibrations for a comparison surface of magnesium oxide are given in figure 10. Note that, over the range illustrated, there is a nearly linear relation between differences in wedge settings, ΔY, ΔG, and the corresponding differences in chromaticity coordinates Δx, Δy. This calibration has been carried out for about 100 widely differing comparison surfaces. It has been found that the calibration is chiefly a function of the chromaticity coordinates, x,y, of the comparison surface; so calibrations for comparison surfaces intermediate in chromaticity to those already calibrated may usually be found satisfactorily by interpolation.

Because of the large patterned field of high luminance and the convenience of the brightness adjustment, this instrument takes full advantage of the ability of the observer to detect small chromaticity

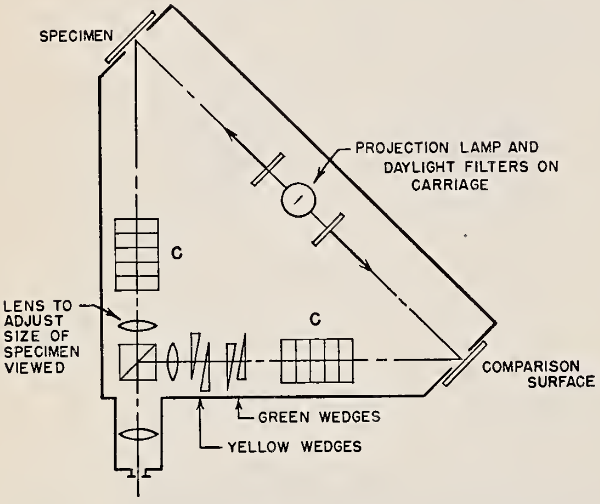


FIGURE 9. *Schematic diagram of the chromaticity-difference colorimeter..*

Both specimen and comparison surface are illuminated perpendicularly by artificial daylight conveniently variable by movement of the lamp carriage. The specimen is viewed through the Lummer-Brodhun cube and two lenses, the comparison surface is viewed by total reflection in the cube through two double wedges, one of yellowish, one of greenish glass. Auxiliary filters of the same kinds of glass may be inserted either in the specimen beam or the comparison beam. Adjustment of the viewing lens permits the observer to focus approximately on the dividing plane of the Lummer-Brodhun cube, the back half of which has etched on it a double-trapezoid pattern giving the well-known contrast field for enhancement of color differences. The power of the second lens determines the size of specimen area from which light is visible. *C* represents the containers for the auxiliary yellow and green filters.

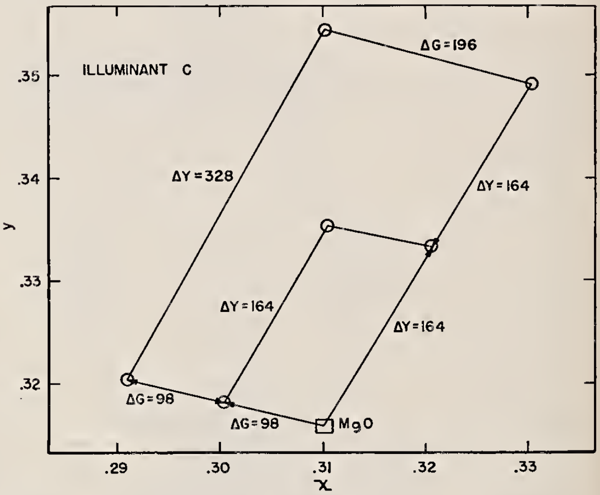


FIGURE 10. *Calibration of the yellow and green wedges of the chromaticity-difference colorimeter for near whites and near grays in terms of the (x,y)-chromaticity diagram.*

The illuminant is standard source *C*. Note that in this region of the diagram an increase of 100 units of the yellow wedge corresponds to an increase in x-coordinate by about 0.0060, and an increase in y-coordinate by about 0.0100. An increase of 100 units of the green wedge corresponds to a decrease in the x-coordinate by about 0.0100, and an increase in the y-coordinate by about 0.0020. These calibration coefficients depend somewhat on the spectral composition of the energy reflected from the comparison surface to produce any given chromaticity, but they are determined approximately by the chromaticity itself.

differences. If specimen and comparison surfaces are similar in spectral composition, the settings for match may be repeated generally within 0.001 in chromaticity coordinates, x,y. For chromaticity differences not much larger than this, it is advantageous to use the interchange method instead of the substitution method. In this method the unknown specimen is compared first directly with the standard, then the two are interchanged and a second setting made. No comparison surface is required, and the uncertainty of the determination is one-half that obtained by the more rapid substitution method. If the surfaces be not spectrally similar, or if they be viewed through many yellow and green filters, a faint projection of the macular pigment may appear in the photometric field. Such an appearance lowers the precision of setting somewhat and is also correlated with failure of the result to agree with indirect colorimetry by way of the spectrophotometer [118]. This failure may often be by as much as 0.003 in x or y.

The chromaticity-difference colorimeter has the further disadvantage of requiring a skilled operator. The calibrations, too, are time-consuming; and the instrument is not applicable to some spectrally selective specimens because nearly homogeneous energy is too little changed in spectral composition by passage through the wedges.

This colorimeter has been used in setting up a color standard for ruby mica [63], for inspection of working standards, transparent and opaque, for conformity to a master standard (such as the Kitchen and Bathroom Accessory Colors), and for general colorimetry by difference both for fluorescent and nonfluorescent specimens [118].

3. Photoelectric tristimulus colorimeters

If three photocells could be adjusted, as by glass filters, so that their responses were proportional throughout the visible spectrum to some linear combination (as in formula 3) of the standard ICI distribution curves (see figure 2), then they could be used to test whether any two light beams have the same color according to eq. 2, and could be made to yield direct measurements of tristimulus values, X,Y,Z [35, 43]. There have been several good attempts to build a photoelectric tristimulus colorimeter suitable for general use [15, 49, 50, 111, 138], but with incomplete success [51, 133]. Figure 11 shows the degree to which filters designed by Hunter [50] to duplicate the ICI standard observer and simultaneously to adjust a projection lamp to ICI illuminant C have succeeded. If A, G, and B represent the settings obtained for a specimen relative to those for a standard magnesium-oxide surface with the amber, green, and blue filters, respectively, approximate tristimulus values, X,Y,Z, may be found as:

$$\left.\begin{aligned} X &\cong 0.80A+0.18B, \\ Y &\cong 1.00G, \\ Z &\cong 1.18B \end{aligned}\right\} \qquad (11)$$

Approximate chromaticity coordinates, x,y, may then be found in the usual way ($x=X/(X+Y+Z)$, $y=$

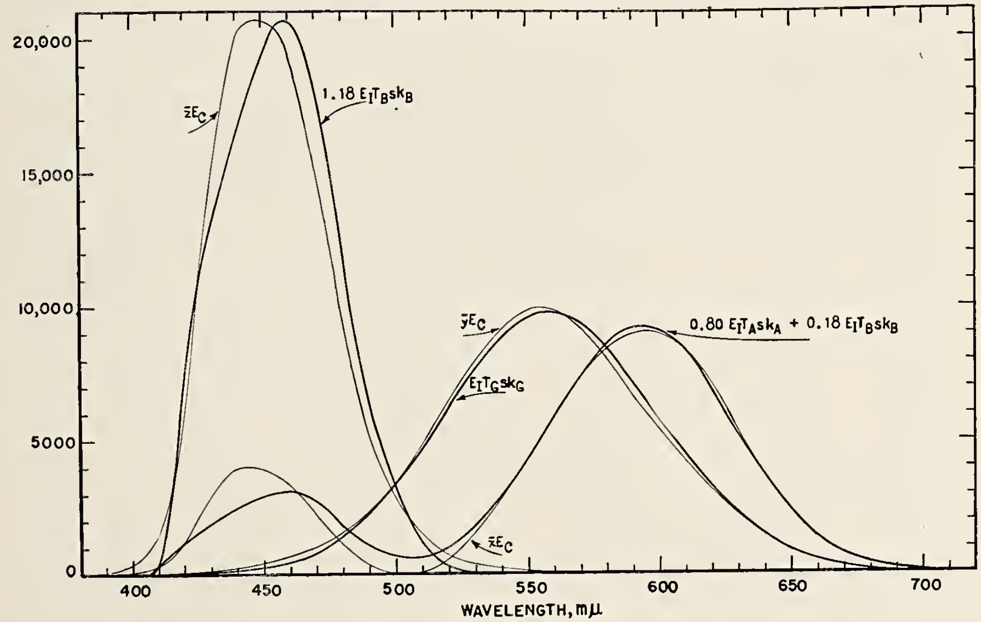


FIGURE 11. *The tristimulus values of the spectrum of standard source C according to the ICI standard observer compared to the spectral sensitivities of the source-filter-photocell combinations designed by Hunter [50] for photoelectric tristimulus colorimetry with three filters.*

Note that the duplication of the yE_c-function is fairly good, but the short-wave portions of the other two functions are more poorly duplicated. Departures of this amount limit the usefulness of these filters to the measurement of color differences between nonmetameric or slightly metameric pairs.

$Y/(X+Y+Z)$). Figure 12 shows the discrepancies that this use of the Hunter filters introduce. These discrepancies are roughly proportional to the distance from the point representing the magnesium-oxide standard, and are frequently larger than 0.02 in x or y, that is, more than 10 times a reasonable chromaticity tolerance for most colorimetric work.

For the comparison of specimens of similar spectral composition (such as near-white surfaces), however, the degree of duplication of the ICI standard observer shown in figure 11 is sufficient. Figure 13 refers to the small rectangle near the center of figure 12 and indicates the degree of agreement between measurements by means of the chromaticity-difference colorimeter and photoelectric tristimulus colorimetry by means of the multipurpose reflectometer. Figure 13 indicates that the discrepancies are less than 0.001 in x or y for comparison of near-white surfaces with magnesium oxide. Figure 14 illustrates an application of photoelectric tristimulus colorimetry to near-whites in which the instrument is used to test white casein paints for one of the requirements of Federal Specification TT-P-23a. Another application of considerable importance is evaluation of the yellowness index:

$$\text{Yellowness index} = (A-B)/G. \tag{12}$$

This index yields a scale of yellowness increasing from zero for the standard magnesium-oxide surface or any equally nonselective surface to positive values for yellowish surfaces and negative values for bluish

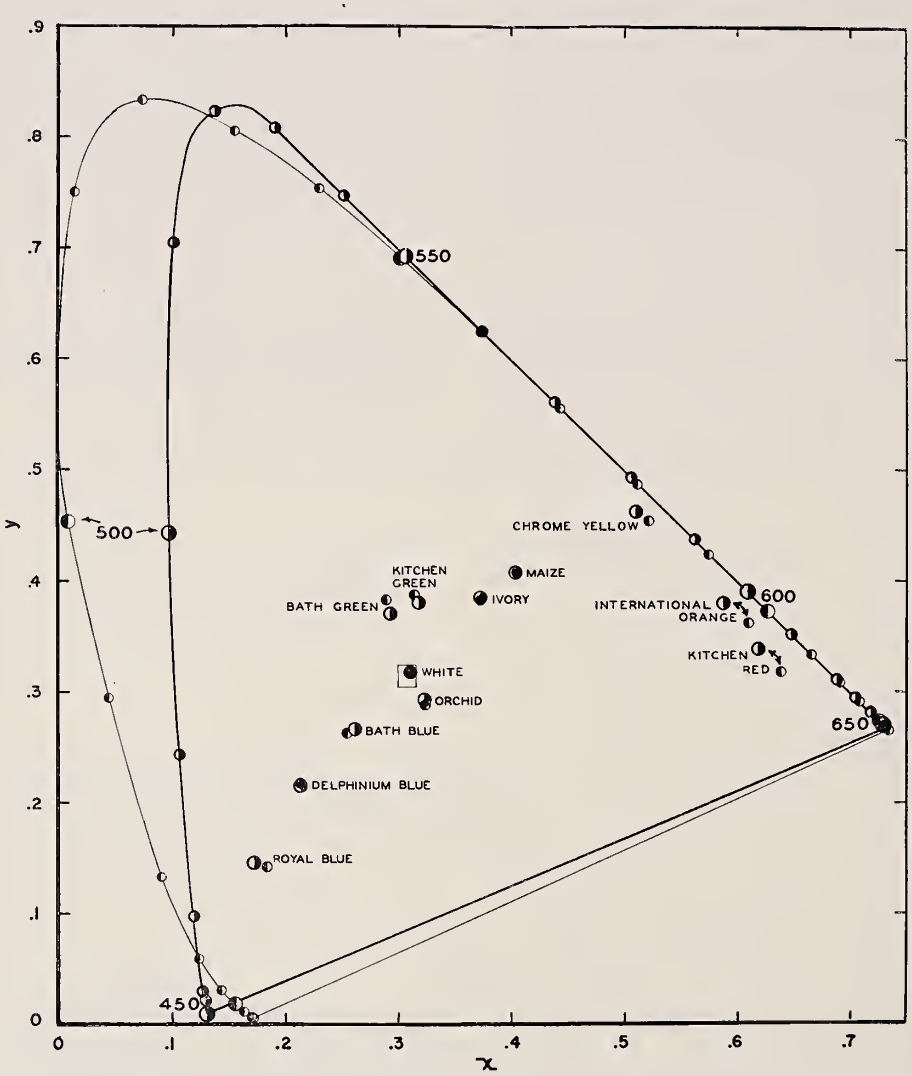


FIGURE 12. *Discrepancies in chromaticity introduced by the use of the source-filter-photocell combinations shown in figure 11 (Hunter [150]).*

The white (MgO) point is common to both plots, and in general the discrepancies increase with distance from this point. Nearly all of these discrepancies far exceed the least differences tolerable in colorimetry of industrial products.

◐ From spectrophotometer and ICI observer.

◑ From multipurpose reflectometer.

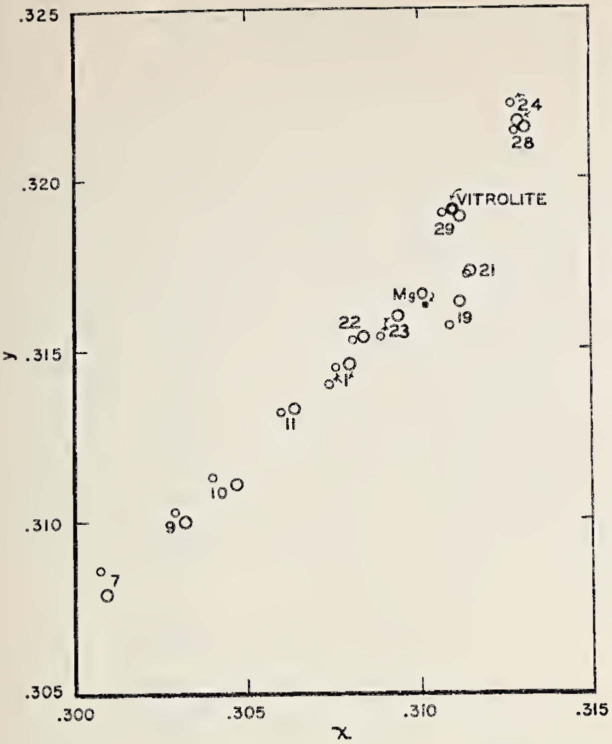


FIGURE 13. *Comparison of photoelectric tristimulus colorimetry (fig. 11) of a series of near-white specimens of vitreous enamel with visual colorimetry of them by means of the chromaticity-difference colorimeter (fig. 9).*

The point representing the Vitrolite plaque used as standard is common to both plots. Note that for these near whites the discrepancies are smaller than 0.0010 in x- or y-coordinate. Errors of this size are nearly always tolerable in the colorimetry of industrial products.
o From subtractive colorimeter.
O From multipurpose reflectometer.

surfaces. Photoelectric tristimulus colorimetry is listed as Method 425.2 in Federal Specification TT-P-141a dealing with test methods for paint, varnish, lacquer, and related materials.

If the limitations of photoelectric tristimulus colorimetry are appreciated, the method is most useful in product-control colorimetry of nonfluorescent specimens by difference from a working standard. A set of 11 gray standards, ranging from black to white, has been developed at this Bureau for this purpose. These standards apply to the colorimetry of nearly nonselective specimens under the standard angular conditions of 45° illumination and normal viewing, or the equivalent. The precision of the method is comparable, though perhaps not quite equal, to the best that can be done by eye. No unusual qualifications or extended special training is required for an operator, and compared to visual colorimetry, or to indirect colorimetry by the spectrophotometer, the results are obtained very rapidly.

Photoelectric reflectometers and transmissometers of proved practical value are commercially available, as are also the filters separately.[2] They have been used for the comparison of paint vehicles, solutions, petroleum products, glasses, painted panels, papers, textiles, ceramics, plastics, and a wide variety of fabricated specimens.

In general, satisfactory agreement can be expected in using such photoelectric tristimulus colorimeters, for the determination of small chromaticity differences between nonmetameric pairs. And even for measurement of fairly sizeable nonmetameric chromaticity differences, such as analyzed spectrophotometrically in the upper portion of figure 15 (BPB 8/2 vs. MgO, BG 7/4 vs. BG 6/4), and small chromaticity differences with a moderate metameric component, such as shown in the lower left portion of figure 15 (Y1 vs. Y2), the discrepancy is in the neighborhood of 0.002 in x or y, which is negligible for many purposes. However, for highly metameric pairs, such as shown in the lower right portion of figure 15, the discrepancy may be expected to be in the neighborhood of 0.02 in x or y, just as it is for large chromaticity differences.

4. Perceptibility of Color Differences

In the interpretation of the importance of chromaticity differences based upon separation of the points representing the two chromaticities in the (x,y)-diagram, a warning is necessary. This diagram is considerably expanded in the green portion relative to the other portions, much as the Mercator projection of the earth's surface is expanded near the poles. Thus, two points separated by a given distance in the green portion of this diagram correspond

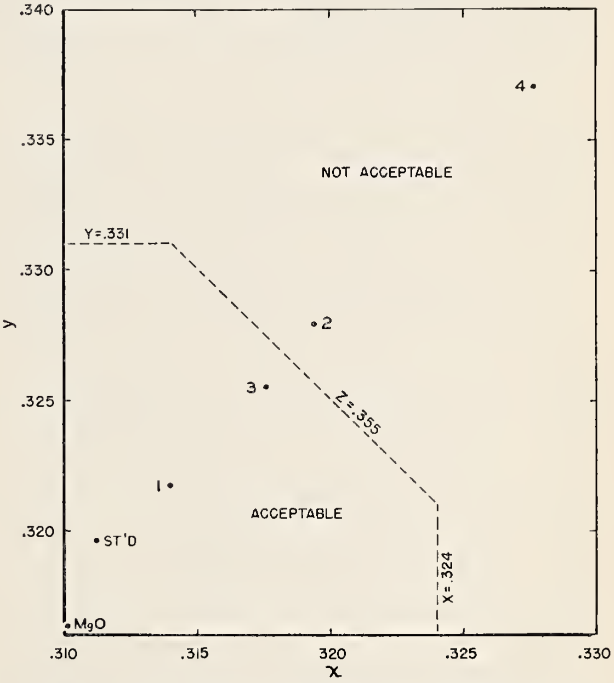


FIGURE 14. *Application of photoelectric tristimulus colorimetry to test white casein paints for conformity to the requirements of Federal Specification TT-P-23a (Hunter [50]).*

Samples 1 and 3 meet the requirements; samples 2 and 4 do not.

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[2] Gardner Laboratories, 4723 Elm St., Bethesda, Md. Photovolt Corp., 95 Madison Ave., New York, N. Y.

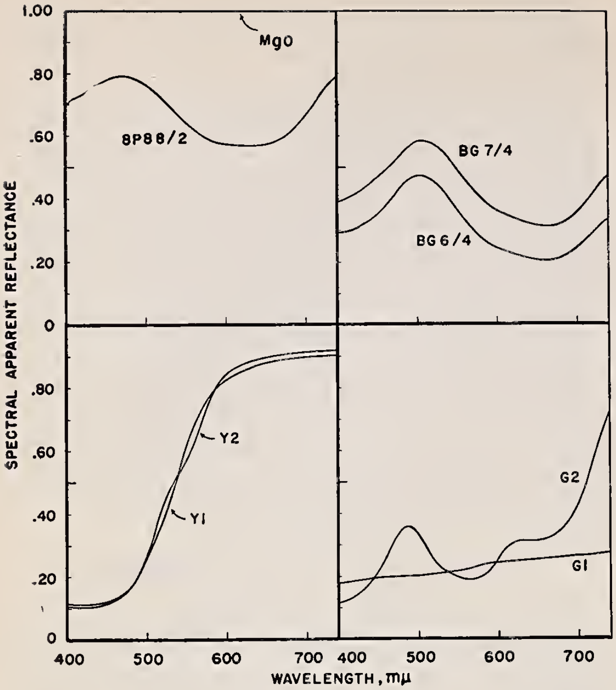

Figure 15. *Illustration of degrees of metamerism (Hunter [50]).*

The upper-right diagram shows the variation of reflectance with wavelength for a pair of Munsell papers (5BG 7/4 and 5BG 6/4) yielding a considerable, though nonmetameric, color difference. The upper left diagram (N 10/, 10B 8/2) also illustrates a color difference of considerable size but slight metamerism. The two lower diagrams refer to small color differences; that on the left, slightly metameric; that on the right, strongly. The presently available filters permit useful application of photoelectric tristimulus colorimetry to moderately large nonmetameric color differences, but not to very large color differences nor to strongly metameric differences.

to chromaticities that are much harder to distinguish under ordinary viewing conditions than two chromaticities separated by the same amount in other portions of the diagram. Furthermore, the bluish purple portion of the diagram is correspondingly compressed. Figure 3 indicates approximately this distortion; for example, the area designated "green" is more than 10 times as large as that designated "purplish blue" but actually refers, within perhaps a factor of 2, to the same number of distinguishable chromaticities. A more detailed and accurate indication of this distortion is given by the system of ellipses shown on figure 16. Under moderately good observing conditions, the distances from the central point of each ellipse to any point on its boundary correspond approximately to one hundred times the chromaticity difference just perceptible with certainty. These ellipses were drawn from a review of the literature in 1936 [57, 60], and subsequent extensive work published by Wright in 1941 [143, 144] and by MacAdam in 1942 [79, 80], has corroborated the essential correctness of the indicated chromaticity spacing. Figure 16 not only indicates approximately the extent to which the green portion of the diagram is expanded, and the bluish purple compressed, but also indicates from the considerable departures of the ellipses from circles that, in general, the perceptive importance of a distance on the (x,y)-diagram depends not only on the position of the central point but also notably on the direction of the deviation from it.

If the properties of an observer be expressed with reference to other sets of primaries than the ICI standard by transformation in accord with formula 3, the chromaticity spacing yielded by the resulting Maxwell triangles may be made to vary widely. There have been several attempts so to choose primaries that the chromaticity spacing corresponds to its perceptibility, that is, yields uniform chromaticness scales [17, 50, 57, 77, 108, 124]. There have also been a number of fairly successful attempts to produce diagrams to serve the same purposes by other means [3, 28, 88, 89, 122, 125]. Figure 17 shows the uniform-chromaticness-scale (UCS) triangle according to Judd [57]. If the chromaticity coordinates of a color are x,y, in the standard ICI system, the color would have chromaticity coordinates, r,g, in this uniform-chromaticness-scale triangle in accord with eq. 13:

$$\left.\begin{aligned} r&=\frac{2.7760x+2.1543y-0.1192}{-1.0000x+6.3553y+1.5405}\\ g&=\frac{-2.9446x+5.0323y+0.8283}{-1.0000x+6.3553y+1.5405} \end{aligned}\right\}. \tag{13}$$

On this diagram the ellipses of figure 16 would be equal tangent circles.

Figure 18 shows the rectangular uniform-chromaticness-scale (RUCS) diagram set up by Breckenridge and Schaub [17] by modification of the UCS diagram so as to place the point representing the equal-energy stimulus at the origin of the coordinate system. Since this stimulus under most observing conditions is perceived to have a nearly achromatic or neutral color, the coordinate axes of the RUCS system may be said to divide the chromaticity diagram approximately into four ranges of hues. The first quadrant corresponds to the hue range yellow to green; second quadrant, green to bluish purple; third quadrant, bluish purple to red, and fourth quadrant; red to yellow. The connection of the RUCS coordinates, x'',y'', to the ICI standard chromaticity coordinates, x,y, is given in two steps by way of intermediate coordinates, x',y'.

$$\left.\begin{aligned} x'&=\frac{0.82303x+0.82303y-0.82303,}{1.00000x-7.05336y-1.64023}\\ y'&=\frac{3.69700x-5.07713y-1.36896.}{1.00000x-7.05336y-1.64023} \end{aligned}\right\}. \tag{14}$$

$$\left.\begin{aligned} x''&=0.0750-x';\\ y''&=y'-0.5000. \end{aligned}\right\}. \tag{15}$$

Tables have been prepared for carrying out this transformation by interpolation with uncertainties less than one in the third decimal [94a]. The RUCS diagram, because of its use of rectangular coordinates, and because the significance of the coordinates in terms of color perception under ordinary condi-

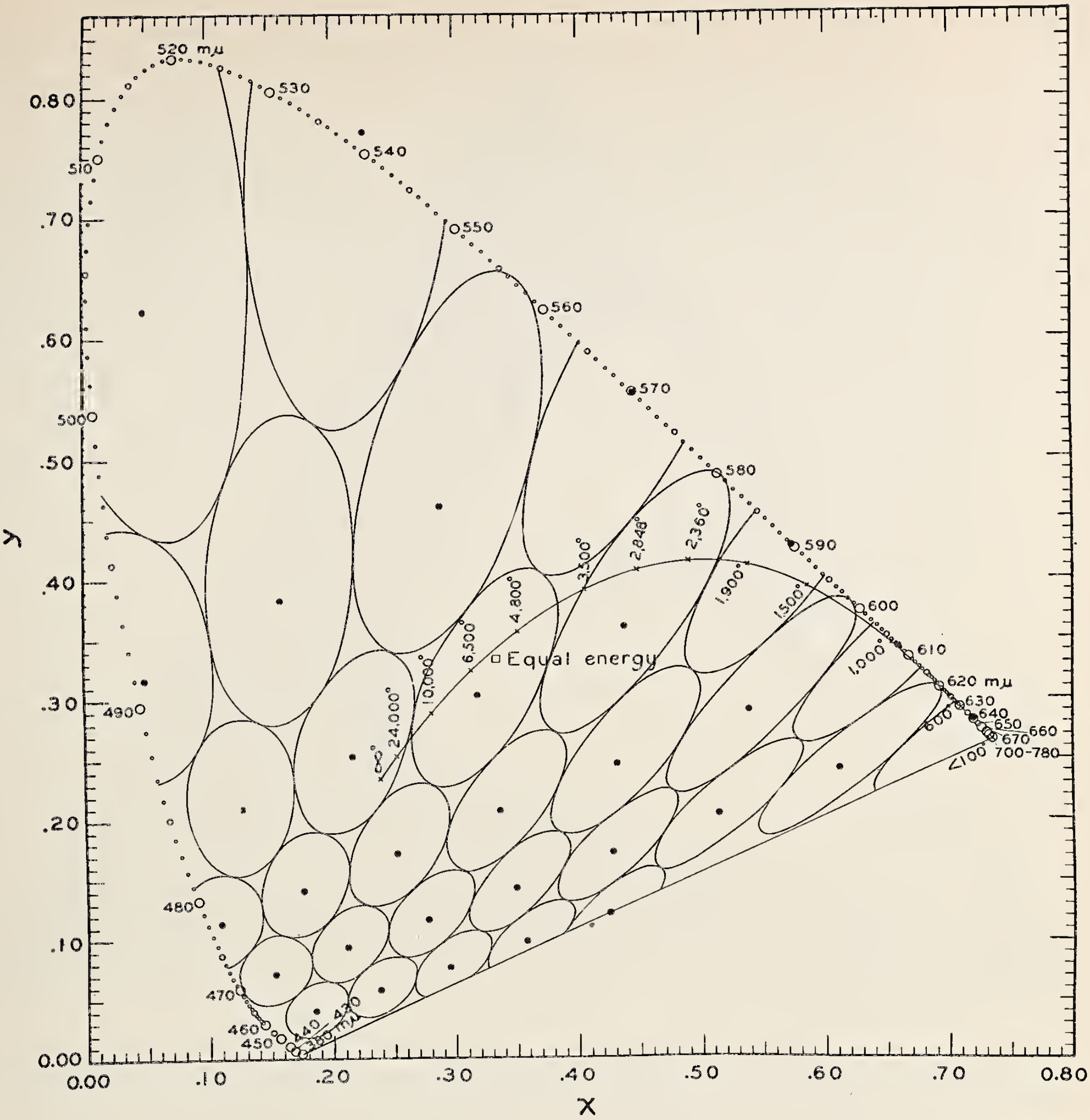

FIGURE 16. *Approximate perceptibility of chromaticity differences on the (x,y)-chromaticity diagram.*

The distances from points on the boundary of each ellipse to the indicated point within it all correspond approximately to one hundred times the chromaticity difference just perceptible with certainty under moderately good observing conditions.

tions is easy to grasp, has advantages for engineering applications. It was put forward as a suggestion for possible future adoption by international agreement.

Figure 19 shows the "alpha-beta" diagram set up by Hunter [50, 124] by modification of the UCS diagram to adapt it for convenient use with the multipurpose reflectometer and to take into account chromaticity-spacing data collected since 1936 [72, 79, 143]. These new data essentially corroborate the UCS diagram but indicate that the perceptibility of chromaticity differences for purple colors in the sense corresponding to changes parallel to the purple boundary (straight line connecting the extremes of the spectrum locus) is less relative to other differences by about a factor of two than is shown by the UCS diagram. The connection of the "alpha-beta" coordinates, α,β, with the ICI standard chromaticity coordinates, x,y, is given by formula:

$$\left.\begin{aligned}\alpha&=\frac{2.4266x-1.3631y-0.3214}{1.0000x+2.2633y+1.1054}\\ \beta&=\frac{0.5710x+1.2447y-0.5708}{1.0000x+2.2633y+1.1054}\end{aligned}\right\}. \qquad (16)$$

To a sufficient approximation for small-difference colorimetry these chromaticity coordinates, α, β, can be found from the settings A, G, B, obtained by using the Hunter amber, green, and blue filters, respectively, of the multipurpose reflectometer [49. 50], by the simple relation:

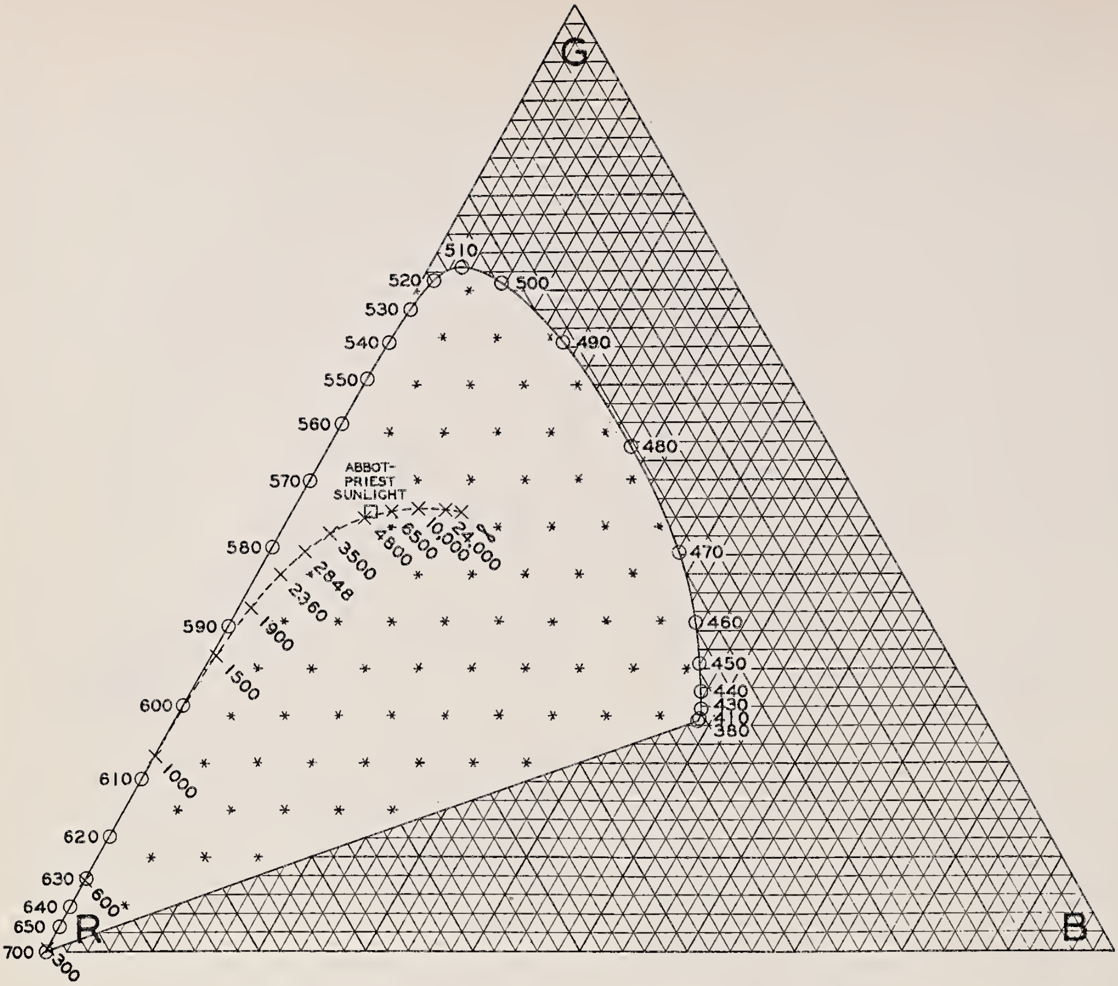

FIGURE 17. *The uniform-chromaticness-scale (UCS) diagram.*

Points on the spectrum locus are indicated in millimicrons. Points representing the chromaticities of light from a small hole in an inclosure are indicated by the temperature of the walls in degrees Kelvin. The ellipses of figure 16 correspond to a family of equal tangent circles on this diagram. The least perceptible chromaticity difference under the most favorable observing conditions corresponds on this diagram to a change in *r*-, *g*-, or *b*-coordinate by about 0.0004.

$$\left.\begin{aligned}\alpha &\cong \frac{A-G}{A+2G+B}\\ \beta &\cong \frac{0.4(G-B)}{A+2G+B}\end{aligned}\right\}. \qquad (17)$$

The least chromaticity difference perceptible with certainty under the most favorable observing conditions (luminance 50 or more millilamberts, field size 8° or more, convenient luminance adjustment about the condition of match) corresponds on each of the three uniform-chromaticness-scale diagrams just discussed to about 0.0004.

(a) NBS Unit of Color Difference

An empirical formula for estimating the perceptibility of color differences has been developed [61, 62] on the assumption that surface colors can be represented by a three-dimensional figure in which length of a straight line is a close measure of the perceptibility of the difference between the colors represented by its extremes. In such a surface-color space the constant-lightness planes (representing colors of surfaces of constant luminous reflectance) would be perpendicular to the lines representing changes in lightness at constant chromaticness. The basic assumption thus permits a calculation of the color difference, ΔE, in terms of the chromaticness and lightness components, $\Delta E_c, \Delta E_l$, by Euclidean geometry, thus:

$$\Delta E = \left[\overline{\Delta E}_c^2 + \overline{\Delta E}_l^2\right]^{1/2}. \qquad (18)$$

The size of the lightness component, ΔE_1, is evaluated by changes in the square-root of the luminous directional reflectance, thus:

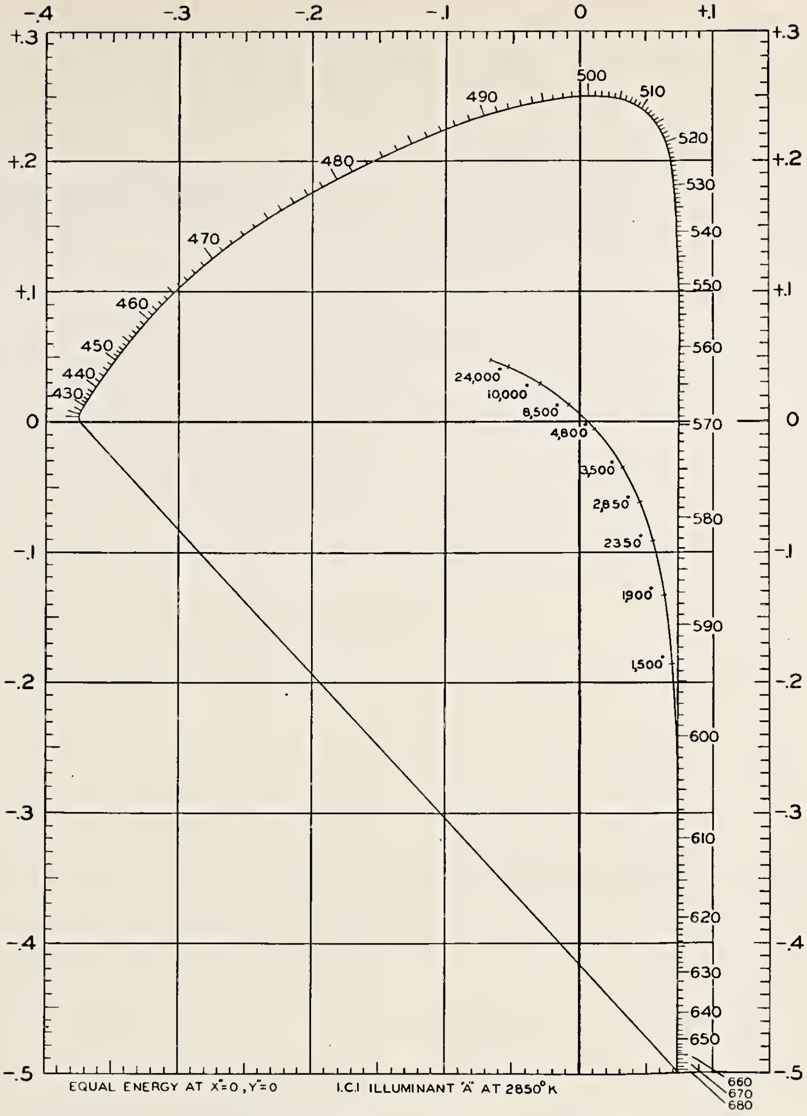

FIGURE 18. *The rectangular uniform-chromaticness-scale (RUCS) diagram derived by Breckenridge and Schaub [17].*

The chromaticity spacing is very similar to that of the UCS diagram (fig. 17).

$$\Delta E_l = k_1 \Delta(Y^{1/2}). \qquad (19)$$

The size of the chromaticness component, ΔE_c, is approximately proportional to distance ΔS on a uniform-chromaticness-scale diagram, except that under usual observing conditions two very dark colors cannot be distinguished as easily as two light colors having the same chromaticity difference. The component is evaluated therefore as:

$$\Delta E_c = k_2 Y^{1/4} \Delta S, \qquad (20)$$

where Y is an average of the luminous directional reflectances of the two surfaces, and ΔS is the distance on a uniform-chromaticness-scale diagram between the points representing the two chromaticities.

The constants k_1 and k_2 must conform to the relative importance of the lightness and chromaticness components and also serve to establish the unit of color difference. Preliminary studies have shown that the perceptibility of differences in luminous directional reflectance depends importantly on the conspicuousness of the line dividing the two fields, but the perceptibility of chromaticity differences is comparatively independent of this factor. The constant, k_1, is therefore known as the "proximity factor". Table 10 gives preliminary values for the proximity factor, k_1, adjusted so that color differences of less than one unit (known as the "NBS unit") are perceptually unimportant in most commercial transactions. The constant, k_2, depends importantly on the angular subtense of the specimens to be compared. For ΔS evaluated as

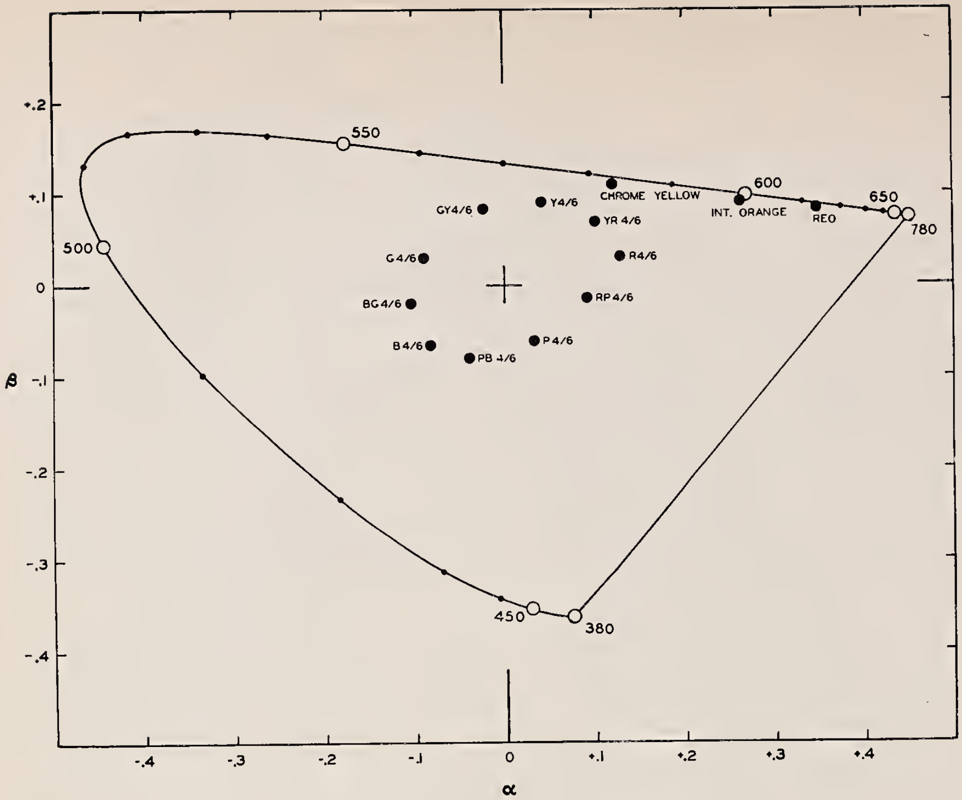

FIGURE 19. *The alpha-beta diagram derived by Hunter* [50].

The chromaticity spacing of this diagram is in somewhat better correspondence with perceptibility of chromaticness differences between extended surface colors than those of figures 17 and 18. The dots indicate ten members of the Munsell hue scale at value 4 and chroma 6.

$$\left[(\overline{\Delta\alpha^2}+\overline{\Delta\beta^2})\right]^{1/2},$$

and for test panels of the usual size (about 3 by 5 in.), this constant, k_2, should be taken at about 700. The expression for color difference in NBS units may be written:

$$\Delta E=\left\{\left[7\,Y^{1/4}\left(\overline{\Delta\alpha^2}+\overline{\Delta\beta^2}\right)^{1/2}.10^2\right]^2+\left[k_1\Delta(Y^{1/2})\right]^2\right\}^{1/2}. \quad (21)$$

From formula 21 it may be seen that the NBS unit is considerably larger than the color difference just perceptible with certainty. By setting $\Delta E=1$ for light surface colors ($Y^{1/4}$ approaching 1.00) of the same luminous reflectance ($\Delta Y^{1/2}=0$), and assuming $\Delta\beta=0$, we find: $\Delta\alpha=0.0037$, which is about 10 times the least chromaticity difference perceptible with certainty under the most favorable conditions. By means of the chromaticity-difference colorimeter [61] and the multipurpose reflectometer [49], color differences can be evaluated in NBS units with an uncertainty of about 0.2 or 0.3 of a unit, even though based on single settings on the two specimens. It is customary practice, however, to make a repeat determination chiefly as a check on clerical errors.

TABLE 10. *Values of the proximity factor, k_1, for various observing conditions*

Observing conditions	k_1
Specimens separated by a very narrow or nonexistent dividing line	120
Specimens separated by a contrasting, but narrow dividing line	90
Specimens separated by a broad patterned area contrasting with the areas being compared	40
Specimens evaluated for whiteness without visual reference to any standard	20
Convenient value for average condition, specimens separated by a narrow line	100

The NBS unit has been used in the inspection of working standards of color (such as the Kitchen and Bathroom Accessory Colors) for conformity to a master standard [93, 94], in the purchase specification and test of various types of paint, and in studies of the performance of accelerated weathering machines [26]. In general, this unit evaluated by way of the multipurpose reflectometer is in agreement with the results of visual judgment. Isolated disagreements have been traced (a) to failure of the source-filter-photocell combination to duplicate the standard observer (see fig. 11), (b) to failure of the "alpha-beta" diagram to yield perfectly uniform chromaticness scales, (c) to a tendency of some inspectors to pay preponderant attention to hue differences and neglect lightness and saturation differences, and (d) to the employment by inspectors of angular conditions of illumination and view different from the standard conditions (45°-normal) used in the measurements themselves.

The angular conditions most often used in inspection correspond approximately with the standard conditions (45°-normal) insofar as the light from the window is incident on the specimens, placed horizontal, at angles roughly centering on 45° and insofar as the direction of view is roughly normal. They differ importantly, however, insofar as most inspecting rooms supply a small quantity of diffused illumination in addition to the roughly directional component from the window. If the specimens to be compared are glossy, the color of each is composed of a mixture of the light diffusely reflected in accord with the measurement combined with light reflected specularly from the ceiling of the inspection room. It has been found particularly for dark specimens that better correlation is obtained with visual estimates of color differences made under these usual inspecting conditions if a correction is made for the specularly reflected light. A convenient approximate correction is obtained if the right-hand member of eq 21 is multiplied by a "gloss factor", f_g, computed as:

$$f_g = Y/(Y+0.025), \qquad (22)$$

where Y refers to luminous directional reflectance either of the standard or of the sample, whichever yields the larger value.

(b) Indices of Hue, Lightness, and Saturation

Inspection of manufactured materials and goods and the rejection of some of them as off-color are very frequently carried out on a basis somewhat different from the strict perceptibility of the color difference between specimen and standard. The inspector will say of an accepted specimen that it is off-color considerably more than another that he rejects, but that this kind of color difference is not objectionable. Departures of the color toward middle gray are usually viewed with comparative disfavor. Such colors are called muddy or dirty. Thus a slight departure from a near-white standard toward gray is more likely to result in a rejection than two or three times such a departure in the opposite direction. On the other hand, the lightest of a group of blacks is usually penalized as gray or "rusty" and the darkest thought of as a "good, clean black". If the standard color be highly chromatic, specimens perceived to have colors of slightly less saturation are rejected, but those even considerably more saturated may be accepted. Some kinds of goods (such as raw cotton) are judged almost entirely on the basis of lightness alone, with saturation and hue secondary. Others, such as dyes, whose price varies roughly with the reciprocal of the amount required to produce a standard color, are judged primarily on saturation. Still others, such as goods finished by application of a colorant (pigment or dye), are judged primarily on conformity to the hue of the color standard. Lightness differences can be overlooked because they appear to the customer to be ascribable to local differences in the illumination. Saturation differences are not too serious, because if the customer looks steadily at the object he will see the saturation of the color decline anyway. Furthermore, rather large changes in lightness and saturation of one element in a design can be tolerated from the standpoint of color harmony, but hue changes are likely to be most injurious. A hue difference also gives the customer the impression that the wrong pigment or dye has been chosen, that some mistake has been made. For these reasons many inspectors are more critical of hue variations than they are of departures from standard lightness and saturation.

In an effort to provide the basis for numerical specification of color tolerances based upon these considerations, indices of hue, lightness, and saturation and their differences in NBS units have been developed [50]. The lightness index, L, is given by eq 23:

$$L = Y^{1/2}. \qquad (23)$$

The hue index, H, is given by eq 24:

$$H = \tan^{-1}\beta/\alpha. \qquad (24)$$

The saturation index, S, is given by eq 25:

$$S = [(\alpha^2+\beta^2)Y^{1/2}]^{1/2}. \qquad (25)$$

The simplicity of the hue and saturation indices in terms of the "alpha-beta" coordinates arises from the definition (eq 16) which places illuminant C and all blacks, whites, and grays illuminated by it at the origin of the coordinates.

The lightness component of a color difference, ΔE_1, is given in NBS units by eq 19 and table 10. The saturation component is given in NBS units by eq 26:

$$\Delta E_s = 700 Y^{1/4} \Delta(\alpha^2+\beta^2)^{1/2}. \qquad (26)$$

The hue component of the color difference, ΔE_h, is given in NBS units by eq 27:

$$\Delta E_h = 12.2 Y^{1/4} (\alpha^2+\beta^2)^{1/2} \Delta H, \qquad (27)$$

where ΔH is found in degrees from eq 24. In eq 26 and 27, as in eq 20, the quantities that do not refer to differences are found as the average for the two specimens whose difference it is desired to evaluate.

An example of the application of eq 27 is afforded by purchases of bituminous emulsion paint by the War Department under U. S. Army Tentative Specification No. T-1213, which stated that some lightness and saturation difference could be tolerated on weathering but no hue differences. Paint was ruled not to comply with this specification, if, from a standard exposure test, it changed in hue by more than 1.0 NBS unit, or if the total color change (eq 21) exceeded 4.0 NBS units.

(c) Index of Whiteness

The whiteness of a specimen may be taken to be proportional to the degree to which it is perceived

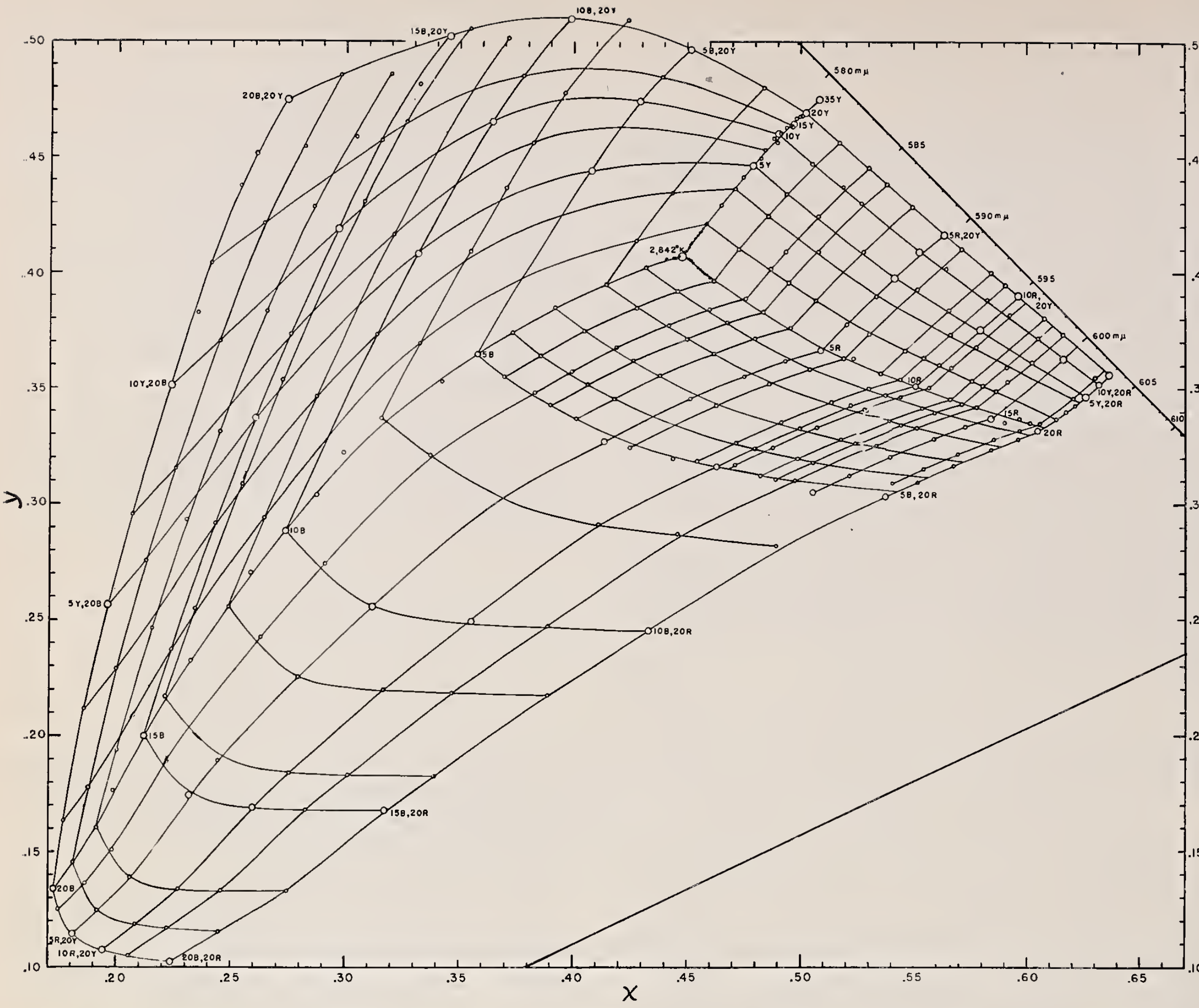

FIGURE 20. *Chromaticities of two-part combinations of Lovibond red, yellow, and blue glasses illuminated by standard source A (gas-filled incandescent lamp).*

Parts of the purple boundary and the spectrum locus (578 to 611 mμ) of the standard (x,y)-chromaticity diagram are also shown. The lines are intended to conform as closely as possible to smooth scales defining the Lovibond color system; they do not in general pass exactly through the centers of the circles representing the actual combinations of Lovibond glasses measured.

to approach in appearance an ideal white standard. If, as is usual, the magnesium-oxide reflectance standard [113] is used as the standard white, a convenient index of whiteness, W, may be based on the relation

$$W \equiv 1 - \frac{\Delta E_{\text{MgO}} \text{ to specimen}}{\Delta E_{\text{MgO}} \text{ to black}} . \qquad (27)$$

This relation gives a scale of whiteness from 1.00 for the magnesium-oxide reflectance standard to zero for black, and has been found [58, 59] to correlate well with visual estimates of whiteness of near-white specimens when the color difference, ΔE, is computed from eq 21 with the proximity factor, k_1, equal to 20; see table 9. Since the index is applicable only to near-white specimens, it can be computed from eq 28:

$$W = 1 - \{[30(\alpha^2+\beta^2)^{1/2}]^2 + [(1.00-Y)/2]^2\}^{1/2}. \qquad (28)$$

It has been found that the chief cause for disagreements among visual estimates of whiteness arises from discrepancies from one grader to another as to what shall be the standard white [58, 59]. If some whiteness standard other than magnesium oxide is desired, eq 27 should be rewritten relative to that standard and used to derive a substitute for eq 28.

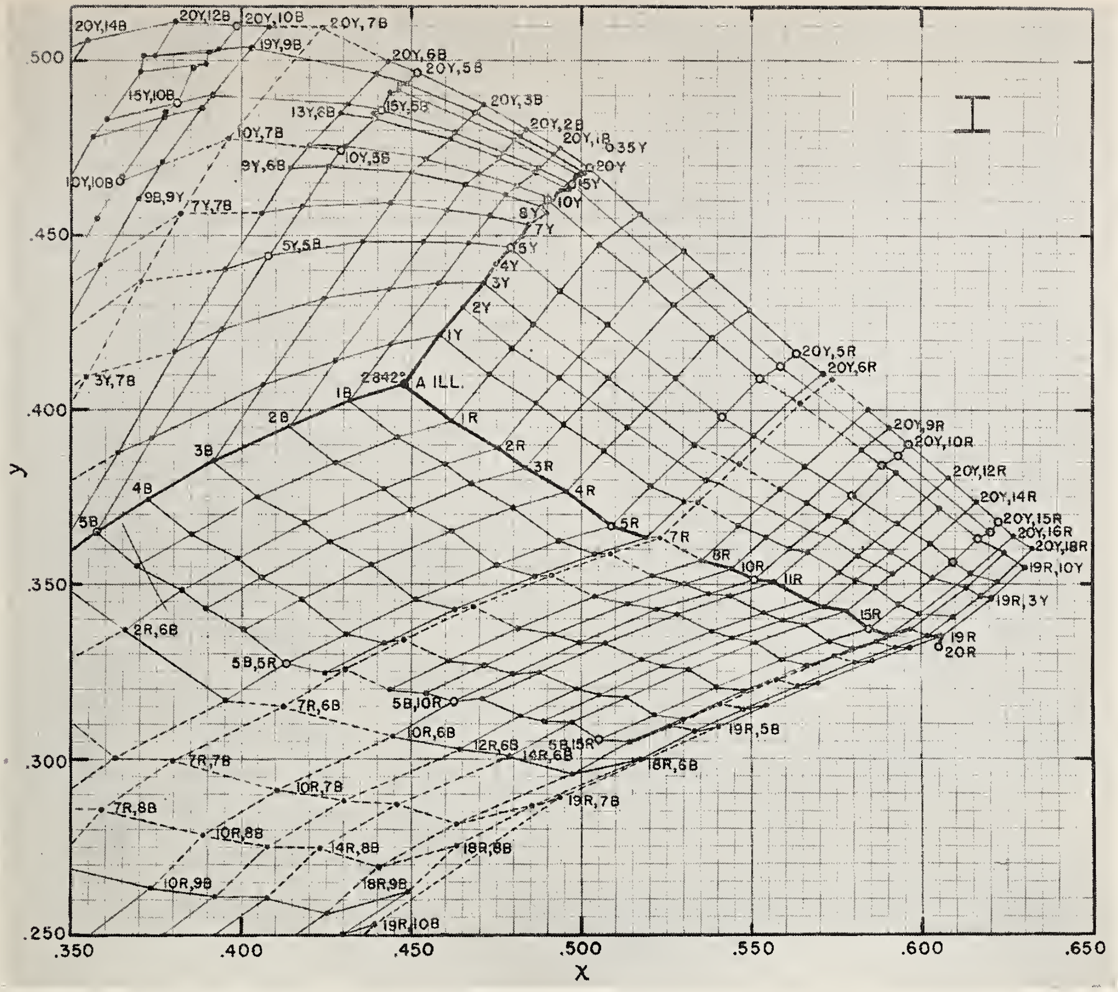

Figure 21. *Chromaticities of two-part combinations of Lovibond red, yellow and blue glasses from the set (BS9940) kept as a reference at the National Bureau of Standards (Haupt and Douglas [46]).*

Combinations judged to be significantly nonrepresentative of the Lovibond system are represented by points joined by dashed lines.

IV. Material Standards of Color

Because of the convenience of material standards of color, they are often used in commerce in preference to specification according to the more fundamental ICI system. Material standards may be carried from place to place and, if the colors are sufficiently closely spaced in the neighborhood of the unknown color, the nearest match may be found by visual comparison. The color specification consists of identifying the particular member of the system yielding a match for the color to be specified. We deal here both with systems of material standards of such scope that a considerable fraction of the colors possible in nonself-luminous objects find in the system a fairly close representation, and with a few special small groups of material standards for particular purposes.

1. Transparent Media

Color systems based upon transparent media take advantage of the fact that it is possible with a fixed illuminant to control the color of the transmitted light over a wide range by introducing varying amounts of three absorbing materials. This is done by permitting the light to pass through two or more elements of the absorbing medium instead of through a single element and is called subtractive combination, or mixture, because the action of each element is to subtract a certain fraction of each part of the spectrum of the incident light. The color specification consists of the number of unit elements required to produce the color match.

(a) Lovibond Glasses

The Lovibond color system consists of three sets of colored glasses, red, yellow, and blue [72, 73, 74], the principal coloring materials being gold, silver, and cobalt, respectively. These coloring materials are confined to a thin layer of glass flashed onto an essentially clear glass support. The unit of the scale defined by each set is arbitrary, but the three units are related by being adjusted so that for observation by daylight subtractive combination of one unit of each of the red, the yellow, and the blue scales results in a filter perceived as neutral or achromatic. Each scale is exemplified by many glasses, each glass being marked with the number of unit glasses to which it is equivalent. If more than one glass of the same coloring material is used in combination, the number of unit glasses to which the combination is equivalent is found simply by adding the numerals marked on each individual glass of the combination. Although the original purpose of the Lovibond color system was to aid in the color control of beer, these glasses are widely used today for other products such as vegetable oils, lubricating oils, and paint vehicles.

A spectrophotometric analysis of the Lovibond color system was made by Gibson and Harris [33], and a scale of the red glasses used in combination with the 35-yellow glass has been constructed by Priest and Gibson [34, 45], having the same unit as the original Lovibond red scale but embodying a closer approach to the principle that the Lovibond numeral should indicate the number of unit glasses to which the single glass bearing the numeral is equivalent. Red glasses are calibrated in terms of the Priest-Gibson scale at the National Bureau of Standards and at the Electrical Testing Laboratories. The latter has also undertaken to adjust red glasses to slightly lower numerals, if desired, by reducing the thickness of the flashed layer slightly [27]. Tintometer, Ltd., makers of the Lovibond glasses, have computed from the published spectrophotometric measurements [33] the chromaticity coordinates, x,y, of all of the colors of the Lovibond system [123] produced by illuminating the glasses with ICI standard illuminant B. Computation of these colors for illuminants A and C has been carried out at the National Bureau of Standards [46]. Figure 20 shows a plot of these results for illuminant A, and figure 21 shows to a larger scale the most used section of this plot.

(b) Arny Solutions

The Arny solutions consist of groups of solutions whose concentrations are adjusted to produce the color match. The required concentrations are the specifications of the color. The most-used group is a triad consisting of half-normal aqueous solutions of cobalt chloride (red), ferric chloride (yellow), and copper sulfate (blue) with 1-percent hydrochloric acid. This group produces all colors except deep blue and deep red; it is supplemented by a triad of ammoniated aqueous solutions of cobalt, chromium, and copper salts, and by a pair consisting of aqueous solutions of potassium permanganate and potassium dichromate [12, 13, 14]. Mellon and Martin [87] have reported the spectral transmittances for a number of solutions for colorimetric standards, including the Arny solutions at 3 or 4 concentrations, each for the spectral range 440 to 700 mμ. By extrapolation of these data it would be possible to find approximately the tristimulus values and chromaticity coordinates, x,y, on the standard coordinate system adopted in 1931 for these solutions just as was done by Mellon [86] for the coordinate system used in America before the international agreement. In this way it is possible to transform color specifications from the standard system into the required concentrations of the Arny solutions, and the reverse. Kelly [67] has selected groups of Arny-solution combinations adapted to color grading because of uniform spacing and variation of one color attribute (hue or saturation) at a time. The Arny solutions are used in the 11th edition of the U. S. Pharmacopoeia as standards for the color of cod-liver oil and in carbonization tests with sulfuric acid for 28 organic compounds.

2. Opaque Media

If the set of material standards are pigmented or dyed surfaces, no automatically convenient notation, such as suggested by additive combination of lights or subtractive combination of absorbing elements, is available. Any systematic aspect of the color specification must be derived from the method of identifying the various members of the set of colored surfaces serving as standards.

(a) Color Dictionaries

From a color dictionary are obtained definitions of color names in terms of material standards. The primary aim is therefore to provide an array of named surface colors adequate for the purpose, and any arrangement or organization of the colors serves only the secondary purpose of assisting the user to find the one that most nearly matches.

The Maerz and Paul Dictionary of Color [81] is the foremost authority on color names. It contains about 7,000 different samples of color printed on semiglossy paper, and there are listed about 4,000 color names that are keyed to one or another of the color samples. These names are drawn from usage in many fields: paint, textile, ceramic, scientific, technical, and artistic. The samples are also identified by plate, column, and row, and because of their large number and fairly uniform color distribution it is usually possible to find among them a sample approaching what is called a "commercial color match" for any given uniform opaque surface. On this account the Maerz and Paul Dictionary finds a considerable application as a collection of color standards quite separate from its primary function of defining color names. There are noticeable color differences between corresponding samples in different copies of the dictionary, but the differences have been held to a satisfactorily small amount by exceptional care in the printing process.

The accepted authority for color names in the textile and allied industries is the Textile Color Card Association of the United States. The TCCA has issued nine editions of a standard color card since 1915, the current edition [129] containing 216 color samples of pure-dye silk; furthermore, the TCCA issues to its members several seasonal cards each year. All colors of these standard and seasonal cards are identified by name and cable number. The standard colors have been measured at the National Bureau of Standards by spectrophotometric and colorimetric procedures; and luminous reflectance, Y, relative to magnesium oxide, and chromaticity coordinates, x,y, for illuminant C have been published [118].

A color dictionary much used for the specification of the colors of flowers, insects, and birds was prepared in 1912 by Ridgway [120]. This outstanding pioneer work contains about 1,000 named color samples of paper painted by hand. Each chart shows columns of colors of the same dominant wavelength progressing from each chromatic color at the middle of the column toward white at the top, and toward black at the bottom; and there are five series of such columns, each one encompassing the entire hue circuit, but at different purities. Many of the names were coined at the time of publication to fill in gaps in popular color nomenclature, and so have not much descriptive value. Each sample is arbitrarily identified by column, row, and series, however; and there is an alphabetical list of the color names giving this identification.

(b) Ostwald Color System

The notation of the Ostwald system is based on the properties of idealized pigment surfaces having spectral reflectance constant at a certain value between two complementary wavelengths and reflectance constant at a certain other value at other parts of the spectrum [31]. The *full colors* are those that have the low value of spectral reflectance equal to zero and the high one equal to 100 percent. The difference between these two reflectances for other idealized pigment surfaces is the *fullcolor content,* the value of the low reflectance is the *white content,* and the difference between the high reflectance and 100 percent is the *black content.*

The complete Ostwald notation consists of a number from 1 to 24 and two letters from *a* to *p*. The number indicates dominant (or complementary) wavelength on an arbitrary but approximately uniform perceptual scale, and is called *Ostwald hue number.* The first letter indicates white content, *a* being a white content of 89.13 percent, which is as near to 100 percent as is practicable for usual pigment-vehicle combinations, and other letters in alphabetical sequence indicating decreasing white content on a logarithmic scale. The second letter indicates black content, *a* being a black content of 10.87 percent, which is as near to zero as is practicable, and other letters in alphabetical sequence indicating increasing black content on a logarithmic scale. The logarithmic scales were thought by Ostwald to insure uniform color scales, but this is true only to a rough approximation. Since the percentage of white (w) content, black (b) content and fullcolor (c) content must necessarily add up to 100, no explicit indication of the latter is required. The notation *aa* thus indicates the practical white of the system ($w=89.13$, $b=10.87$, $c=0.00$). This notation, *aa,* however, is usually written simply as *a*; and notations for grays and black are likewise abbreviated to a single letter. The Ostwald constant-hue triangle has the pigment approximation to white (*aa*) at the top, the pigment approximation to black (*pp*) at the bottom, and at the other angle of the equilateral triangle there is the pigment representation of the fullcolor (*pa*) of that particular Ostwald hue. The colors arranged in slant lines parallel to the top of the triangle have constant black content and are called *isotones.* The colors arranged in slant lines parallel to the bottom of the triangle have constant white content and are called *isotints.* The colors arranged in vertical lines parallel to the black-white series have constant fullcolor content and are called *isochromes.* The isochromes are colors of constant dominant wavelength and purity. Thus the isochromes vary in luminous reflectance but are of constant chromaticity; on this account they are said to form *shadow series.*

The Ostwald ideas have been a considerable aid in thinking about color relationships on the part of those who duplicate colors by mixtures of chromatic pigments with white and black pigments, and they have served as a guide in the selection of combinations of such colors to produce pleasing effects. However, the use of these idealized pigment surfaces as a basis for a system of colorimetry has been hampered by the fact that actual pigment surfaces approximate them rather poorly, and by the fact that not all actual pigment surfaces can be color matched by one of these ideal surfaces. Still, color charts made up more or less in accord with the Ostwald principles have been widely used for color standards and for the selection of harmonizing colors [54, 109, 110, 128]. Of these the Jacobson Color Harmony Manual [54] is preeminent not only because of its technical excellence, but also because Foss [31] has given a clear statement of which of the somewhat contradictory Ostwald principles were followed in its construction, and Granville and Jacobson [38] have made a spectrophotometric study of the color chips, and have published luminous reflectance, Y, and chromaticity coordinates, x,y, for every chip. These chips are therefore valuable for use in colorimetry by difference from a working standard, and the fact that the chip is a member of an orderly arrangement of colors facilitates the selection of a working standard for any particular purpose. The master standards of this collection measured by Granville and Jacobson have been deposited at the National Bureau of Standards.

(c) Munsell Color System

The basis of the Munsell system is description of colors perceived to belong to surfaces in terms of hue, lightness, and saturation. Each such tridimensional description can be represented by a point plotted in a space diagram known as the *surface-color solid;* see figure 22. In the surface-color solid, the central axis represents the grays extending from black at the bottom to white at the top. *Lightness* of a chromatic (nongray) color determines the gray to which it is equivalent on this scale. Lightness is represented in the color solid by distance above the base plane. *Hue* determines whether a color is perceived as red, yellow, green, blue, purple, or some intermediate; it is represented in the color solid by angle about the central axis. *Saturation* indicates the degree of departure of a surface-color perception from the gray of the same lightness; it is represented by perpendicular distance from the central gray axis.

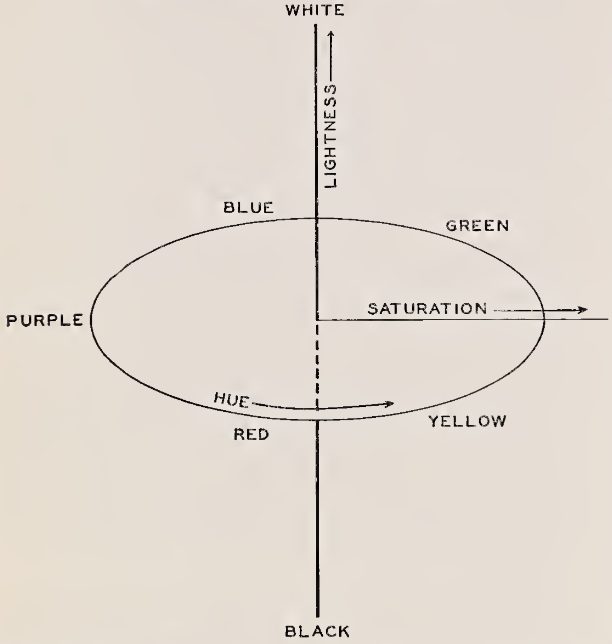

Figure 22. *Dimensions of the psychological surface-color solid.*

In this solid each point represents the color perception of a surface. White is at the top, black at the bottom, with grays intermediate forming the black-white axis. Variations parallel to this axis are lightness variations. Hue is represented by angle about this axis. Saturation is represented by perpendicular distance from the axis.

The Munsell color system specifies a surface color by giving for usual viewing conditions its position on more or less arbitrary hue, lightness, and saturation scales having perceptually nearly uniform steps [92]. The Munsell term corresponding to lightness is Munsell value; that for saturation is Munsell chroma, and that for hue is Munsell hue.

Munsell value is zero for the ideal black surface having luminous reflectance equal to zero, and it is ten for the ideal white diffusing surface having luminous reflectance equal to one. Munsell chroma is expressed in arbitrary units intended to be perceptually of the same size regardless of value and hue. The strongest known nonfluorescent pigment colors have chromas of about 16; neutral grays have zero chroma, as do black and white. Munsell hue is expressed on a scale intended to divide the hue circuit (red, yellow, green, blue, purple, back to red) into 100 perceptually equal steps. According to one convention, the 100 Munsell hues are identified simply by a number from 1 to 100, and on this scale hues that differ by 50 are nearly complementary; that is, combine by rotary mixture to yield near gray. The most common convention, however, is to divide these 100 hues into 10 groups of 10 hues each, and to identify each group by initials indicating the central member of the group, thus: red R, yellow red YR, yellow Y, green yellow GY, green G, blue green BG, blue B, purple blue PB, purple P, and red purple RP. The hues in each group are identified by the numbers 1 to 10. Thus, the most purplish of the red hues is designated as 1R, the most yellowish as 10R, and the central hue as 5R, or often simply as R; see figure 23. The transition points (10R, 10YR, 10Y, and so on) between groups of hues are also sometimes designated by means of the initials of the two adjacent hue groups, thus R-YR=10R, YR-Y=10YR, Y-GY=10Y, and so on (see figures 23 and 24).

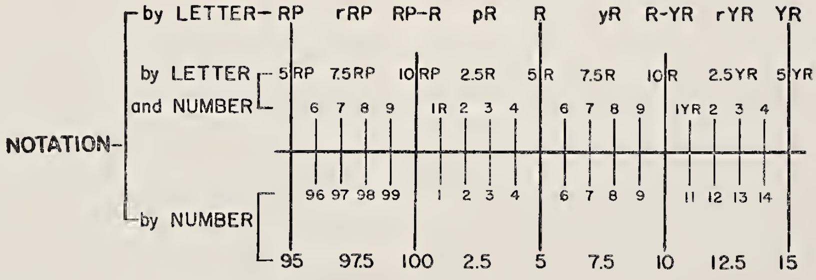

Figure 23. *Hue notation in the Munsell system.*

The relation between the letter, the number, and the combination letter and number designations of Munsell hue are shown here for a portion of the hue circuit. By means of the combination letter-number method, more widely used than the other designations, the third intermediate set of hues are designated: 7.5RP, 2.5R, 7.5R, 2.5YR, and so on.

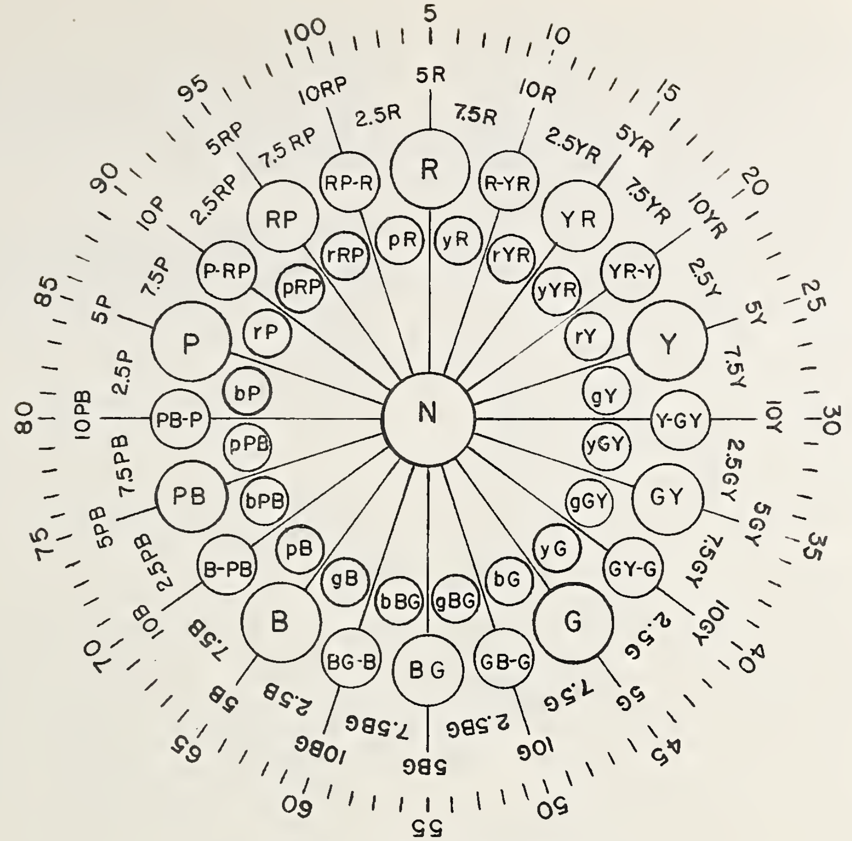

Figure 24. *Designations of Munsell hue.*

The Munsell hue scale is based upon five principal hue names: red, yellow, green, blue, and purple, and it makes use also of the intermediates: yellow-red, green-yellow, blue-green, purple-blue, and red-purple. A second set of intermediates enlarges the hue scale to 20; and a third to 40. The system of letter abbreviations for these 40 hues, shown here, is cumbersome and confusing and has largely been supplanted by a combination number and letter designation (5Y, 10Y, 5GY, 10GY, and so on). A purely numerical system has also been proposed (outer circle) but is little used.

The Munsell notation is commonly written: Hue Value/Chroma, that is, the hue notation, such as 6R, then the value, such as 7, and finally the chroma, such as 4, the latter two being separated by a shilling mark: 6R 7/4. More precise designations are given in tenths of the arbitrary steps of the scales, thus: 6.2R 7.3/4.4. The grays are indicated by the symbol N for neutral followed by the value notation, thus: N 7/ or N 7.3/; the chroma, being zero for neutrals, is not specifically noted.

There have been two representations of the Munsell system published, the original Atlas in 1915 [90], now chiefly a collector's item, and the Munsell Book of Color [91]. This book consists of rectangles of hand-painted paper mounted on charts in a loose-leaf binding. The neutrals form a one-dimensional color scale extending from N 1/ to N 9/. Each chromatic sample, of which there are about 1,000, takes its place on three color scales: a hue scale, a value scale, and a chroma scale; and the spacing of these scales is intended to be perceptually uniform. The pocket edition, adapted for determining Munsell notation of unknown colors by visual comparison, consists of 40 constant-hue charts, so called because all of the samples on each chart have the same Munsell hue, and are chosen with the intention of yielding under usual observing conditions colors all perceived to have the same hue. These samples are arranged in rows and columns, the rows being chroma scales at constant Munsell value, the columns being value scales at constant Munsell chroma. Comparison of an unknown color with these two families of scales gives by interpolation the Munsell value and Munsell chroma of the unknown. Interpolation between adjacent constant-hue charts gives the Munsell hue. Table 11 gives the Munsell book notations of the Kitchen Green (SKC–15) and the Orchid (SBC–20) master standards found in this way. Unknowns not too far outside the range of the Munsell charts may be evaluated with some reliability by extrapolation along the value and chroma scales.

To facilitate the comparison of the unknown color with those of the paper rectangles on the Munsell charts, either the unknown color must be brought into juxtaposition with the rectangle and held in nearly the same plane, or, if the form of the unknown prevents such juxtaposition, two masks of thin cardboard having rectangular openings to fit the paper rectangles should be used. One mask should be placed over the unknown color; the other over one or another of the Munsell colors in succession to obtain the interpolated Munsell book notation. Kelly [64, 65] made effective use of a form of mask with three rectangular openings particularly adapted to comparisons involving powdered chemicals and drugs viewed through a cover glass. It is advantageous to have the color of the mask fairly close to that of the unknown, particularly in Munsell value; that is, if the unknown color is dark, the mask should be of a dark color also. Use of a light mask for a dark color prevents the observer from making as precise a visual estimate as he can make with a mask more nearly a color match for the unknown.

Because of the visual uniformity of the scales, the estimates of Munsell notation for unknown colors within the color range of the charts have a reliability corresponding to the use of a much larger collection of unequally spaced color standards. On this account the pocket edition of the Munsell Book of Color is widely used as a practical color standard for general purposes.

The standard, or library, edition shows the same colors as the pocket edition, but it shows them on constant-value charts in a polar coordinate system, and on constant chroma charts in a rectangular coordinate system, as well as on constant-hue charts. This edition has full explanatory matter in the text and is adapted particularly for teaching color. It is too bulky for convenient practical use in determining the Munsell notation of an unknown color.

The samples of the 1929 Munsell Book of Color have been measured by means of the spectrophotometer twice independently with generally concordant results [36, 68]. In both of these studies luminous reflectance, Y, relative to magnesium oxide and chromaticity coordinates, x,y, of the roughly 400 samples were computed for illuminant C. Glenn and Killian [36] have also published the dominant wavelength and purity for each of the colors; and Kelly, Gibson, and Nickerson [68] have published specifications (X,Y,Z; x,y,z) for three additional illuminants (illuminant A, Macbeth daylight [108], and limit blue sky). Furthermore, they have published a series of (x,y)-chromaticity diagrams showing the position of the Munsell colors for each of the Munsell values from 2/ through 8/. Figure 25 is their diagram for Munsell value 6/. From these diagrams, it is possible to find the Munsell book notation corresponding to any given values of luminous reflectance and chromaticity coordinates (Y,x,y). Table 11 shows such values for the Kitchen Green and Orchid master standards. The samples of the 1942 supplement to the Munsell Book of Color together with many special standards put out by the Munsell Color Company, have been measured spectrophotometrically by Granville, Nickerson, and Foss [37]. These Munsell standards, together with those of the 1929 Munsell Book of Color, number over 1,000, and comprise the largest systematic set of color standards of known luminous reflectance and chromaticity coordinates ever made. These standards are commercially available separately in disk form and on large sheets, and they make practical the general colorimetry of opaque specimens not only by disk mixture (section II, 2 (a)) but also by difference from a standard (section III).

Master standards of the Munsell Book of Color are on file at the National Bureau of Standards, and as repaintings of the various colors are issued this file is kept up to date.

TABLE 11. *Munsell book notations of the Kitchen Green and Orchid master standards determined, (1) by visual interpolation along the color scales of the Munsell Book of Color, and (2) by interpolation from the chromaticity coordinates (x,y) and luminous reflectance (Y) given in Table 5*

The Munsell renotations found from Y,x,y, are also shown

	Munsell book notation		Munsell renotation
Color standard	From visual interpolation	From chromaticity coordinates and luminous reflectance of table 5	
Kitchen Green	9GY 6.2/3.8	9.2GY 6.2/4.0	9.6GY 6.3/4.4
Orchid	2RP 6.1/3.6	2.1RP 6.0/3.6	1.3RP 6.2/4.2

From reverse application of the charts, luminous reflectance and chromaticity coordinates of an unknown color may also be found quickly and with an accuracy sufficient for many purposes by obtaining first the Munsell book notation of the unknown by visual interpolation and then transforming it by reference to (x,y)-interpolation charts (like figure 25) based on the complete set of standards.

The Munsell color standards may also be used, though less conveniently, in the colorimetry of light-transmitting elements (gelatine films, crystal and glass plates, solutions, and so on); and, conversely, such elements may be given Munsell book notations from their ICI specifications by means of interpolation charts like figure 25. In the application of these charts to light-transmitting specimens, Munsell value is found from luminous transmittance in a way analogous to its usual dependence on luminous reflectance. Table 18, to be discussed presently in another connection, shows Munsell book notations so derived from the luminous transmittances, T, and chromaticity coordinates, x,y, of the glass standards of the ASTM Union colorimeter. The last four book notations given are relatively uncertain, because the colors to be specified are far outside the range of the Munsell standards.

The spacing of the Munsell colors has been examined in detail by a subcommittee of the Colorimetry Committee of the Optical Society of America [96, 97]. This Committee work confirmed the many local irregularities in spacing revealed by the spectrophotometric studies (see figure 25) and established the need for some more general but minor adjustments to make the colors of the Munsell charts correlate more perfectly under ordinary observing conditions (adaptation to daylight, gray to white surrounding field, and so on) with the surface-color solid. The subcommittee found it possible from this study to recommend specifications (Y,x,y) on the

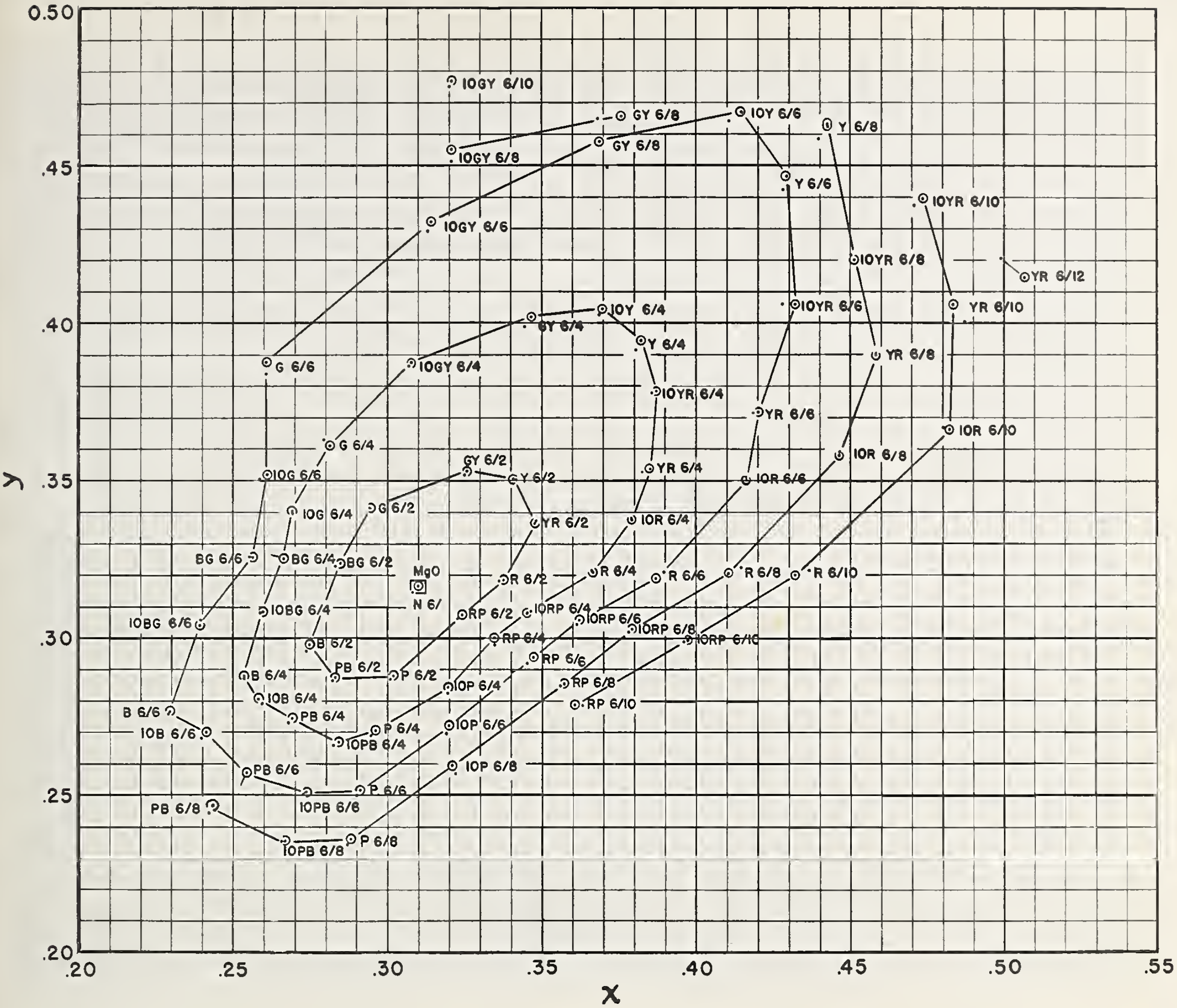

FIGURE 25. *Chromaticities on the (x,y)-diagram found by Kelly, Gibson, and Nickerson [68] for the colors of the 1929 Munsell Book of Color [91] including the first and second intermediate hues at value 6/.*

Points representing colors of constant Munsell chroma are connected by straight lines. Note that these chroma loci are roughly elliptical but show local irregularities corresponding to technical imperfections in the book.

standard coordinate system defining an ideal Munsell system. They have also given to every Munsell standard a revised notation, called the *Munsell renotation,* indicating exactly in what way and how much each color chip deviates from the ideal. Furthermore, the recommended definition of the ideal Munsell system has been extended beyond the color ranges covered by the present Munsell charts so as to include all colors theoretically producible from nonfluorescent materials [76] under illuminant C. The relation between luminous reflectance, *Y,* and Munsell renotation value is given in table 12 for every tenth of a value step. The OSA Subcommittee on the Spacing of the Munsell Colors has published a table giving the same connection for every hundredth of a value step [97]. Note that N 0/ corresponds to $Y_V=0$, N 9.91/ corresponds to magnesium oxide ($Y_V=100\%$) and N 5.0/ corresponds to $Y_V=19.8\%$ (quite different from 50%). Figure 26 shows one of the (x,y)-chromaticity charts (that for Munsell renotation value equal to 6.0) defining the ideal system. From these charts, combined with the data given in table 12, it is possible to find the revised Munsell notation for any color specified in terms of luminous reflectance, *Y,* and chromaticity coordinates *x,y.* Note that they agree in a general way, though not perfectly, with the notations found by direct visual comparison with the Munsell Book of Color. A large part of the discrepancies are ascribable to local irregularities of the color spacing of the Munsell charts. These irregularities have been eliminated by the Munsell renotations. For the two

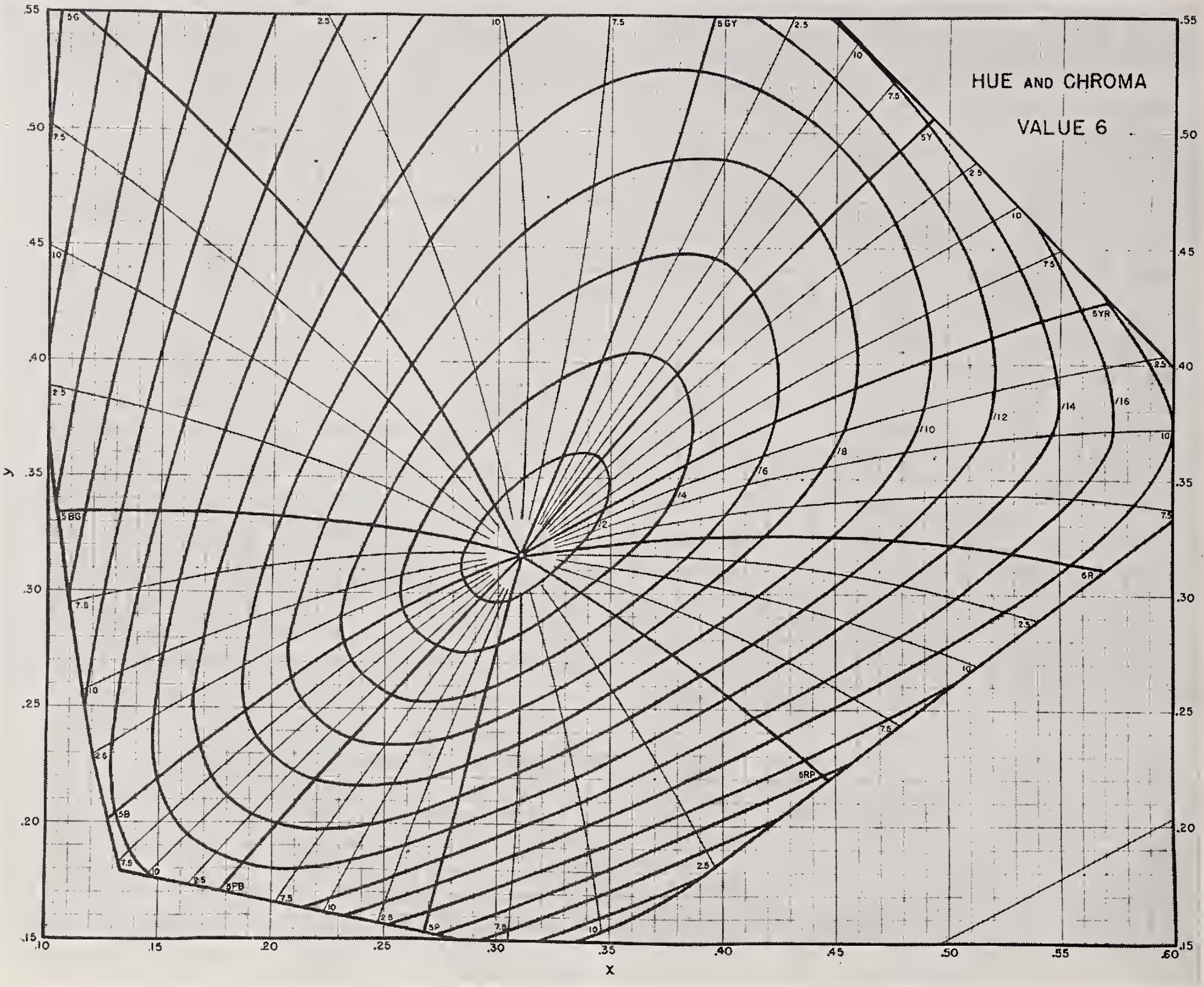

FIGURE 26. *The definition of Munsell renotation hue and chroma for Munsell renotation value equal to 6/.*

The definition is given in terms of the (x,y)-chromaticity diagram. The curved lines radiating from the point $(x=0.310,\ y=0.316)$ define the chromaticities for Munsell renotation hue; and the ovals surrounding this point define the chromaticities for Munsell renotation chroma. These definitions are carried out to the theoretical limits evaluated by MacAdam [76], beyond which the chromaticities can be produced in nonfluorescent specimens only by reducing the value below 6/. These definitions of hue and chroma developed by a subcommittee of the Optical Society of America [97] are free from the local irregularities shown by the actual colors of the Munsell book (see fig. 25).

colors of table 11 it will be seen from columns 2 and 3 that the adjustments required to correct the irregularities of the Munsell charts are larger than the uncertainties of finding the Munsell notation by visual interpolation (compare cols. 1 and 2) but are of the same order of magnitude. And, in general, it may be stated that the Munsell renotation of a color will differ from its book notation by fairly small but perceptible amounts. The Munsell Color Co. has agreed to accept the findings of the OSA Subcommittee as a guide in its future preparation of color standards. On this account the present small discrepancies between Munsell book notation and Munsell (ideal) renotation may be expected to decrease progressively as the standards are repainted.

Munsell renotations, such as shown in table 11, have a unique usefulness as color specifications. Because of their definition in terms of the standard ICI coordinate system, they are capable of nearly the precision of the Y,x,y form of specification, and like that form they may be extended to apply to all object colors, both opaque and transparent objects. For opaque surfaces, Y is luminous directional reflectance relative to magnesium oxide; for transparent objects, Y is luminous transmittance relative to the same thickness of air; for solutions, Y is

TABLE 12. *Luminous reflectance (Y_V) corresponding to various Munsell renotation values (V)*

V	Y_V	V	Y_V	V	Y_V	V	Y_V	V	Y_V
	Percent		*Percent*		*Percent*		*Percent*		*Percent*
0.0	0.000	2.0	3.126	4.0	12.00	6.0	30.05	8.0	59.10
.1	.120	2.1	3.391	4.1	12.66	6.1	31.23	8.1	60.88
.2	.237	2.2	3.671	4.2	13.35	6.2	32.43	8.2	62.71
.3	.352	2.3	3.968	4.3	14.07	6.3	33.66	8.3	64.57
.4	.467	2.4	4.282	4.4	14.81	6.4	34.92	8.4	66.46
.5	.581	2.5	4.614	4.5	15.57	6.5	36.20	8.5	68.40
.6	.699	2.6	4.964	4.6	16.37	6.6	37.52	8.6	70.37
.7	.819	2.7	5.332	4.7	17.18	6.7	38.86	8.7	72.38
.8	.943	2.8	5.720	4.8	18.02	6.8	40.23	8.8	74.44
.9	1.074	2.9	6.128	4.9	18.88	6.9	41.63	8.9	76.53
1.0	1.210	3.0	6.555	5.0	19.77	7.0	43.06	9.0	78.66
1.1	1.354	3.1	7.002	5.1	20.68	7.1	44.52	9.1	80.84
1.2	1.506	3.2	7.471	5.2	21.62	7.2	46.02	9.2	83.07
1.3	1.667	3.3	7.960	5.3	22.58	7.3	47.54	9.3	85.33
1.4	1.838	3.4	8.471	5.4	23.57	7.4	49.09	9.4	87.65
1.5	2.021	3.5	9.003	5.5	24.58	7.5	50.68	9.5	90.01
1.6	2.216	3.6	9.557	5.6	25.62	7.6	52.30	9.6	92.42
1.7	2.422	3.7	10.134	5.7	26.69	7.7	53.94	9.7	94.88
1.8	2.642	3.8	10.734	5.8	27.78	7.8	55.63	9.8	97.39
1.9	2.877	3.9	11.356	5.9	28.90	7.9	57.35	9.9	99.95
								10.0	102.56

luminous transmittance relative to the same thickness of distilled water or solvent. For any of these objects, Munsell renotation value, V, may be found from Y in accord with table 12. Because of their close correlation with the color solid, Munsell renotations are capable of being quickly understood. Thus the renotation 1.3 RP 6.2/4.2 for Orchid indicates from the letters *RP* that the specimen is a red-purple, from the value 6.2 that it is somewhat lighter than middle gray, and from the chroma 4.2 that it is a moderately saturated color inclining somewhat toward gray. Munsell renotation hue and chroma serve more adequately for object colors the purposes formerly served by dominant wavelength and purity. Munsell renotation hue correlates significantly better under ordinary conditions of daylight observation with the hue of the perceived color than does dominant wavelength; and Munsell renotation chroma is by far superior to purity in its correlation to saturation of the perceived color.

This correlation with the color-perception solid does not, however, necessarily hold under all observing conditions. Ordinarily this Orchid standard will be perceived to have a moderate to weak reddish purple color, but it is not so perceived under all conditions. If this specimen be viewed next to a brilliant magenta color, such as may be provided by a fluorescent fabric, it will be perceived to take on a color of different hue and much reduced saturation, such as might be described as a bluish gray. Thus the lightness, hue, and saturation of the color perception depend upon the surroundings and upon the adaptive state of the eye; and lightness, hue, and saturation are taken correctly to be psychological terms. But the Munsell renotation refers only to the light that is reflected from the specimen, and stays constant regardless of these changes in observing conditions. It is therefore a psychophysical characterization of the specimen according to the light reflected from it just as are luminous reflectance, Y, and chromaticity coordinates, x,y, from which it can be derived, and dominant wavelength and purity, which it is coming to supplant.

Another advantage of expressing spectrophotometric results in the form of the Munsell renotation is that the amount and kind of the color difference between two specimens can be found immediately from the two renotations in an easily understandable form. One NBS unit of hue difference from eq 27 is equivalent to approximately 0.25 Munsell hue step at chroma 10, or to 2.5 Munsell hue steps at chroma 1. One NBS unit of saturation difference from eq 26 is equivalent to about 0.15 Munsell chroma step; and one NBS unit of lightness difference from eq 19 corresponds to about 0.10 Munsell value step. If the two colors differ in more than one dimension, the number of NBS units may be estimated by taking the sum of the NBS units in each dimension, thus:

$$\Delta E = (C/5)(2\Delta H) + 10\Delta V + 6\Delta C. \qquad (29)$$

Or if, as often happens, hue differences are of greater concern than lightness and saturation differences, the Nickerson [101, 103] index of color difference, I, may be used

$$I = (C/5)(2\Delta H) + 6\Delta V + 3\Delta C, \qquad (30)$$

where C is Munsell chroma, and ΔH, ΔV, ΔC, are the differences between the two colors in Munsell hue, value, and chroma respectively.

V. One-Dimensional Color Scales

There are many tests analogous to the comparison of a solution of an unknown amount of a constituent with a series of suitably prepared standard solutions to find the concentration of the specimen. In these tests the colors of the unknowns exhibit a one-dimensional change with concentration, and although this change may be complicated in terms of luminous transmittance and chromaticity coordinates [86], a suitably spaced series of standards over this range of colors will yield the desired concentration either by actual match with one of the standards, or by visual interpolation among them. Such a series of standards is said to constitute a *color scale*. The ideal material from which to make the standards is the unknown constituent itself; in this way there is guaranteed not only a perfect color match at some point along the scale, but also a perfectly nonmetameric match so that variations of the illuminant or individual-observer variations are generally unimportant.

However, if the unknown is impermanent, it may become necessary to try to duplicate the desired colors in a more permanent medium. Glass is a frequent choice because of its generally superior permanence. Some degree of metamerism has then to be tolerated, because the standards have coloring constituents not a perfect spectral match for the unknown. It is also rare that a perfect job of color matching for any standard illuminant and observer is done. The observer is then faced with what is often a difficult, and sometimes an impossible, task.

He must estimate the position of the unknown color on the scale, and oftentimes it will seem to him that the unknown color is not equal to any of the standard colors nor intermediate between any two of them. The concepts in terms of which the observer perceives these color differences then come into play. If he judges the color difference between the two luminous areas presented to him in terms of hue, brightness, and saturation, as is fairly common, he could estimate the position of the unknown on the color scale as the point on the scale yielding the same hue, or as that yielding the same brightness, or the same saturation; or he could try to estimate the point on the scale yielding the closest color match, or he could disregard brightness differences and try to estimate the point on the scale yielding the closest chromaticity match. The determination becomes an estimate based on what criterion of equivalence is used by the observer, and it depends upon his mental furniture in an essentially indescribable way. In spite of these drawbacks, a good color scale is a useful timesaver, as long as it is not used in attempts to provide a one-dimensional solution to what is essentially a multi-dimensional problem.

Judgments of position on the color scale according to equality of brightness can be expected to correspond to luminous transmittance. Judgments according to equality of hue agree well with Munsell renotation hue for ordinary conditions of observation; see curved lines of figure 3 separating the areas corresponding to the various hue names, also figure 26. Judgments according to equality of saturation agree well with Munsell renotation chroma; see figure 26. If the brightness variation along the scale is slight compared to the chromaticity variation, judgments are likely to be according to nearest chromaticity match. Such judgments agree well with the point on the scale found by taking the shortest distance on a uniform-chromaticity-scale diagram between the point representing the unknown color and the line representing the scale; see figures 17, 18, and 19. If there is primary variation of both luminance and chromaticity, no reliable way of estimating the nearest color match has yet been developed. According to the OSA Colorimetry Committee [108] "the complete experimental clarification of this problem is one of the major programs yet to be undertaken in the field of colorimetric research".

1. Color Temperature

Perhaps the most widely used one-dimensional color scale is that of color temperature for classifying light sources. The *color temperature* of a light source is the temperature at which the walls of a furnace must be maintained so that light from a small hole in it shall yield the chromaticity of the source to be specified. The color scale thus consists of the series of lights producible by closed-cavity radiation and is specified by temperature on the absolute scale (degrees Kelvin). Working standards of color temperature may consist of an incandescent lamp operating at a fixed voltage combined with a series of amber or blue filters, like the Davis-Gibson [22] liquid filters or the Lovibond blue glasses; but by far the most common way of producing these chromaticities over moderate ranges of color temperature is by variation of the voltage applied to an incandescent lamp. The locus of these chromaticities (the so-called Planckian locus) is shown on figures 3a, 5, 7, 17, and 18. If the chromaticity of the light source is close to, but not exactly equal to, any of the Planckian chromaticities, still it is possible to correlate a color temperature with the source by taking the nearest chromaticity match. Figure 27 shows this correlation [60]. The isotemperature lines, which cut the Planckian locus at varying angles, are all such as to be perpendicular to this curve on figure 17. Since color temperature specifies only the chromaticity of a light, there are many spectral compositions corresponding to the same color temperature. Color temperature of an illuminant is therefore an incomplete and unreliable indication of the rendering of the colors of objects illuminated by it, or of the photographic effect of the illuminant. To make color temperature a perfectly valid basis for comparing two lights, it must also be shown that they are spectrally similar; thus, incandescent lamps may be usefully compared by means of color temperature, and fluorescent lamps with about the same admixture of mercury spectrum may also be so intercompared, but comparison of incandescent lamps with fluorescent, by color temperature, is often not valid.

2. Lovibond Grading of Vegetable Oils

The Lovibond glasses are used extensively in the grading of vegetable oils in this country. McNicholas [85] has shown from the spectral transmittance of representative samples of refined cottonseed oils, also oil of sesame, peanut, soybean, corn, rape, and olives, that these oils owe their colors to independently varying amounts of two groups of coloring matters commonly found in plants, a group of brown pigments, and a group of green (chlorophyll compounds). For commercial purposes the colors of these oils are mostly graded by the number of Lovibond red units required in combination with Lovibond 35-yellow to color match a 5¼-in. layer of the oil. Figure 28 shows the spectral transmittance of a group of cottonseed oils (solid curves) and one oil of other plant origin (dotted curve), all of which have color grades between 11.0 and 13.0 Lovibond red units on the 35-yellow plus red scale. Figure 28 also shows the spectral transmittances of these limiting Lovibond glass combinations (solid curves with solid circles). It will be noted first that the oils all make metameric pairs with the Lovibond glass combinations. A standard light source is thus required, and a certain amount of individual-observer difference may be expected in grading oils by this method. Since there are two main groups of pigments in the oils, it is also to be expected that no one-dimensional grading system, like the scale of 35-yellow plus some

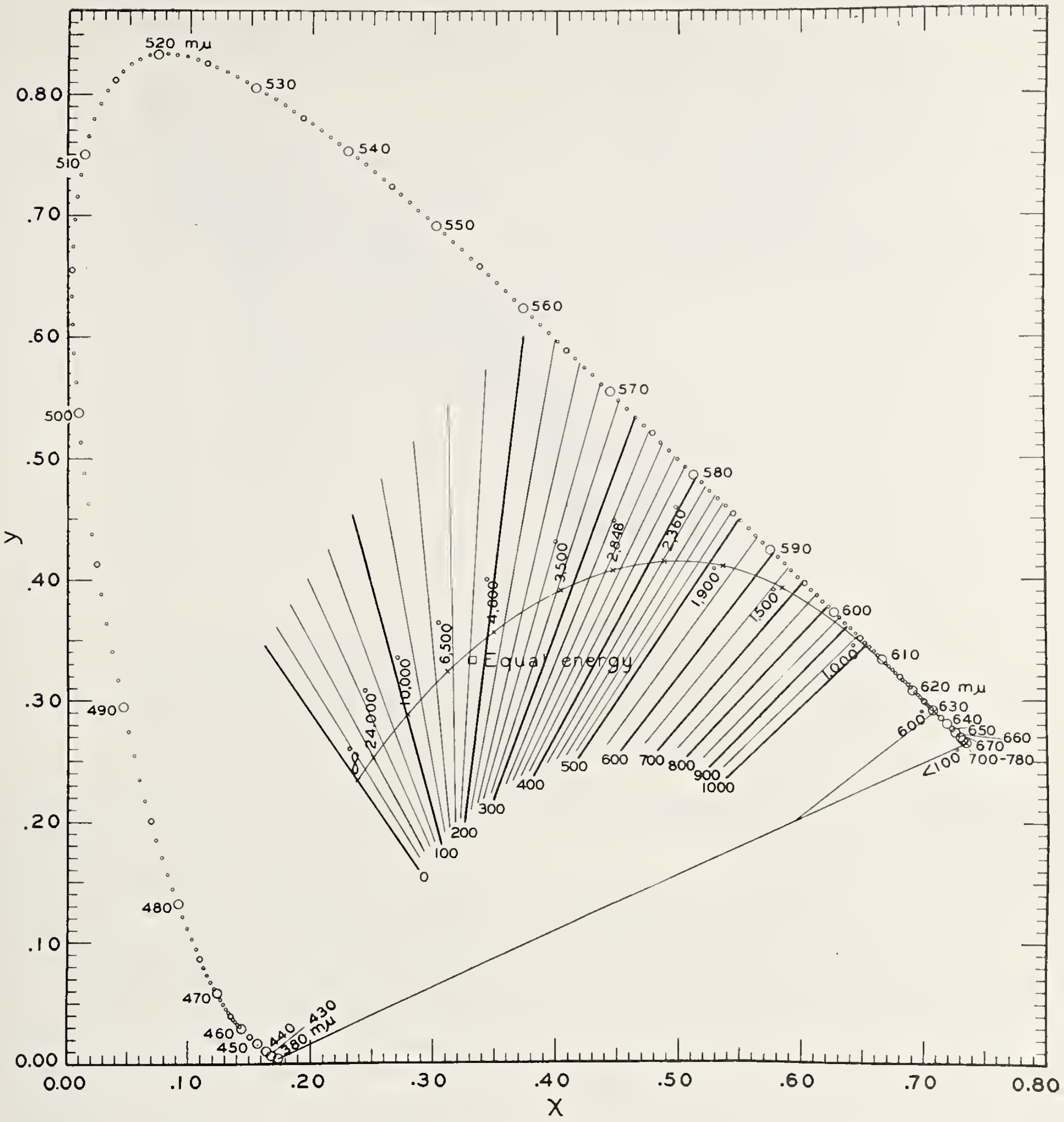

FIGURE 27. *Correlated color temperature for chromaticities represented near the Planckian locus on the (x,y)-chromaticity diagram.*

The straight lines intersecting the Planckian locus are identified by the reciprocal color temperature in microreciprocal-degrees (μrd) of their intersection with the Planckian locus. Points on the locus, itself, are identified by color temperature in degrees Kelvin. The straight lines, if transformed to the UCS triangle (fig. 17), would be orthogonal to the Planckian locus.

amount of red, will yield color matches. Computation of the tristimulus values from the spectral-transmittance curves and plotting on the (r,g)-Maxwell triangle (see fig. 29 due to McNicholas [85]) reveals this spreading of the chromaticity points over an area instead of along a single line, and it shows further that the Lovibond 35Y plus R locus is too far from the spectrum locus to be a good match for the center of this area except for Lovibond red less than 3 units. There is some doubt, therefore, what Lovibond grade corresponds to a given spectral-transmittance curve of figure 28. McNicholas chose to base the decision on nearest chromaticity match, and made use of the uniform-chromaticity-scale diagram shown in figure 17. The short straight lines on figure 29 intersecting Lovibond 35Y plus R locus at somewhat oblique angles would be perpendicular to the corresponding locus on figure 17.

In spite of the difficulties of determining the Lovibond grade of a vegetable oil, the method is well suited to indicate quality and salability, and it has been used for more than 30 years. Because of the difficulties, however, and the further disadvantage of being dependent upon a foreign source of glass standards, there has been fairly continuous agitation and research work intended to lead to another, perhaps photoelectric, method. But the Lovibond color scale, 35Y plus R, is so ingrained in the purchase

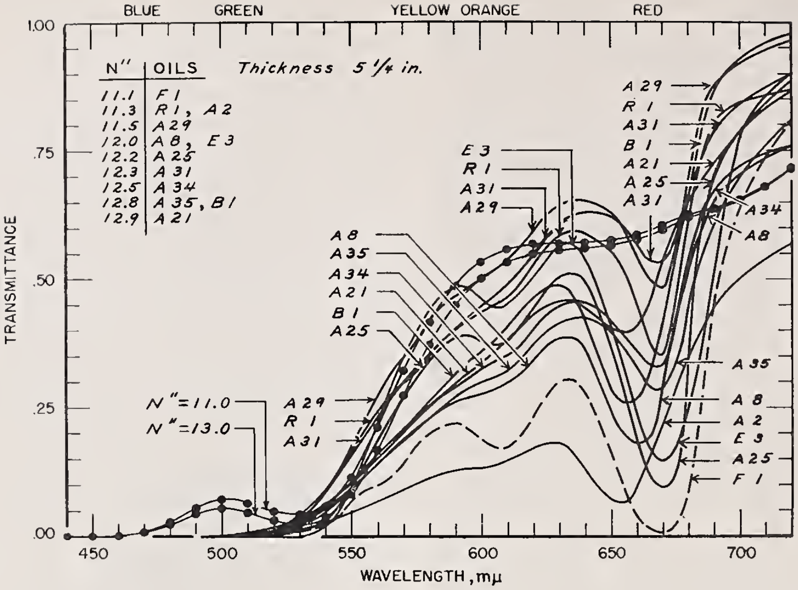

Figure 28. *Spectral transmittance of a group of vegetable oils (solid and dashed curves) compared to the spectral transmittance (solid circles) of combinations of Lovibond yellow and red glasses (35Y 11R and 35Y 13R) to which they are closely equivalent in trade practice (McNicholas [85]).*

It will be noted that each oil combined with each glass combination makes a rather strongly metameric pair; that is, the brown and green pigments in the oils are spectrally quite dissimilar to the colorants (silver, yellow, gold ruby) of the glasses.

and sale of vegetable oil in this country that it may well be that a substitute method will still have to give results expressed on that scale.

3. Union Colorimeter Grading of Lubricating Oils

For more than 20 years the color of lubricating oils and petrolatum has been graded by comparison with the colors of 12 glass standards [10]. The petroleum product in a 33-mm layer and the standard are illuminated by artificial daylight produced by combining an incandescent lamp of color temperature approximately 2,750°K with a filter of Corning Daylite glass specially selected to have spectral transmittances within specified tolerances and further such as to have, for standard illuminant A, luminous transmittance, T_w, and chromaticity coordinates x,y,z, within the limits:

T_w	0.107 to 0.160
x	.314 to 0.330
y	.337 to 0.341
z	.329 to 0.349

The specimen holder, the magazine containing the glass color standards, the artificial daylight assembly, and a viewing diaphragm defining the direction of view are mounted together to form a portable instrument known as the Union colorimeter.

Table 13 gives the Lovibond analysis of the glass color standards [10], the luminous transmittance, T_w, and chromaticity coordinates, x,y, for illuminant C [124], and the color names used by the National Petroleum Association. Figure 30 shows by large circles on the (x,y)-diagram the chromaticities of these glass color standards compared to those of 22 petroleum products (small circles) measured by Diller, De Gray, and Wilson [24]. It will be noted from figure 30 that the Union colorimeter glasses yield good chromaticity matches for the petroleum products represented except for ASTM color numbers 1½ to 3½, in which range the oil points fall between the spectrum locus and the Union locus. Note also that the oil points and the Union locus above ASTM color number 5 fall on the spectrum locus; that is, the standard observer would find that each such glass standard and each such oil color matches some part of the spectrum between 605 and 640 mμ. It is possible that some improvement could be achieved by choosing glass standards of higher excitation purity for the color numbers 2, 2½, 3, and 3½; such a change would make it easier for an observer to decide which glass standard is closest in chromaticity to such oils as are represented on figure 30. However, the difficulty that has probably prevented the Union colorimeter from becoming the basis of a standard method is that the luminous transmittances of lubricating oils fail to correlate well with their chromaticities; that is, no single line in the color solid can represent these colors. This essential difficulty would not be overcome by any choice of standards in a one-dimensional series. If, as seems probable from the words "lighter" and "darker" used in describing the method [10] and

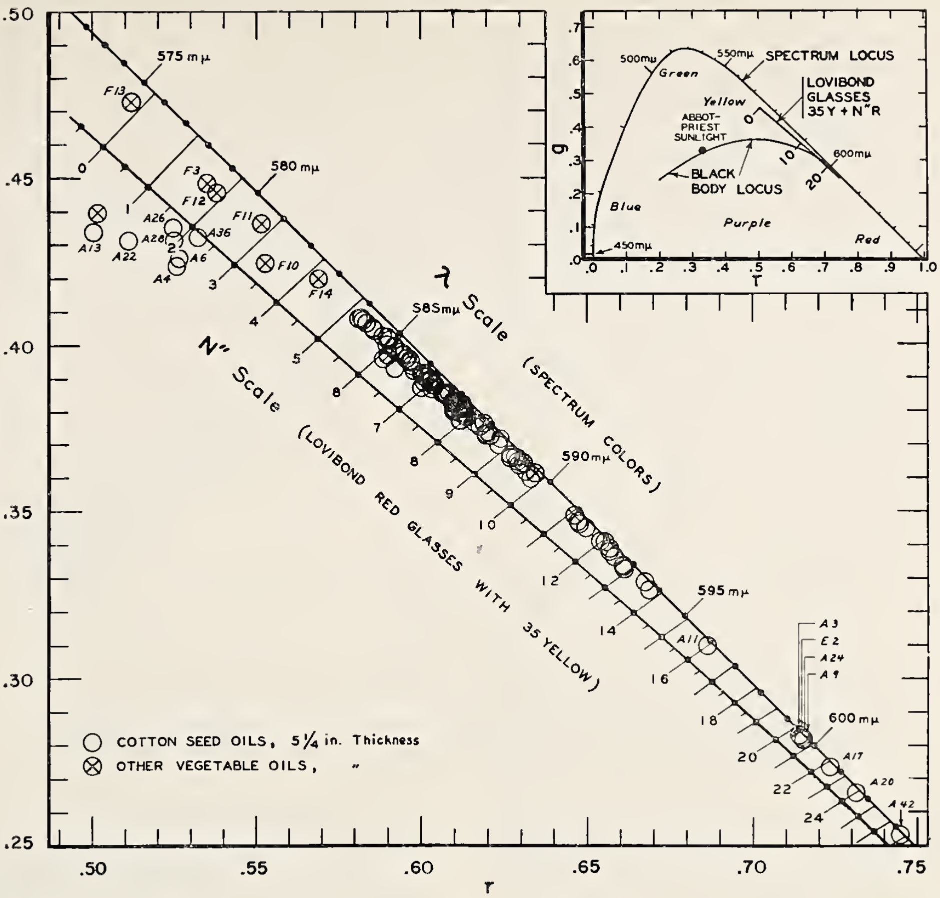

FIGURE 29. *Chromaticities of vegetable oils compared to those of the combinations of Lovibond yellow and red glasses, by means of which they are graded.*

The insert shows the entire (r,g)-chromaticity diagram used by McNicholas [85]; the main diagram shows only the chromaticity range covered by the vegetable oils. Note that the chromaticities of the oils are represented between the spectrum locus and the locus of 35-yellow plus N''-red Lovibond glasses by which they are graded. The slightly oblique lines extending from the spectrum locus through the 35-yellow N''-red locus indicate the nearest chromaticity matches to the latter. These lines if plotted on the UCS triangle (fig. 17) would be perpendicular to the 35-yellow N''-red locus.

from a proposed photoelectric method of test [7], the chief intent is to specify the lightness or darkness of the oil color, it would be more helpful to bring together the photometric fields to be compared by a suitable prism eye-piece so that brightness differences could be more readily detected; or even use a photometer such as the Martens (fig. 8) to determine by comparison with the glass standards the luminous transmittance of the oil. The Union colorimeter, however, in spite of occasional important ambiguity of setting is very useful, and the Union color scale is well known in the petroleum industry.

4. Saybolt Chromometer Grading of Refined Petroleum

The grading of naphthas, kerosines, and so on, has for many years been carried out by comparison of the color of rather thick layers (up to 20 in.) of the refined oil with the colors of a set of three color standards made of yellowish glass. The Saybolt chromometer [8] is a device for carrying out this comparison. It consists of an artificial daylight lamp meeting the same requirements given above for the ASTM Union colorimeter, a graduated tube for the oil specimen, a holder for the glass color standards, and a prism eyepiece to

TABLE 13. *ASTM Union Colorimeter Standards, Lovibond analysis, luminous transmittance, T_w, and chromaticity coordinates, x, y, for standard illuminant C, and NPA color names*

ASTM color number	Lovibond anaylsis			Luminous transmittance, T_w	Chromaticity coordinates		National Petroleum Association names
	Red	Yellow	Blue		x	y	
1 . . .	0.12	2.4	. . . .	0.751	0.349	0.382	Lily white.
1½. .	.60	8.0	. . . .	.654	.400	.446	Cream white.
2 . . .	2.5	26.0	. . . .	.443	.472	.476	Extra pale.
2½. .	4.6	27.0	. . . .	.365	.498	.457	Extra lemon pale.
3 . . .	6.9	32.0	. . . .	.287	.525	.440	Lemon pale.
3½. .	9.4	45.0	. . . .	.211	.556	.423	Extra orange pale.
4 . . .	14.0	50.0	0.55	.096	.591	.400	Orange pale.
4½. .	21.0	56.0	.55	.065	.620	.376	Pale.
5 . . .	35.0	93.0	. . . .	.036	.653	.347	Light red.
6 . . .	60.0	60.0	.55	.017	.676	.323	Dark red.
7 . . .	60.0	106.0	1.8	.0066	.684	.316	Claret red.
8 . . .	166.0	64.0	. . . .	.0020	.714	.286	. . . .

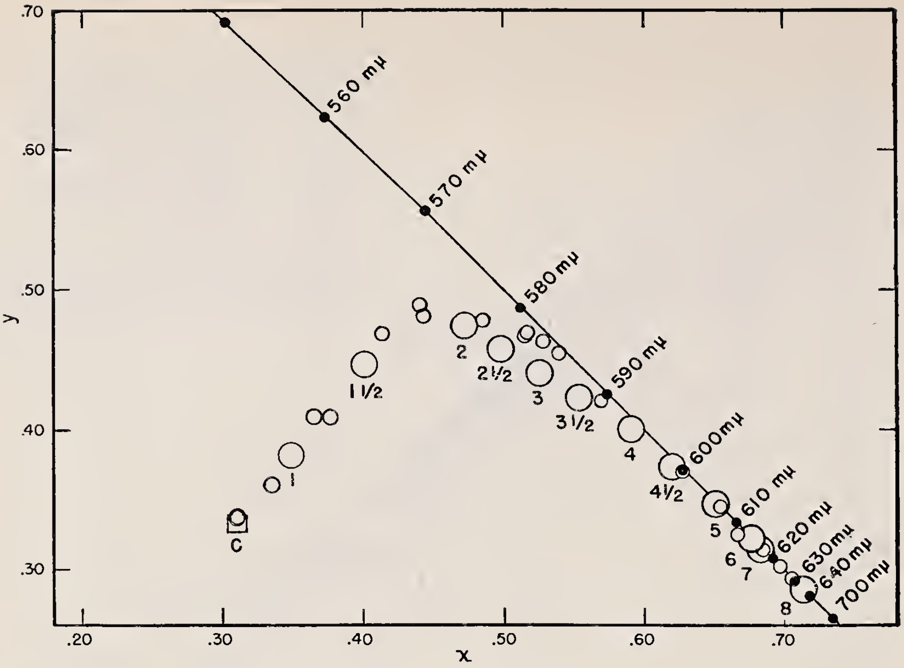

Figure 30. *Chromaticities of the ASTM Union colors [10] compared to those of a group of petroleum products measured by De Gray, Diller, and Wilson [24].*

A portion of the spectrum locus (550 to 700 mμ) of the (x,y)-chromaticity diagram is also shown. It will be noted that between Union 1½ and 3½ the chromaticities of these petroleum products tend to be represented by points falling between the Union color locus and the spectrum locus.

bring into juxtaposition the two fields to be compared. There is an open ungraduated tube beneath the holder for the glass standards that serves to duplicate to a degree on the standard side of the instrument the effect of light multiply reflected within the specimen tube. The glass standards consist of two whole disks, and one "one-half" disk; these standards must have luminous transmittances, T_w, and chromaticity coordinates, x,y, for illuminant C, as follows:

	Whole disks	One-half disks
T_w	0.860 to 0.865	0.888 to 0.891
x	.342 to 0.350	.327 to 0.331
y	.367 to 0.378	.344 to 0.350
z	.272 to 0.291	.319 to 0.330

Oils having colors closely resembling that of distilled water are graded on the Saybolt chromometer by comparison with the half disk, more yellowish oils by one or two whole disks. The depth of oil yielding the closest color match with the glass disk is found by a prescribed procedure [8], and the color of the oil specimen is designated by a number defined from the disk used and the required depth of oil. Table 14 gives this definition.

In spite of the fact that the prism eyepiece brings the fields to be compared into juxtaposition, and so facilitates the detection of differences in brightness, the change in luminous transmittance with thickness of these refined oils is so slight that it is much less easily detected than the corresponding chromaticity change. The settings of the Saybolt chromometer therefore probably depend essentially on nearest chromaticity match. If the oil sample is turbid, however, not even an approximate match can be obtained, and the method may be inapplicable. In these cases a thickness of the specimen yielding a chromaticity match is much darker than the standard, and no reliable setting of depth of sample to yield nearest color match can be found.

Table 14. *Definition of Saybolt chromometer numbers from the depths of the specimen and the number of disks used to produce the nearest color match*

Number of disks	Depth of oil	Saybolt number	Number of disks	Depth of oil	Saybolt number
	in.			*in.*	
½	20.00	+30	2	5.75	+5
½	18.00	+29	2	5.50	+4
½	16.00	+28	2	5.25	+3
½	14.00	+27	2	5.00	+2
½	12.00	+26	2	4.75	+1
1	20.00	+25	2	4.50	0
1	18.00	+24	2	4.25	—1
1	16.00	+23	2	4.00	—2
1	14.00	+22	2	3.75	—3
1	12.00	+21	2	3.625	—4
1	10.75	+20	2	3.50	—5
1	9.50	+19	2	3.375	—6
1	8.25	+18	2	3.25	—7
1	7.25	+17	2	3.125	—8
1	6.25	+16	2	3.00	—9
2	10.50	+15	2	2.875	—10
2	9.75	+14	2	2.75	—11
2	9.00	+13	2	2.625	—12
2	8.25	+12	2	2.50	—13
2	7.75	+11	2	2.375	—14
2	7.25	+10	2	2.25	—15
2	6.75	+9	2	2.125	—16
2	6.50	+8	......	......	......
2	6.25	+7	......	......	......
2	6.00	+6	......	......	......

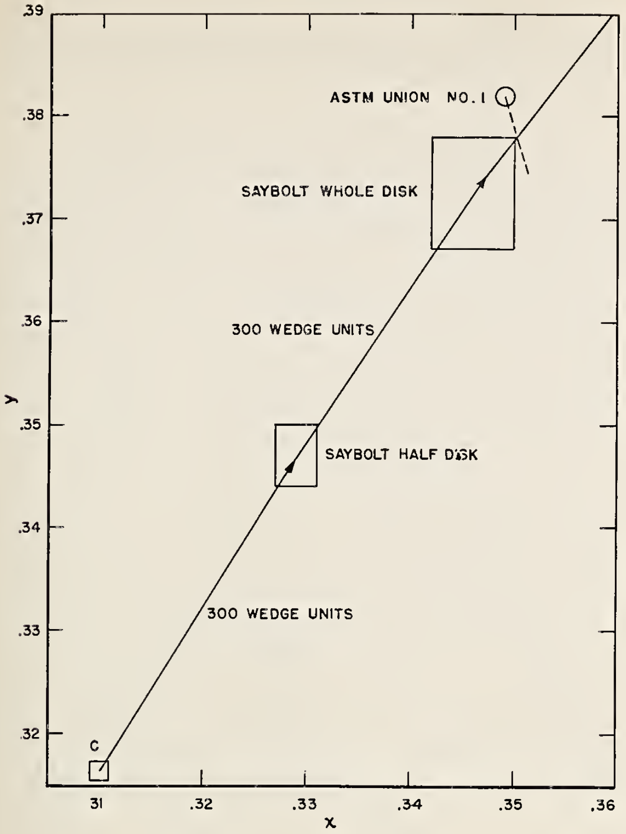

FIGURE 31. *Chromaticities of the half disk and whole disk of the Saybolt chromometer compared with that of Union color 1 and with those produced by the yellow wedge of the chromaticity-difference colorimeter (see fig. 10).*

The dotted line serving to evaluate the Saybolt color corresponding to Union color 1 would be perpendicular to the solid line that it intersects if they were plotted on the UCS triangle (fig. 17).

Figure 31 shows a portion of the (x,y)-chromaticity diagram on which have been plotted the rectangles corresponding to the chromaticity tolerances for Saybolt half disks and whole disks (see above). It will be noted that the tolerances are fairly wide so that variations of plus or minus 10 percent of the chromaticity change caused by introduction of the disk into the daylight beam are permissible. From table 14 it may be seen that a whole disk is never equivalent to as much as a difference of 10 in Saybolt color; so if, as seems likely, the criterion of match is nearest chromaticity, the tolerances are equivalent to less than 1 in Saybolt color.

On figure 31 is shown by a circle the point representing Union Color No. 1. It is of interest to inquire what would be the Saybolt color of an oil having ASTM Color No. 1, because we could then write down an interconversion of the two color scales. This may be done on the assumption that the settings are made by the criterion of nearest chromaticity. Since the Saybolt chromometer makes use of variable depths of oil specimen, we must first find out how the chromaticity of an oil specimen expressed on the (x,y)-diagram varies with its thickness. An approximate answer is supplied by the calibration of the yellowish wedge of the chromaticity-difference colorimeter (fig. 9). The spectral transmittances of this wedge resemble fairly well those of refined petroleum oils. Figure 31 shows two vectors each corresponding to 300 yellow wedge units on the chromaticity-difference colorimeter and the beginning of a third. The first vector starts at the point representing illuminant C, and it is seen that a difference of 300 wedge units is equivalent within the prescribed tolerances to the Saybolt half disk. The second vector starts at the end of the first and indicates that adding 300 additional wedge units bridges the chromaticity differences between the Saybolt half disk and the Saybolt whole disk. The third vector commences at the end of the second and shows, in agreement with figure 30, that increasing thickness of a light-absorbing medium like refined petroleum oil gives a slightly curving line on the (x,y)-diagram with a decreasing scale as thickness is increased. The point representing Union Color No. 1 does not fall exactly on this line, but the dotted line drawn from this point intersecting the third vector corresponds to the perpendicular to the vector plotted on a uniform-chromaticity-scale triangle (fig. 17) and indicates the closest chromaticity match. The direction of the dotted line may also be found with good approximation from the family of ellipses shown on figure 16; it is the direction of the line connecting the center (Union Color No. 1) of an ellipse of this family with the point of tangency to the vector. From the intersection of the dotted line with the vector it may be seen that the chromaticity of Union Color No. 1 approaches most closely to that of about 1.10 Saybolt disk. An oil of Union Color No. 1 will therefore normally match 1.00 Saybolt disk in a thickness of $33.0/1.10=30.0$ mm, and it will match two whole disks in a thickness of 60.0 mm or 2.365 in. Reference to table 14 shows that such an oil would have Saybolt color equal to about −14; this corresponds nearly to the yellowest oil indicated on the Saybolt scale. Thus, the Saybolt scale serves to fractionate the first unit of the Union colorimeter scale. If it is permitted us to evaluate colors on the Union scale in accord with the fraction, f, of the depth of a standard solution such that unit depth matches Union Color No. 1, then we may find the Union Color, f, corresponding to any Saybolt number from the number, n, of disks used and the depth, D, defining the Saybolt number (table 14) by the relation

$$f=2.365n/2D=1.182n/D. \qquad (31)$$

Table 15 shows the Union Color, f, corresponding to each Saybolt color in accord with eq 31. Correlations obtained from actual comparison of specimens to the two sets of standards may differ somewhat from that indicated in table 15 if the criterion of nearest chromaticity match be not followed exactly, or if the observer departs considerably from the ICI standard.

TABLE 15. *Union color, f, corresponding to Saybolt color number from eq 31*

Saybolt number	Union color, *f*	Saybolt number	Union color, *f*	Saybolt number	Union color, *f*
+30	0.030	+15	0.225	0	0.526
+29	.033	+14	.243	—1	.557
+28	.037	+13	.262	—2	.592
+27	.042	+12	.286	—3	.631
+26	.049	+11	.305	—4	.652
+25	.059	+10	.326	—5	.677
+24	.066	+9	.350	—6	.701
+23	.074	+8	.364	—7	.727
+22	.084	+7	.376	—8	.758
+21	.098	+6	.393	—9	.789
+20	.110	+5	.411	—10	.823
+19	.124	+4	.430	—11	.860
+18	.143	+3	.450	—12	.901
+17	.163	+2	.474	—13	.947
+16	.189	+1	.498	—14	.996
......	......	......	......	—15	1.05
......	......	......	......	—16	1.11

5. U. S. Color Standards for Rosin

Gum rosin has been graded by color for more than 50 years. Up to 1914 the color standards were made of rosin itself in spite of the relative impermanence of its color, and from 1914 to 1936 standards composed of combinations of Lovibond glasses were used. Brice [18] has described the selection of the present 12 official standards composed of two components of colored glass combined with one component of clear glass, all three cemented together with Canada balsam. The various combinations are given letter designations denoting the grades of rosin delimited by them and have legal status under the Naval Stores Act. The cemented face of the clear glass in each combination is fine ground so as to duplicate the slight turbidity characteristic of molded samples of rosin that commonly contain traces of fine dirt. The chromaticities are shown by solid circles on figure 32, and it will be noted that they cover approximately the same chromaticity range as ASTM Union Color 1½ to 8. The chromaticity spacing was adjusted by means of the UCS triangle of figure 17 so as to progress regularly from small steps for yellow rosins to steps of about four times the initial size for reddish orange rosins.

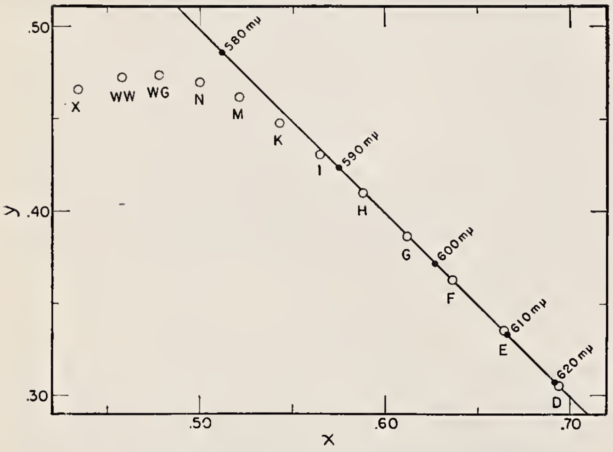

FIGURE 32. *Chromaticities of the US Color Standards for Rosin according to Brice [18].*

A part of the spectrum locus (580 to 620 mμ) of the (*x*,*y*)-chromaticity diagram is also shown. This series of chromaticities is similar to that of the Union colors (fig. 30), except that it has been adjusted for progressively greater spacing toward the red end of the scale by making use of the UCS triangle (fig. 17).

6. Color Standards for Paint Vehicles

Many special color scales have been set up for the specification of paint vehicles (varnishes, linseed oil, tung oil, and so on). A solution of nickel sulfate and iodine [19] is used in Great Britain to define the darkest color permissible for spar varnish. A color comparator having 18 glass color standards made by Hellige, Inc., has been used for similar purposes. The Pfund color grader, made by the Munsell Color Company, compares a variable thickness of the unknown specimen with a variable thickness of a carbon-yellow glass [113]. The standard is wedge-shaped, and the cell for the specimen is likewise wedge-shaped. The Parlin (or Cargille) color standards consist of a set of 35 solutions. The first ten are Hazen platinum-cobalt solutions [47] developed originally to measure the color of natural waters and still used for that purpose under the name of APHA (American Public Health Association) standards [6]. The remainder of the Parlin color standards are caramel solutions. They have been adopted tentatively by the ASTM (Designation D 365-39T) for testing the color of soluble nitrocellulose base solutions. The Pratt and Lambert Color Standards are varnish mixtures calibrated against the Pfund color grader. The DuPont colorimeter employs six glass plates as color standards, together with a wedge of the same glass permitting a continuous variation of color between the standards. The Hellige-Stock-Fonrobert colorimeter has three sets of nine glass disks each. The unit of color is 1 mg of iodine in 100-ml of potassium iodide solution [32]. The Gardner color standards consist of 18 combinations of red and yellow Arny solutions. Gardner has determined the Arny and Lovibond specification for the nearest matches for all of the above-mentioned sets of color standards and has obtained also the nearest equivalents in terms of potassium dichromate solutions [32]. From these nearest equivalents it is possible to express color specifications given by any of these means in terms of any other of them.

The British Paint Research Station has recommended [130] combinations of Lovibond glasses for color grading oils and varnishes in 1-in. thickness. Some of the combinations involve colorless (or blue) glasses to be combined with the varnish or oil to match red and yellow glasses, and a device facilitating the setting up of such combinations is also recommended. The Lovibond glasses are mounted in a slide, and the two photometric fields to be compared are brought into juxtaposition by mirrors. For oils two scales, a reddish and a greenish, are provided; and for both oils and varnishes care has been taken to find glass combinations that yield a luminance as well as a chromaticity match for the average commercial product.

VI. General Methods

Standard methods of testing, specifying, and describing colors have been adopted by various organizations. We give here summaries of most such methods intended to apply to color in general.

1. TAPPI Standards T 216 m-47 and T 442 m-47

The Technical Association of the Pulp and Paper Industry has used since January, 1940, the Official Standards T 216 m-47, Spectral Reflectivity and Color of Pulp, and T 442 m-47, Spectral Reflectivity and Color of Paper [127]. The method is to determine by means of a spectrophotometer the spectral directional reflectance relative to magnesium oxide of a sufficient number of thicknesses of the pulp or paper sample that doubling the number of thicknesses will not cause appreciable change in the reflectance at any wavelength. This quantity for brevity is termed spectral reflectivity, and serves to specify the optical properties of pulp and paper that are responsible for the color, subject to the limitation that the color may depend upon the manner of illumination and viewing, which must be made a part of the report. The tristimulus specification of the color computed from the spectral reflectivity according to the ICI standard observer and colorimetric coordinate system is used. The 30 selected-ordinate method of calculation and luminous reflectivity (called visual efficiency) and chromaticity coordinates (called trichromatic coefficients) are suggested.

2. ASTM Standard D 307-44

The American Society for Testing Materials has used since 1944 the Standard Method of Test for Spectral Characteristics and Color of Objects and Materials, D 307-44 [9]. The method consists of spectrophotometry plus computation based upon the 1931 ICI standard observer and colorimetric coordinate system. Three kinds of test specimens are dealt with (a) light transmitting, (b) nonopaque reflecting, and (c) completely opaque reflecting. For these three kinds of test specimens the spectrophotometer yields (a) spectral transmittance (called spectral transmission), (b) spectral directional reflectance (called spectral apparent reflectance), and (c) spectral directional reflectivity (called spectral apparent reflectivity), respectively. Application of the standard luminosity function (y_λ of the standard observer) to these data yields respectively: (a) luminous transmittance, (b) luminous directional reflectance, and (c) luminous directional reflectivity. For transparent specimens the incident light is used as a standard relative to which the transmitted light is evaluated; for reflecting specimens the standard is either the ideal completely reflecting completely diffusing surface, or it is an opaque layer of magnesium oxide prepared by collecting the smoke from magnesium burning in air. The chromaticities of all three types of specimens are specified by the chromaticity coordinates (called trichromatic coefficients), x and y. The manner of illumination and viewing must also be included in the report, together with an identification of the illuminant to which the color designation of the test specimen applies; that is, whether it is the standard illuminant A, B, or C, or some other illuminant.

3. Federal Specification TT-P-141a

Since June 16, 1944, there has been in force Federal Specification TT-P-141a for sampling and test methods for paint, varnish, lacquer, and related meterials [30]. Three general methods of color measurement are included.

(a) Method 425.1, Color specified by trichromatic coefficients (from spectrophotometric curve). This method is identical with ASTM Standard D 307-44 except for some abridgment and inclusion of recommended angular conditions of illumination and view. For transmission measurements illumination at 0° and viewing at 180° is specified. For reflection measurements, either the illumination should be at 45° from the perpendicular and viewing at 0° or the illumination should be at 0° with diffuse viewing. The standard for reflection measurements is an opaque layer of magnesium oxide, use of the ideal perfectly reflecting perfectly diffusing surface not being permitted.

(b) Method 425.2, Color specified by trichromatic coefficients (from photoelectric tristimulus settings). In this method the unknown specimen is compared to a standard known to be permanent and spectrally similar to the specimen. From previous spectrophotometric examination by method 425, the tristimulus values, X,Y,Z, of the standard are also known. From the settings, A, G, and B, obtained for the specimen relative to those for a standard magnesium-oxide surface with the amber, green, and blue filters, respectively, of the photoelectric colorimeter, the tristimulus values of the specimen are obtained from eq 11, and from these are then obtained the chromaticity coordinates, x,y (called trichromatic coefficients). The Hunter multipurpose reflectometer and Photovolt Lumetron colorimeter are mentioned as suitable instruments.

(c) Method 623, Color difference (or color change) from photoelectric tristimulus measurements. Settings A, G, and B are made in accord with method 425.2 for both of the specimens whose color difference it is desired to evaluate. The color difference is found from a modification of eq 21 as follows:

$$\Delta E=[(L_t-L_s)^2+(a_t-a_s)^2+(b_t-b_s)^2]^{1/2}, \quad (32)$$

where for each panel

$$L=10G^{1/2}$$
$$a=70G^{1/2}(A-G)/(A+2G+B),$$
$$b=28G^{1/2}(G-B)/(A+2G+B).$$

This method applies only to panels of similar spectral characteristics. However, it is not necessary that the standard used have spectral characteristics similar to either.

4. ISCC-NBS Method of Designating Colors

A method devised at the request of the American Pharmaceutical Association and the U. S. Pharmacopoeial Convention for designating the colors of drugs and chemicals is also used for general purposes. The plan of the method was worked out by the Inter-Society Color Council, and the details were developed at the National Bureau of Standards; the method is therefore referred to as the ISCC-NBS method of designating colors [64, 65]. This method provides a designation for every color perceived as belonging to an object (either an opaque surface, or a light-transmitting layer), and it has been extended to the colors of self-luminous areas by Kelly [66]; see figure 3. The number of color designations was purposely made small (slightly over 300) for the sake of simplicity. Since about ten million surface

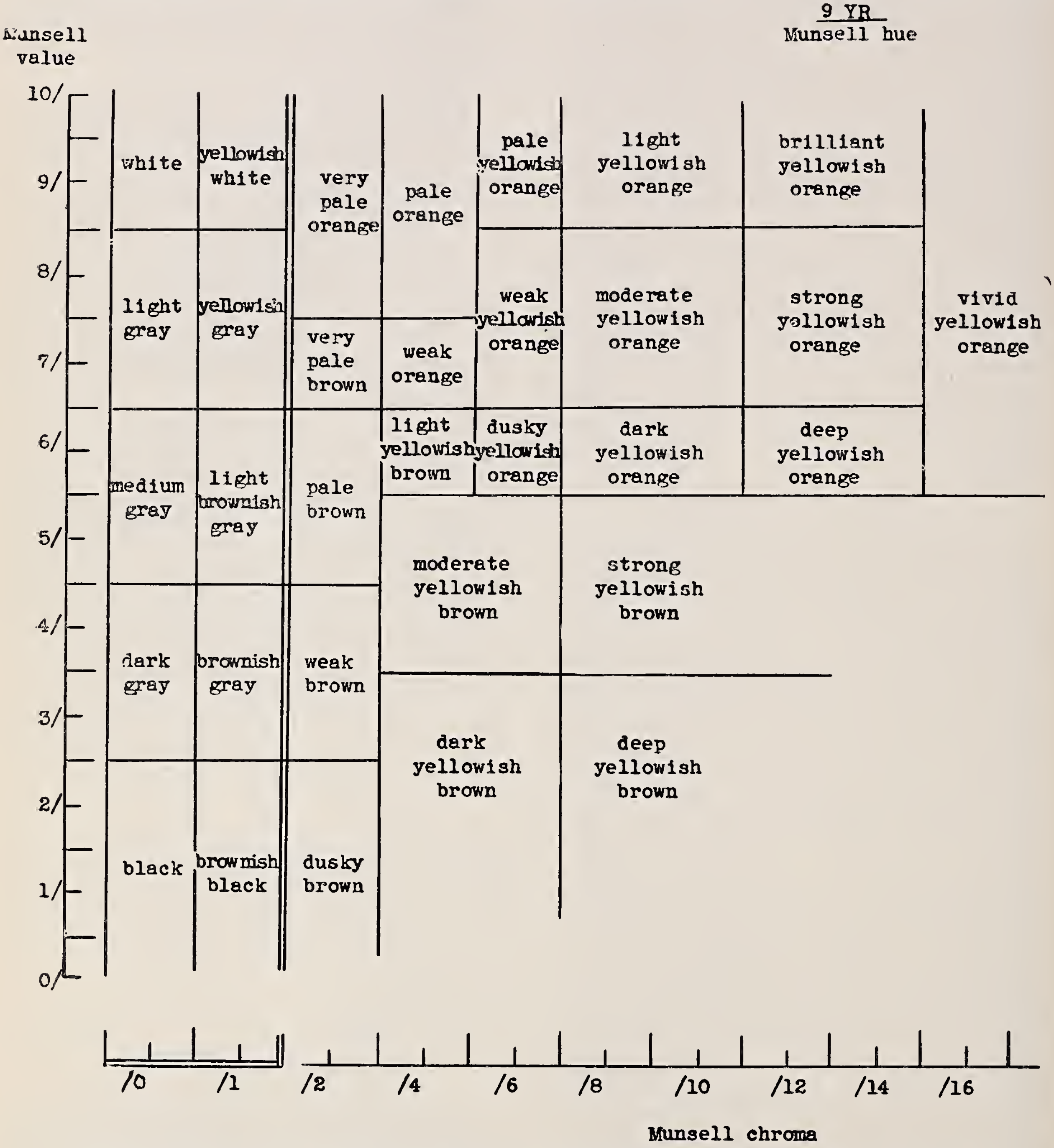

FIGURE 33. *Illustration of method of defining color designations in terms of Munsell book notations.*
The designations for all colors of Munsell book notation hue 9YR are shown in terms of Munsell value and chroma.

colors can be distinguished by the normal human eye with optimum observing conditions, the ISCC-NBS method falls far short of supplying a different designation for each distinguishable color, or even for all colors (numbering perhaps half a million) considered to be commercially different.

The plan of the method is to divide the surface-color solid (see fig. 22) arbitrarily into about 300 compartments, and assign a designation to each in as good conformity as possible to color nomenclature currently used in art, science, and industry. The compartments embracing the black-white axis are given the designations black, dark gray, medium gray, light gray, and white. The compartments adjacent to these are given similar designations formed by adding an adjective indicating the hue, such as yellowish white, dark purplish gray, or greenish black. All other compartments take designations consisting of a hue name (red, orange, yellow, green, blue, purple, pink, brown, olive) preceded by modifiers (light, dark, weak, strong) indicating the lightness and saturation of the perceived color. Table 16 shows the complete list of modifiers used in the ISCC-NBS system; note that *vivid* is an abbreviation for *very strong; pale,* an abbrevation for *light, weak; deep,* for *dark, strong,* and so on.

The boundaries between the groups of colors known by these designations have been adjusted to accord as closely as possible with common usage and have been expressed in terms of Munsell book notation. Figure 33 shows one of the charts indicating these boundaries, and figure 34 shows the shape of the compartments selected for colors of purple hue.

TABLE 16. *Modifiers of hue names in the ISCC-NBS method of designating colors*

Very pale (very light, weak) . .	Very light . . .	Very brilliant (very light, strong) . . .	
Pale (light, weak)	Light	Brilliant (light, strong) . . .	
Weak	Moderate	Strong	Vivid (very strong).
Dusky (dark, weak)	Dark	Deep (dark, strong) . . .	
Very dusky (very dark, weak) . .	Very dark . . .	Very deep (very dark, strong) . . .	

The ISCC-NBS designations are not to be considered a substitute for numerical designations of color resulting from application of a suitable colorimetric method, but they do supply a certain precision to ordinary color designations that has previously been lacking. These color designations are used in the current edition of the National Formulary [5] throughout and in some of the monographs of the U. S. Pharmacopoeia [132]. They have been used for describing the colors of building stone [69] and soils [119] and for a considerable variety of research purposes such as, for example, the description of mica colors after heat treatment [48]. They have been used throughout the text of this circular on colorimetry. Nickerson and Newhall [104] have found the Munsell book notations for the central color of each compartment assigned an ISCC-NBS color designation, and have recommended a system of abbrevations: see table 17.

TABLE 17. *Abbreviations for the terms used in the ISCC-NBS color designiations*

B=blue	lt=light	pk=pinkish
b=bluish	med=medium	R=red
Bk=black	mod=moderate	r=reddish
Br=brown	O=orange	str=strong
br=brownish	o=orange (adj)	v=very
d=dusky	Ol=olive	viv=vivid
dk=dark	ol=olive (adj)	Wh=white
G=green	P=purple	wk=weak
g=greenish	p=purplish	Y=yellow
Gr=gray	Pk=pink	y=yellowish

The ISCC-NBS designations are generally unsuited for use in sales promotion in which it is important to avoid any suggestion of weakness or adulteration. Many products have weak colors, but it is considered to be poor salesmanship to say it right out, weak red or weak green. Instead, names like Rose Ash, or Cedar, or Almond Green, are used. To illustrate this point the Munsell book notations corresponding to the ICI specifications for the glass standard of the ASTM Union colorimeter (see table 13) have been found by interpolation and extrapolation on charts like figure 25, and the corresponding ISCC-NBS color designations have been read [64] and listed in table 18. It will be noted that these designations give a satisfactory description of the colors of petroleum oil, but the term, brown, suggests mud or impurities, and would probably never be applied by an oil dealer to his own product for sales promotion; compare these designations with the corresponding names that have arisen in trade practice and used by the National Petroleum Association (see first col. of table 18). Furthermore, a light colored oil suggests extreme purity; so names emphasizing the paleness of the color tend to become current in trade; note the first eight names in the NPA list. The ISCC-NBS color designations are intended to be as descriptive as possible; they are not intended to supplant color names adopted for sales promotion.

TABLE 18. *Munsell book notations and ISCC-NBS Color Designations for the Union colors for petroleum products*

NPA names	ASTM Union Color number	Munsell book notation	ISCC-NBS Color Designation
Lily white	1	10Y 9.6/3.8	Pale greenish yellow
Cream white	1½	8Y 8.8/8.0	Light greenish yellow
Extra pale	2	3Y 7.2/12	Strong yellow
Extra lemon pale. .	2½	10YR 6.0/12	Deep yellowish orange
Lemon pale	3	7YR 5.7/13.5	Strong orange
Extra orange pale. .	3½	4YR 5.0/14	Deep orange
Orange pale	4	2YR 3.2/12	Strong brown
Pale	4½	1YR 2.6/12	Strong reddish brown
Light red	5	1YR 2.2/9.5	Deep reddish brown
Dark red	6	1YR 1.3/8.5	Dark reddish brown
Claret	7	1YR 0.6/4.5	Dusky reddish brown
. . . .	8	1YR 0.2/1.4	Reddish black

The ISCC-NBS method has been approved for color description of drugs and chemicals by the dele-

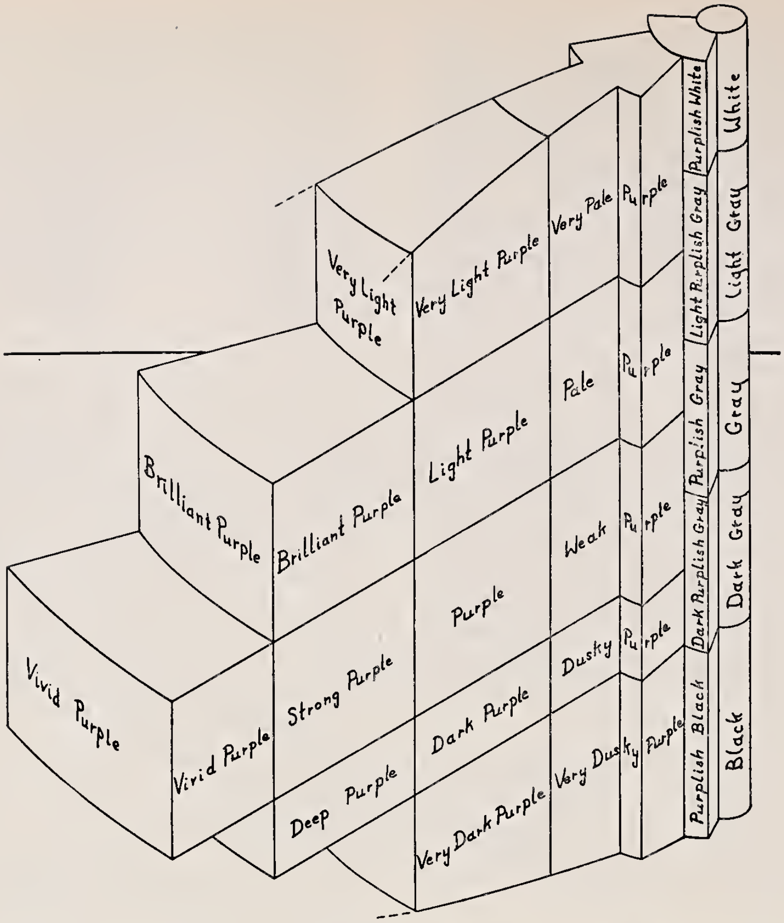

FIGURE 34. *Illustration of the tridimensional nature of the definitions of color designations.*

The approximate shape of the blocks of the surface-color solid corresponding to the various designations of purple colors in the ISCC-NBS method of designating colors. Figure 33 gives the definitions for only one vertical plane through a system of blocks such as this.

gates of nine national societies to the Inter-Society Color Council; and it has been recommended for general use by the American Standards Association [11] in the American War Standard for the Description and Specification of Color Z44-1942.

5. ASA War Standard Z44-1942 for Specification and Description of Color

In 1942 there was approved and published by the American Standards Association an American War Standard for the Specification and Description of Color under the designation ASA Z44-1942 [11]. This standard is of importance because it points the way toward agreement on basic procedures in colorimetry. Hence the text, with foreword and notes deleted, and with literature references brought up to date, is reproduced here verbatim:

> *Purpose.* To recognize and recommend a basic method for the specification of color, and to facilitate its popular interpretation.
>
> *Provisions.* (1) The spectrophotometer shall be recognized as the basic instrument in the fundamental characterization of color.
>
> (2) Color specifications computed from spectrophotometric data shall be found by means of the standard observer and coordinate system adopted in 1931 by the International Commission on Illumination [20, 44, 56].
>
> In the absence of a special reason for adopting some other illuminant in reducing spectrophotometric data, standard ICI Illuminant C, representative of average daylight, shall be used [20, 44, 56].
>
> The basic specifications of color shall consist of the tristimulus value, Y, and the trichromatic coefficients, x and y, of the ICI coordinate system, or they shall consist of the tristimulus value, Y, and the dominant wavelength and purity [44, 56].
>
> (3) For the popular identification of color, material standards may be used. The only system of material standards that has been calibrated in terms of the basic specification is represented by the 1929 edition of the Munsell Book of Color [36, 68, 91]. The use of this book is recommended wherever applicable to the specification of the color of surfaces. Approximate identifications of Munsell hue, value, and chroma may be secured by direct visual comparison with the samples in the 1929 Munsell Book of Color. When the most accurate visual comparisons are needed, the mask method [64, 65] is recommended. Wherever more exact Munsell notations are desired, they shall be found from the basic specification, Y, x, and y, by interpolation among the smoothed curves [97] for Munsell hue, value, and chroma.
>
> (4) A descriptive name according to the ISCC-NBS

system of color designation [64, 65] may be derived from the Munsell notation. This name is recommended wherever general comprehensibility is desired and precision is not important. The use of color names for color specification is not recommended.

The National Bureau of Standards cooperated in the drafting of this ASA standard. We find that it summarizes well our principal techniques for the measurement, specification, and description of color.

TABLE 19. *Relative spectral irradiance of ICI illuminants A, B, and C for colorimetry*

Wave-length	E_A	E_B	E_C	Wave-length	E_A	E_B	E_C
$m\mu$				$m\mu$			
380	9.79	22.40	33.00	580	114.44	101.00	97.80
385	10.90	26.85	39.92	585	118.08	100.07	95.43
390	12.09	31.30	47.40	590	121.73	99.20	93.20
395	13.36	36.18	55.17	595	125.39	98.44	91.22
400	14.71	41.30	63.30	600	129.04	98.00	89.70
405	16.15	46.62	71.81	605	132.70	98.08	88.83
410	17.68	52.10	80.60	610	136.34	98.50	88.40
415	19.29	57.70	89.53	615	139.99	99.06	88.19
420	21.00	63.20	98.10	620	143.62	99.70	88.10
425	22.79	68.37	105.80	625	147.23	100.36	88.06
430	24.67	73.10	112.40	630	150.83	101.00	88.00
435	26.64	77.31	117.75	635	154.42	101.56	87.86
440	28.70	80.80	121.50	640	157.98	102.20	87.80
445	30.85	83.44	123.45	645	161.51	103.05	87.99
450	33.09	85.40	124.00	650	165.03	103.90	88.20
455	35.41	86.88	123.60	655	168.51	104.59	88.20
460	37.82	88.30	123.10	660	171.96	105.00	87.90
465	40.30	90.08	123.30	665	175.38	105.08	87.22
470	42.87	92.00	123.80	670	178.77	104.90	86.30
475	45.52	93.75	124.09	675	182.12	104.55	85.30
480	48.25	95.20	123.90	680	185.43	103.90	84.00
485	51.04	96.23	122.92	685	188.70	102.84	82.21
490	53.91	96.50	120.70	690	191.93	101.60	80.20
495	56.85	95.71	116.90	695	195.12	100.38	78.24
500	59.86	94.20	112.10	700	198.26	99.10	76.30
505	62.93	92.37	106.98	705	201.36	97.70	74.36
510	66.06	90.70	102.30	710	204.41	96.20	72.40
515	69.25	89.65	98.81	715	207.41	94.60	70.40
520	72.50	89.50	96.90	720	210.36	92.90	68.30
525	75.79	90.43	96.78	725	213.26	91.10	66.30
530	79.13	92.20	98.00	730	216.12	89.40	64.40
535	82.52	94.46	99.94	735	218.92	88.00	62.80
540	85.95	96.90	102.10	740	221.66	86.90	61.50
545	89.41	99.16	103.95	745	224.36	85.90	60.20
550	92.91	101.00	105.20	750	227.00	85.20	59.20
555	96.44	102.20	105.67	755	229.58	84.80	58.50
560	100.00	102.80	105.30	760	232.11	84.70	58.10
565	103.58	102.92	104.11	765	234.59	84.90	58.00
570	107.18	102.60	102.30	770	237.01	85.40	58.20
575	110.80	101.90	100.15	775	239.37	86.10	58.50
580	114.44	101.00	97.80	780	241.67	87.00	59.10

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