UNITED STATES DEPARTMENT OF COMMERCE • Charles Sawyer, *Secretary*
NATIONAL BUREAU OF STANDARDS • E. U. Condon, *Director*



# Testing of Hydrometers

*by* Elmer L. Peffer and Mary G. Blair

National Bureau of Standards Circular 477
Issued September 1, 1949
[Supersedes Circular C16]

For sale by the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C.
Price 10 cents

## Contents

# Testing of Hydrometers[1]

By Elmer L. Peffer[2] and Mary G. Blair[3]

**Abstract**

This Circular discusses the design and construction of hydrometers, with particular reference to hydrometers that are to be submitted to this Bureau for test. The various scales that are commonly used for hydrometers are defined and recommendations given for subdividing and marking them. These scales include density, specific gravity, degrees Baumé, degrees API, percentage by weight, percentage by volume, percentage proof spirit, Brix, Balling, and some others. The relations between some of the arbitrary scales (for example, API) and specific gravity are stated.

The circular outlines the procedure of testing hydrometers that are submitted to the Bureau and discusses the forms of certificates and reports issued as a result of these tests. Instructions and other helpful information about submitting hydrometers for test are given.

## I. Introduction

The object of this Circular is to outline to manufacturers and users desirable features of design and construction of hydrometers, that will improve their usefulness and facilitate the testing of them on a uniform basis; to define briefly the various scales used for hydrometers and to specify the conditions that should be fulfilled by hydrometers submitted to this Bureau for test.

The recommendations on the construction of hydrometers apply particularly to those instruments that are to be submitted to this Bureau for certification or report.

In some cases the scale with which a hydrometer is fitted may refer to a temperature that is different from the temperature usually used as a reference for the liquid concerned. In such cases intercomparisons may be facilitated by use of the temperature correction and conversion tables published in Circular C19, Standard density and volumetric tables.

## II. Features of Hydrometer Construction

There are two general types of hydrometers, namely, hydrometers proper and hydrometers combined with a thermometer, called thermohydrometers.

### 1. Classes of Hydrometers

With reference to the indication or scale, hydrometers may be classified as follows:

(a) Density hydrometers, indicating density of a specified liquid, at a specified temperature, in specified units; for example, in grams per milliliter.

(b) Specific gravity hydrometers, indicating the specific gravity of a liquid, at a specified temperature, in terms of water at a specified temperature as unity.

(c) Percentage hydrometers, indicating, at a specified temperature, the percentage of a substance in solution with water; for example, the percentage of salts in a sample of sea water, or the percentage of sugar in a water mixture.

(d) Arbitrary scale hydrometers, indicating concentration or strength of a specified liquid referred to an arbitrarily defined scale at a specified temperature (Baumé hydrometers and API hydrometers).

### 2. Recommendations on Construction

#### (a) Materials and Form

Hydrometers should be made of smooth, transparent glass, free of bubbles, striae, or other imperfections. The glass should be of a kind that will adequately resist the action of chemical reagents and also have suitable thermal qualities to permit its use over the range of temperatures to which it may be subjected. It should be thoroughly annealed before final adjustment and sealing.

The total length of any hydrometer should not exceed 45 cm, and, for convenience in use, a length of 35 cm or less is desirable.

[1] Mr. H. S. Bean of the Bureau has been responsible for the necessary revision of the original draft of this Circular as prepared by the authors and has added the Appendix.

[2] Deceased.

[3] Present address U.S. Department of Agriculture, Southern Regional Laboratory, New Orleans, La.

835729°—49

All sections perpendicular to the axis should be circular, such that the outer surface is symmetrical about the vertical axis. There should be no uneven or unnecessary thickening of the walls and no abrupt changes nor constrictions that would hinder thorough cleaning.

The stem should be uniform in cross section with no perceptible irregularities. The top of the stem should be neatly rounded without unnecessary thickening. It should extend above the top of the scale at least 1.5 cm and should continue cylindrical below the scale for at least 3 mm.

Material used for ballast should be confined to the bottom of the instrument, and no loose material of any sort should be inside a hydrometer. The disposition of the weight should be such that the hydrometer will always float with its axis vertical. Bulbs containing shot may be flattened, but in all cases the shot must be confined in the bottom either by a partition or a cement that does not soften at usual temperatures.

Only the best quality of material should be used for scales and designating labels. Paper usually known as first-class ledger paper is suitable for this purpose. The scales and labels should be securely fastened in place by some material that does not soften at the highest temperature to which the instrument will be exposed in use, and which does not deteriorate with time. The scale should be straight and without twist.

The division lines must be perpendicular to the axis of the hydrometer, that is, horizontal when the instrument is floating. They should extend at least one-fourth around the circumference of the stem and be adjacent to or intersect a line parallel to the axis indicating the front of the scale.

The lengths of division and subdivision lines should be so chosen as to facilitate readings. Sufficient lines should be numbered to clearly indicate the reading at any point. The numbers at the ends of the scale intervals should be complete, but those intermediate may be abbreviated.

The graduation lines and inscriptions should be clear and distinct in permanent ink, such as india ink.

The distance between any two adjacent subdivision lines of the scale should never be less than 0.8 mm nor more than 3.0 mm, and a distance of 1.5 to 2.0 mm is recommended.

If flat scales are used, the length and spacing of the graduation lines should conform to recommendations given above.

To facilitate readings near the end of the hydrometer scale, the graduations should be continued a few divisions beyond the ends of the principal interval.

The scale of a hydrometer for density indications should be divided into 0.001, 0.0005, 0.0002, or 0.0001 subdivisions of the main scale reading. Similar values of the scale intervals may be used in specific gravity hydrometers. For percentage or degree indications, the hydrometer scale should be divided into whole, half, fifth, or tenth percents or degrees (never into fourths).

No hydrometer should have more than one scale, and there should be no secondary or auxiliary graduations.

With thermohydrometers, the thermometer element may be contained in the bulb or stem of the hydrometer. In either case, the capillary stem of the thermometer should be parallel to the hydrometer axis. Other desirable features of the thermometer element are: the stem should contain an enlargement that will permit its being heated to 120° C. The space above the mercury should be either evacuated or filled with an inert gas. White-back capillary tubes should not be used. The thermometer scale should be divided into whole or half degrees, and the division lines should extend behind and on both sides of the thermometer capillary. It is desirable that the thermometer scale should include the ice point (0° C). This scale should not extend beyond the cylindrical portion of the containing tube (that is, beyond the bulb or stem of the hydrometer) nor beyond the uniform portion of the thermometer capillary. If within the (hydrometer) stem, it should be above the hydrometer scale.

The hydrometer should be thoroughly dry on the inside when sealed.

#### (b) Inscriptions

The hydrometer scale or a suitable special label should bear an inscription that indicates unequivocally the purpose of the instrument. This inscription should denote the liquid for which the instrument is intended, the temperature at which it is to be used, and the character of the indication, including definition of any arbitrary scale employed.

The designation of standard temperature and reference temperature may be abbreviated, as, for example, sp gr 15°/15° C, meaning that the hydrometer indicates at 15° C the specific gravity of the liquid referred to water at 15° C as unity.

Hydrometers submitted to this Bureau for testing must show upon the scale or label, the maker's name or trade mark, the year of manufacture and an individual identifying number. This last is especially important, since it will be the only means by which the instrument may be associated with its certificate or report. Therefore, manufacturers must be particularly careful not to duplicate their identification numbers, although these numbers may be associated with the year.

#### (c) Tolerances

The greatest error of the hydrometer scale should not exceed the following:

| Smallest subdivision of hydrometer scale | Limit of error |
|---|---|
| Fifth, tenth, or twentieth percents or degrees or any density subdivision. | One smallest division. |
| Whole or half percents of proof spirit. | One smallest division. |
| Whole or half percents or degrees (other than above). | One-half smallest division. |

The greatest error of thermometer scales in thermohydrometers should not exceed the following:

| Smallest division of thermometer scale | Limit of error |
|---|---|
| 2.0° F | 1.0° F |
| 1.0° C | 0.5° C |
| 0.5° C | 0.3° C |

There should be no apparent irregularities in either the hydrometer scale or the thermometer scale.

## 3. Reference Tables for Hydrometer Scales

The tables used as a basis in testing most of the hydrometers submitted to this Bureau are contained in Bureau Circular C19 (6th edition), Standard density and volumetric tables, and are referred to below by the numbers used in that circular.

### (a) Alcoholometers

Alcoholometers may be graduated to indicate the percentage of ethyl alcohol, either by weight or by volume, in mixtures of ethyl alcohol and water, or they may be graduated to indicate percentages of "proof spirit" as defined by the Bureau of Internal Revenue, United States Treasury Department.[1]

### (b) Saccharometers

Saccharometers should be graduated to indicate percentage of sugar according to weight [2] or degrees Baumé at 20° C.

The basis for graduation of saccharometers, standard at 20° C, should be density at 20° C, as given in table 12 of NBS Circular C19.

Balling saccharometers should be graduated to indicate the percentage of sugar by weight at 60° F, and Brix saccharometers the percentage of sugar by weight at the temperature indicated.

Degrees Baumé for sugar is defined by the formula,[3]

$$\text{degrees Baumé at } 20°\text{ C} = 145 - \frac{145}{\text{sp gr } 20°/20°\text{ C}}.$$

### (c) Hydrometers for Sulfuric Acid

The basis for the graduation of hydrometers indicating percentage of sulfuric acid by weight in mixtures of acid with water should be density at 20° C as given in table 13 of Circular C19.

### (d) Baumé Hydrometers

The basis for hydrometers indicating degrees Baumé of liquids lighter than water should be the relation to specific gravity at 60°/60° F= 15.56°/15.56° C given by the formula:

$$\text{degrees Baumé} = \frac{140}{\text{sp gr } 60°/60°\text{ F}} - 130,$$

as given in tables 22 and 23 of Circular C19.

For hydrometers indicating degrees Baumé of liquids heavier than water, the basis should be the relation to specific gravity at 60°/60° F= 15.56°/15.56° C given by the formula

$$\text{degrees Baumé} = 145 - \frac{145}{\text{sp gr } 60°/60°\text{ F}},$$

as given in tables 20 and 21 of Circular C19.

The liquids for standardization of Baumé hydrometers should be, in general, mineral oils for light liquids, mixtures of sulfuric acid and water for heavier liquids, the Thoulet solution for liquids heavier than sulfuric acid.

### (e) Hydrometers for Petroleum Oil

The hydrometers in general use in the petroleum oil industry in the United States are based on the modulus 141.5, which is the basis for the API scale (American Petroleum Institute Scale).

The formula for converting specific gravity to API is as follows:

$$\text{degrees API} = \frac{141.5}{\text{sp gr } 60°/60°\text{ F}} - 131.5,$$

as given in tables 23 and 24 of Circular C19.

### (f) Density Hydrometers

Hydrometers indicating density should be graduated to indicate, at the temperature marked on the scale, the density of liquids in grams per milliliter.

[1] "Proof spirit" is that alcoholic liquor that contains one-half of its volume of pure ethyl alcohol of a specific gravity of 0.7939 at 60° F., referred to water at 60° F. as unity. "Gauging Manual," 1941, U. S. Treasury Department, Bureau of Customs, page 75. (The percentage of proof spirit is in every case twice the percentage of ethyl alcohol by volume at 60° F.)

[2] Both of the terms, "Brix" and "Balling," are interpreted as meaning the percentage, by weight of pure sucrose. The relation between percentage of sugar and density used by this Bureau is that determined by F. Plato. (Wiss. Abh. Kaiserlichen Normal-Eichungs-Kommission **2**, 153; 1900).

[3] See NBS Tech. Pap. T115, New Baumé scale for sugar solutions; and NBS Circular C440, Polarimetry, saccharimetry, and the sugars.

### (g) Specific Gravity Hydrometers

The scales of hydrometers for indicating specific gravity should be graduated to indicate the ratio of the density of the liquid at a specified temperatue (usually 60° F or 20° C) to the density of water at a specified temperature (4° C, 60° F, or 20° C) as unity. Normally these scales will apply to a particular liquid and specific gravity range. The following are some of the common liquids and solutions and the approximate range of their specific gravities.

| Liquids | Range | Liquids | Range |
|---|---|---|---|
| Mineral oil | 0.62 to 1.00 | Caustic soda | 1.00 to 1.55 |
| Ammonia | .85 to 1.00 | Nitric acid | 1.00 to 1.55 |
| Common salt | 1.00 to 1.23 | Sulfuric acid | 1.00 to 1.85 |
| Hydrochloric acid | 1.00 to 1.25 | Thoulet solution ($K_2HgI_4$) | 1.84 to 3.00 |

### (h) Thermohydrometers

Hydrometers containing thermometers should not contain temperature "correction tables", as such tables can, in general, be correct for only a very limited portion of the hydrometer scale. Accurate correction tables for the more common liquids (sugar, sulfuric acid, ethyl alcohol, and mineral oils) are given in Circulars C19 and C410.

## III. Manipulation of Hydrometers

Hydrometers are seldom used for the greatest accuracy, as the usual conditions under which they are used preclude such special manipulation and exact observation as are necessary to obtain high precision. It is, nevertheless, desirable that they be as accurately graduated as is practical so as to avoid, as far as possible, the use of instrumental corrections. To obtain this result, it is helpful to employ certain precautions and methods in standardizing these instruments.

The methods of manipulation described below are, in general, the ones employed at this Bureau in testing hydrometers, and it may be helpful for a maker or user to follow them to such extent as his needs for accuracy require.

### 1. Observing

The hydrometer should be clean, dry, and at the temperature of the liquid before immersing to make a reading.

The liquid in which the observation is made should be contained in a clear, smooth glass vessel of suitable size and shape.

The liquid should be thoroughly mixed by means of a stirrer reaching to the bottom of the vessel. In the hydrometer comparator, this is accomplished by the propeller, *E*, in figure 1.

The hydrometer should be slowly immersed in the liquid slightly beyond the point where it floats naturally and then allowed to float freely.

The scale reading should not be made until the liquid and hydrometer are free from bubbles and at rest.

In reading the hydrometer scale the eye is placed slightly below the plane of the surface of the test liquid; it is raised slowly until the surface, seen as

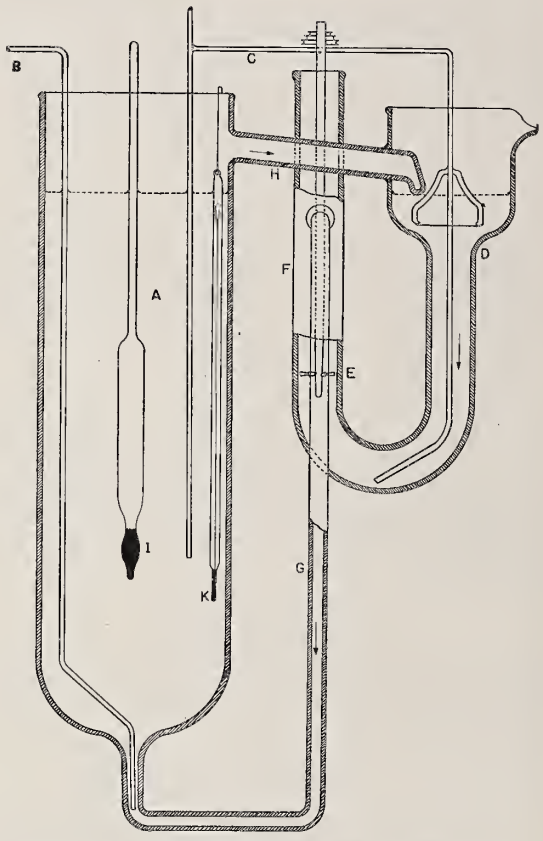


FIGURE 1. *Section of hydrometer comparator.*

*A*, Cylinder containing the test liquid; *B*, glass tube for filling and emptying *A*; *C*, siphon for removing the liquid from *D* into *A*; *E*, propeller that stirs the liquid and raises it in the tube, *F*, making it flow through *G* into *A* and through the cross tube, *H*, into *D*; *I*, hydrometer; *K*, thermometer.

an ellipse, becomes a straight line. The point where this line cuts the hydrometer scale should be taken as the reading of the hydrometer.

In reading the thermometer scale, errors of parallax are avoided by so placing the eye that near the end of the mercury column the portions of the line on either side of the stem and that portion seen through the capillary appear to lie in a straight line. The line of sight is then normal to the stem.

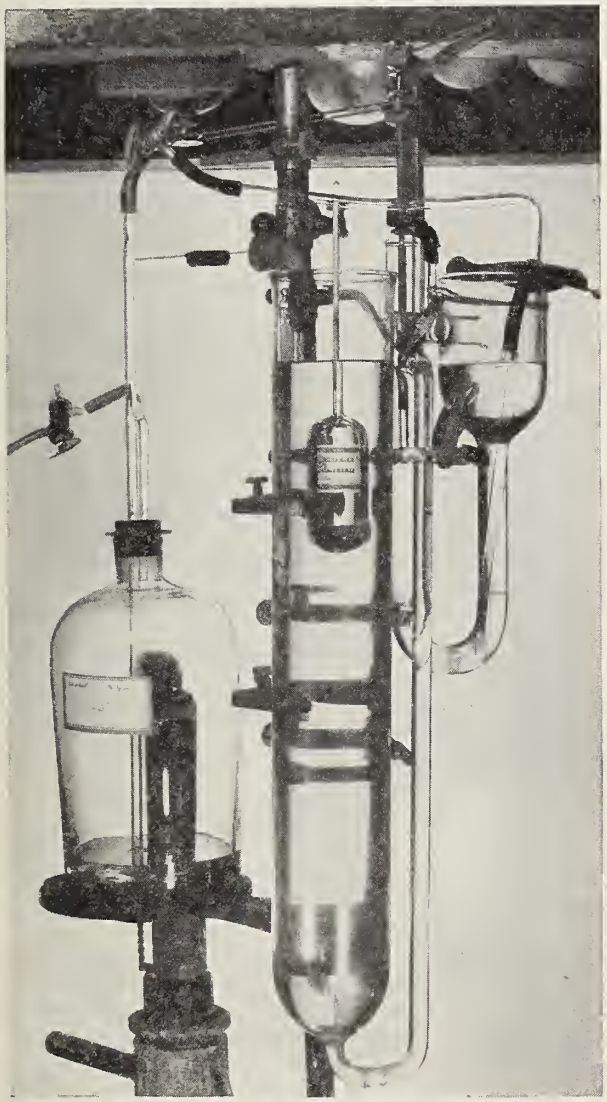

FIGURE 2. *Hydrometer comparator.*

## 2. Influence of Temperature

In order that a hydrometer may indicate correctly the density or strength of a specified liquid it is essential that the liquid be uniform throughout and at the standard temperature.

To insure uniformity in the liquid, stirring is required shortly before making the observation. This stirring should be complete and may be accomplished by a perforated disk or spiral at the end of a rod long enough to reach the bottom of the vessel. Motion of this stirrer from top to bottom serves to disperse layers of the liquid of different density. Or, if using a hydrometer comparator, the stirring can be accomplished by means of the propeller, *E*, in figure 1.

The liquid should be at nearly the temperature of the surrounding atmosphere, as otherwise its temperature will be changing during the observation, causing not only differences in density but also doubt as to the actual temperature. When the temperature at which the hydrometer is observed differs from the standard temperature of the instrument, the reading is not truly the density according to the basis of the instrument or the quality of the liquid according to percentage or arbitrary scale, but a figure that differs from the normal reading by an amount depending on the difference in temperature and on the relative thermal expansions of the instrument and the particular liquid.

If the latter properties are known, tables of corrections for temperature may be prepared for use with hydrometers at various temperatures. Such tables should be used with caution, and only for approximate results when the temperature differs much from the standard temperature or from the temperature of the surrounding air. (See table 28, Circular C19.)

## 3. Influence of Surface Tension

Surface-tension effects on hydrometer observations are a consequence of the downward force exerted on the stem by the curved surface, or meniscus, which rises about the stem and affects the depth of immersion and consequent scale reading.

Because a hydrometer will indicate differently in two liquids having the same density but different surface tensions, and since surface tension is a specific property of liquids, it is necessary to specify the liquid for which a hydrometer is intended.

Although hydrometers of equivalent dimensions may be compared, without error, in a liquid differing in surface tension from the specified liquid, comparisons of dissimilar instruments in such a liquid must be corrected for the effect of the surface tension.

In many liquids spontaneous changes in surface tension occur due to the formation of surface films of impurities, which may come from the apparatus, the liquid, or the air.

Errors from this cause may be avoided by the use of liquids not subject to such changes, although this will require making a correction to the results for surface tension when dissimilar instruments

are compared, as mentioned above. A second method of avoiding these errors is to purify the surface of the test liquid before making an observation, by causing an overflow of the liquid. This method is employed at this Bureau for testing hydrometers in sulfuric-acid, sodium chloride, and alcohol solutions, and is accomplished by causing the liquid to overflow from the part of the apparatus in which the hydrometer is immersed by a small rapidly rotating propeller, which serves also to stir the liquid.

The necessity for such special manipulation is confined to the reading of hydrometers in liquids that are subject to surface contamination. Such, in general, are aqueous solutions or mixtures of acids, alkalies, salts, sugar, and weak alcoholic mixtures. Oils, alcoholic mixtures of strength above 40 percent by volume, and other liquids of relatively low surface tension are not, in general, liable to surface contamination sufficient to cause appreciable changes in hydrometer readings.

### 4. Cleanliness

The accuracy of hydrometer observations depends, in many cases, upon the cleanliness of the instruments and of the liquids in which the observations are made.

In order that readings shall be uniform and reproducible, the surface of the hydrometers, and especially of the stem, must be clean, so that the liquid will rise uniformly and merge into an imperceptible film on the stem.

The readiness with which this condition is fulfilled depends somewhat upon the character of the liquid. Certain liquids, such as mineral oils and strong alcoholic mixtures, adhere to the stem very readily. On the other hand, with weak aqueous solutions of sugar, salts, acids, and alcohol, scrupulous cleaning of the stem is required in order to secure the normal condition.

Before being tested, hydrometers are thoroughly cleaned by dipping in a mixture of concentrated sulfuric acid and fuming sulfuric acid, rinsed with water, and dried by wiping with a clean, lint-free cloth.

If they are to be used in aqueous solutions that do not adhere readily, the stems are wiped with alcohol and dried immediately before each reading.

## IV. Tests Performed by the Bureau

### 1. Instruments Admitted for Test

At present the Bureau is prepared to test the following kinds of hydrometers and thermohydrometers: Alcoholometers, saccharometers, salinometers, lactometers,[4] API hydrometers, Baumé hydrometers, hydrometers indicating density or specific gravity of liquids lighter than water or heavier than water, hydrometers indicating percentage of sulfuric acid.

Instruments other than the above should not be submitted for test without previous arrangement.

### 2. Description of Tests

The testing of hydrometers comprises the following operations: general inspection; testing at three or more scale points; affixing the precision or report stamp; preparation of certificates or reports.

#### (a) General Inspection

The general inspection includes noting whether the scale is fully defined and suitably marked, as described in section II, 2. It also includes noting whether the instrument stem is vertical when the instrument is placed in a test solution.

#### (b) Points Tested

When tests are made at three scale points, the points shall include at least 60 percent of the graduated interval of the scale. Neither of the extreme points shall be farther from the nearest end of the graduated scale than a distance represented by 25 percent of the length of the graduated scale. No two adjacent points shall be farther apart than a distance represented by 50 percent of the length of the graduated scale.

Tests will be made at more than three points when this has been requested by the party submitting the instrument at the time of requesting the test. (An additional fee is charged for each point tested over three.)

#### (c) Precision Stamp and Certificate

If at all points tested the results are within the limits given in the section on tolerances, the official precision stamp, consisting of the letters NBS and year, as shown below, is etched on the instrument, and a certificate is issued giving the corrections to the scale readings as determined by the tests.


#### (d) Report of Corrections

When the results of the test show that at one or more of the points tested the corrections exceed the tolerance for precision grade but do not

[4] The corrections to lactometer readings as reported by the Bureau apply to readings taken at the top of the milk meniscus.

exceed more than one-half of a numbered division (usually five times the precision tolerance), a report will be issued giving the corrections and a statement explaining in what respect the hydrometer does not meet the requirements for a certificate. Hydrometers on which reports are issued will receive the reported stamp, such as shown below, to show that they have been tested by this Bureau and, while not meeting the precision requirements, are of serviceable quality.


If the correction at one or more test points exceeds one-half of a numbered division, or if for some other reason the hydrometer is found to be unsuited for the grade of use intended, neither certificate nor report is issued. Also, there is no NBS stamp applied to the instrument. The results of the test and reason for the action taken may be given in a letter.

#### (e) Weight in Air

In addition to making the tests outlined above, the weight in milligrams of the hydrometer against brass weights in air at 760 mm of mercury pressure and 20° C, will be determined and reported if the sender so requests. (There is an additional charge for this.)

## V. Directions for Submitting Apparatus for Test

### 1. Application for Test

The request for test should be made in writing and should include a complete list of the apparatus and a statement of the nature of the test desired as explained in section IV, 2.

The sender should always examine apparatus carefully before submitting it for test to determine whether or not it complies with the recommendations and is qualified for the test desired

### 2. Identification Marks

Instruments and the packages in which they are shipped should both be plainly marked to facilitate identification, preferably with the name of the manufacturer or shipper, and a special reference number should be given, which should be referred to in the correspondence concerning the test. After the Bureau has acknowledged the request for the test and assigned to it a test number, this test number should be given on any correspondence about the test.

### 3. Shipping Directions

Instruments should be securely packed in cases or packages, which may be used in returning them to the owner. In all cases, transportation charges are payable by the party desiring the test and must be prepaid. Return shipment is made by express, collect unless other arrangements are made in advance. Apparatus must be accompanied by an itemized shipping invoice.

### 4. Breakage

About 3.0 percent of the hydrometers shipped to this Bureau for test are broken when received. In most cases this is due to inadequate protective packing, although rough handling in shipment is undoubtedly a contributing factor. About 0.5 percent of the hydrometers received are broken during the testing operations in the laboratory. There is insufficient information at hand on which to estimate the breakage in return shipments. While the Bureau endeavors to keep the laboratory and return shipment breakage to a minimum, such breakage must be viewed as a part of the cost of submitting hydrometers for test.

### 5. Address

Articles should be addressed, "National Bureau of Standards, Washington 25, D. C." Articles delivered in person or by messenger may be left at the Shipping Room of the Bureau, or at the laboratory if the submitter wishes to discuss the tests in which he is interested. In either case, there should be a written request for the tests desired.

### 6. Remittances

Payment of test fee should be made promptly upon receipt of bill, as certificates or reports are not mailed until the fees have been paid. Remittances may be made by money order or check drawn to the order of the National Bureau of Standards.[5]

Copies of Test Fee Schedule 202.403, "Fees for testing hydrometers and thermohydrometers," may be obtained from the Bureau upon request.

[5] In the case of apparatus submitted by or upon the request of a Federa or State agency, for whom the Bureau is directed to perform tests without charge, a fee bill for testing rejected apparatus and for supplying any necessary identification numbers may be rendered to the concern furnishing the apparatus.

## VI. Appendix, Relations Between Principal Scale Interval and the Dimensions of Hydrometers

Referring to figure 3, let

$V$ = total volume ($cm^3$) of hydrometer from top scale mark to bottom of bulb
$v$ = volume ($cm^3$) of stem per unit length
$m$ = total mass (g) of hydrometer
$D$ = diameter (cm) of the bulb
$d$ = diameter (cm) of the stem
$H$ = length (cm) of bulb
$L$ = length (cm) of principal interval of scale in the stem (that is, distance between the two extreme marked graduation lines)
$h$ = distance (cm) between top of bulb and bottom of scale
$s_1$ and $s_2$ = distances (cm) from the top (zero) scale line to any two readings.

If the hydrometer is placed in a liquid of density $\rho_0$, it will sink to the zero (top) scale mark, hence

$$m=V\rho_0. \tag{1}$$

If placed in a second liquid having a density $\rho_1$, ($\rho_1>\rho_0$) and sinks only to position $s_1$, then

$$m=(V-s_1v)\rho_1. \tag{2}$$

If a third liquid is used having a density $\rho_2$, ($\rho_2>\rho_1$) the reading position will be $s_2$, and

$$m=(V-s_2v)\rho_2. \tag{3}$$

From eq 1 and 2,

$$V\rho_0=(V-s_1v)\rho_1$$

and

$$s_1=\frac{V}{v}\left(\frac{\rho_1-\rho_0}{\rho_1}\right); \tag{4}$$

likewise, from eq 1 and 3,

$$s_2=\frac{V}{v}\left(\frac{\rho_2-\rho_0}{\rho_2}\right), \tag{5}$$

and by eq 4 and 5,

$$s_1-s_2=\frac{V}{v}\left[\left(\frac{\rho_1-\rho_0}{\rho_1}\right)-\left(\frac{\rho_2-\rho_0}{\rho_2}\right)\right]. \tag{6}$$

Now assume that the densities $\rho_1$ and $\rho_2$ are going to be so chosen that the positions $s_1$ and $s_2$ will coincide with the highest and lowest principal graduation lines of the scale. This means that $\rho_1=\rho_0$, the distance to $s_1$ is 0 and to $s_2$ is $L$, and, therefore, eq 6 reduces to

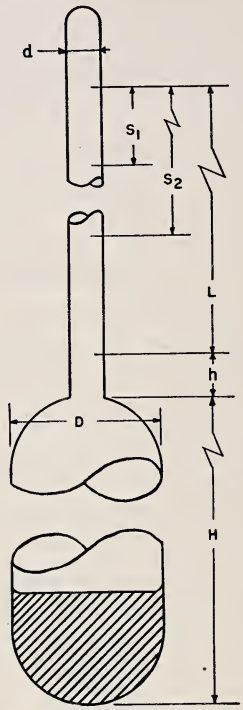


FIGURE 3. *Principal dimensions of a hydrometer.*

$$s_1-s_2=0-s_2=-L=\frac{V}{v}\left[\left(\frac{\rho_1-\rho_1}{\rho_1}\right)-\left(\frac{\rho_2-\rho_1}{\rho_2}\right)\right].$$

or

$$L=\frac{V}{v}\left(\frac{\rho_2-\rho_1}{\rho_2}\right). \tag{7}$$

If the ends of the bulb are assumed to be spherical segments approximately $D/3$ cm in height, the volume, $V$, of the hydrometer is

$$V=\frac{\pi}{4}D^2(H-0.3D)+\frac{\pi}{4}d^2h+\frac{\pi}{4}d^2L. \tag{8}$$

The 0.3 factor in $(H-0.3D)$ is a rounded value

but is sufficiently exact for most purposes. Also, since by definition, its length is unity,

$$v=\frac{\pi}{4}d^2. \tag{9}$$

Therefore,

$$\frac{V}{v}=\frac{D^2}{d^2}(H-0.3D)+h+L \tag{10}$$

and with eq 7

$$L\left(\frac{\rho_2}{\rho_2-\rho_1}\right)=\frac{D^2}{d^2}(H-0.3D)+h. \tag{11}$$

In many cases it may be more convenient to use specific gravity, $g$, in place of density. This will not change the numerical value of the density term; hence, in place of eq 11 we may write

$$L\left(\frac{g_2}{g_2-g_1}\right)=\frac{D^2}{d^2}(H-0.3D)+h. \tag{12}$$

Returning to eq 6, if $\rho_2$ is the density corresponding to the lowest scale mark, then $s_2=L$ and eq 6 may be written

$$L-s_1=\frac{V}{v}\rho_0\left(\frac{\rho_2-\rho_1}{\rho_1\rho_2}\right). \tag{14}$$

Having determined the dimensions of the bulb and stem, we have established the values of $V$ and $v$, so that $V/v$ is a constant. Also, for any given range, $\rho_0$ is a constant. Therefore, the density term $(\rho_2-\rho_1)/\rho_1\rho_2$, governs the distance between the graduation lines on the scale. Furthermore, as $\rho_1$ is increased by equal increments from $\rho_0$ to $\rho_2$, the value of the density term changes by decreasing increments. This means that for equal changes in density the distance between graduation marks on the scale will be smaller near the bottom of the scale than at the top of the scale.

In order to use either eq 11 or 12 to determine the proportions of a hydrometer, it will be necessary to assign values to enough of the variables, or to use additional relations between some of them, so as to make it possible to solve for one variable. One additional relation that may be used is

$$\text{total length}=H+h+L. \tag{13}$$

Thus, we may assign values to total length, $D$, $d$, and $h$. Then for any range of specific gravities that it is desired to have the hydrometer cover, we may solve for $H$ and $L$.

*Example:*

$$\text{Total length}=21.0 \text{ cm}$$
$$D=3.0 \text{ cm}$$
$$d=0.6 \text{ cm}$$
$$h=1.0 \text{ cm}$$
$$g_1=0.95178$$
$$g_2=1.0000$$

Then by eq 13

$$H=20-L$$

and by eq 12

$$19.738L=25(19.91-L)+1$$

or

$$L=11.15 \text{ cm}$$

and

$$H=8.85 \text{ cm}$$

Washington, March 21, 1949.



U. S. GOVERNMENT PRINTING OFFICE: 1949