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[Supersedes Circular C415]

MAGNETIC TESTING

By Raymond L. Sanford

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PREFACE

The first edition of Circular C17, Magnetic Testing, was published in 1909, and the second edition appeared the following year. In these editions primary attention was given to descriptions of apparatus and testing procedure in use at the National Bureau of Standards. As the use of magnetic materials and consequently the importance of magnetic testing continued to increase, many requests were received for information regarding magnetic quantities and the units in which the results of magnetic tests are expressed. The third and fourth editions of the circular, therefore, were enlarged in scope to include brief discussions of magnetic quantities and units.

Circular C415, issued in 1937, superseded Circular C17, and in addition to information on magnetic quantities, units and testing methods gave a brief discussion of magnetic analysis by which the mechanical quality of magnetic materials may be judged from observations of their magnetic characteristics. The present circular has been revised and somewhat enlarged to include descriptions of new methods and apparatus which have been developed since 1937.

E. U. Condon, *Director*.

MAGNETIC QUANTITIES, UNITS, SYMBOLS, AND EQUATIONS

Magnetic Quantities and Units

Quality	Symbol	Name of Unit
Magnetomotive force	$\mathscr{F}$	gilbert
Magnetic flux	φ	maxwell
Magnetic reluctance	$\mathscr{R}$	
Magnetizing force	H	oersted
Magnetic induction	B	gauss
Magnetic permeability	μ	
Intensity of magnetization	J	
Magnetic susceptibility	κ	

Equations

$$\mathscr{F}=0.4\,\pi\,NI$$

$$e=N\frac{d\varphi}{dt}10^{-8}$$

$$\varphi=\mathscr{F}/\mathscr{R}$$

$$H=d\mathscr{F}/dl$$

$$B=\varphi/A$$

$$\mu=B/H$$

$$H=0.4\,\pi\,nI$$

$$J=(B-H)/4\,\pi$$

$$\kappa=J/H$$

N=number of turns
I=current in amperes
e=electromotive force in volts
A=cross section in square centimeters

n=number of turns per centimeter
l=length in centimeters
t=time in seconds

NOTE.—In this Circular the term "csg unit" refers to the csg electromagnetic system.

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ABSTRACT

This Circular gives general information regarding magnetic quantities, the magnetic characteristics of materials, the principles employed in magnetic testing apparatus, and a brief discussion of the theory and application of magnetic analysis.

CONTENTS

I. INTRODUCTION

The work of the National Bureau of Standards in magnetic testing includes the (a) investigation and development of testing methods; (b) testing of magnetic materials; and (c) investigations in the field of magnetic analysis.

Investigations of testing methods are carried out for the purpose of determining the possibilities and limitations of existing methods with particular reference to present requirements. The development of new methods and apparatus is undertaken when possible if existing methods are found to be inadequate.

Magnetic testing is done in connection with investigations involving the magnetic properties of materials. A small amount of magnetic testing is also done for the public. This service is limited in general to tests made for the purpose of checking the accuracy of testing apparatus or for the settling of disputes arising from disagreement between different laboratories. Routine acceptance tests which can be made satisfactorily by commercial testing laboratories should not be requested except for special reasons acceptable to the Bureau. A fee is charged for routine magnetic tests made for the public.

In the field of magnetic analysis, the Bureau is primarily concerned with the study of possible relationships between the magnetic properties of materials and their other physical properties with the end in view of aiding in the development of nondestructive methods of testing. Some attention is also given to the design of special apparatus for the application of magnetic analysis in a practical way.

This Circular gives general information regarding magnetic quantities, the magnetic characteristics of materials, the principles employed in magnetic testing apparatus, and a brief discussion of the theory and application of magnetic analysis.

II. MAGNETIC QUANTITIES AND UNITS

1. MAGNETOMOTIVE FORCE

In magnetic testing, magnetization is produced by means of electric current in windings surrounding the magnetic circuit of which the specimen forms a part. The total magnetizing influence is proportional to the product of the current by the number of turns and is called the magnetomotive force, $\mathscr{F}$. The cgs unit of magnetomotive force is called the *gilbert* and is defined by the equation

$$\mathscr{F}=0.4\pi NI,$$

in which

$\mathscr{F}$=magnetomotive force in gilberts
N=number of turns
I=current in amperes.

2. MAGNETIC FLUX

The effect of a magnetomotive force is to produce, within the medium or material acted upon, a condition called magnetic flux, ϕ. This quantity is characterized by the fact that an electromotive force is induced in a conductor surrounding the magnetic flux during any time that the flux changes in magnitude. The induced electromotive force is proportional to the time rate of change of magnetic flux. The cgs unit of magnetic flux is called the *maxwell* and is defined in terms of induced electromotive force by the equation

$$e=N(d\phi/dt)10^{-8},$$

in which

e=induced electromotive force in volts
N=number of turns surrounding the flux
ϕ=magnetic flux in maxwells
t=time in seconds.

3. MAGNETIC RELUCTANCE

The amount of magnetic flux resulting from a given magnetomotive force acting on a magnetic circuit is determined by the *magnetic reluctance* $\mathscr{R}$, of the circuit. In terms of the so-called "Ohm's law of the magnetic circuit" the relationship is expressed by the equation

$$\phi=\mathscr{F}/\mathscr{R},$$

in which

ϕ=magnetic flux in maxwells
$\mathscr{F}$=magnetomotive force in gilberts
$\mathscr{R}$ =magnetic reluctance in cgs units.

This relationship serves to define the cgs unit of magnetic reluctance.[1]

4. MAGNETIZING FORCE

The total magnetomotive force acting on a magnetic circuit is distributed along its length in a manner determined by the distribution of the magnetizing winding and the reluctance of the circuit. At any given point in the circuit the magnetomotive force per unit length is called the *magnetizing force, H*. At the center of a very long

[1] Previous to 1930 this unit was called the oersted, but the International Electrotechnical Commission, in 1930, adopted the name oersted for the unit of magnetizing force, leaving the unit of reluctance without a name.

uniformly wound solenoid having n turns per centimeter in which a current of I amperes is flowing, the magnetizing force is

$$H=0.4\pi nI.$$

The cgs unit is called the oersted and is defined by the equation

$$H=d\mathscr{F}/dl,$$

in which

H=magnetizing force in oersteds
$\mathscr{F}$=magnetomotive force in gilberts
l=length in centimeters.

5. MAGNETIC INDUCTION

Magnetic induction, B, also called *magnetic flux density*, is the flux per unit area of a section normal to the direction of the flux. The cgs unit is called the gauss and is defined by the equation

$$B=d\phi/dA,$$

in which

B=magnetic induction in gausses
ϕ=magnetic flux in maxwells
A=area in square centimeters.

6. INTRINSIC INDUCTION

That part of the induction in excess of the induction in a vacuum for the same magnetizing force is called the *intrinsic inducaion*, B_i. Numerically,

$$B_i=B-H.$$

7. MAGNETIC PERMEABILITY

The ratio of the magnetic induction in a given medium to the induction which would be produced in a vacuum by the action of the same value of magnetizing force is the *magnetic permeability* of the medium. In the cgs system, the induction in a vacuum is numerically equal to the magnetizing force and consequently the magnetic permeability of a medium is *numerically* equal to the ratio of the magnetic induction to the magnetizing force. Thus

$$\mu=B/H,$$

in which

μ=magnetic permeability
B=magnetic induction in gausses
H=magnetizing force in oersteds.

The term "magnetic permeability" as thus defined applies only to the normal induction curve illustrated in figure 1, and strictly speaking, should be called *normal permeability*. Various other relationships between magnetic induction and magnetizing force are called permeability, for example, incremental permeability.

8. INCREMENTAL PERMEABILITY

If a material is magnetized by the combined action of a steady and an alternating magnetizing force, the ratio of the cyclic change in induction to the corresponding cyclic change in magnetizing force is called the *incremental permeability*, μ_Δ. The incremental permeability depends upon the degree of magnetization produced by the steady component of magnetizing force and the amount of change of induction due to the alternating component of magnetizing force.

9. INTENSITY OF MAGNETIZATION

A unit sometimes used instead of intrinsic induction is called *intensity of magnetization*, J. Its relation to intrinsic induction is

$$J=B_i/4\pi.$$

No name has been given to this unit.

10. MAGNETIC SUSCEPTIBILITY

The ratio of intensity of magnetization to magnetizing force is called the *magnetic susceptibility*, κ.

$$\kappa=J/H.$$

This quantity is generally used to express the properties of feebly magnetic substances and is sometimes referred to as volume susceptibility. Susceptibility referred to unit mass, called mass susceptibility, is obtained by dividing the volume susceptibility by the density of the material and is designated by the symbol χ.

III. MAGNETIC CHARACTERISTICS OF MATERIALS

1. CLASSIFICATION OF MATERIALS

Materials are classified with respect to their magnetic properties as ferromagnetic, paramagnetic, or diamagnetic. Ferromagnetic materials have a variable permeability greater than 1, usually very large and the intrinsic induction approaches a definite limit with increasing values of magnetizing force. They also exhibit the phenomenon of magnetic hysteresis. Paramagnetic materials have a constant permeability slightly greater than 1. Diamagnetic materials have a constant permeability less than 1. The magnetic susceptibility of diamagnetic materials is negative.

2. INDUCTION

The magnetic properties of ferromagnetic materials are commonly expressed in terms of corresponding values of magnetic induction, B, and magnetizing force, H, or quantities derived from them. A typical magnetization curve is shown in figure 1. It is to be noted that the induction is not directly proportional to the magnetizing force. In other words, the magnetic permeability does not have a constant value but depends upon the degree of magnetization. This is one of the distinguishing characteristics of ferromagnetic materials. As the magnetizing force is increased from a zero value, the magnetization

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proceeds in three more or less distinct stages. In the first stage, the induction increases with increase of magnetizing force at a comparatively slow rate. In the second stage, the rate of increase is much greater, while in the third stage the induction increases at a continually decreasing rate. The limits of the three stages are more or less definite according to the nature of the material.

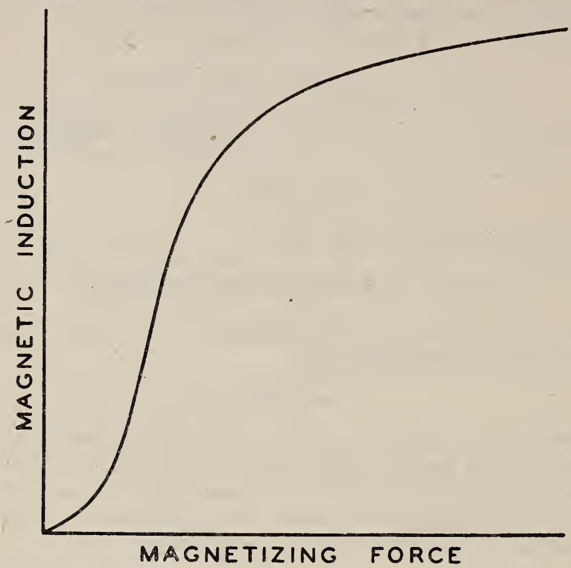

FIGURE 1.—*Typical magnetization curve.*

3. HYSTERESIS

The phenomenon of magnetic hysteresis is illustrated in figure 2. If the magnetizing force is decreased from some value, the induction does not follow the magnetization curve in reverse order but decreases in the manner indicated by the curve. When the magnetizing force has been reduced to zero, a certain induction remains which is called the residual induction, B_r. In order to reduce the induction still further, a magnetizing force in the opposite direction must be applied. The value of reversed magnetizing force required to bring the induction to zero is called the coercive force, H_c. If the magnetizing force is carried from a certain maximum value in one direction to the same value in the other direction and back a sufficient number of times, the induction cycle becomes a closed loop as indicated in the figure. This loop is called a hysteresis loop, and its area is proportional to the amount of energy expended in carrying the material through one complete cycle of magnetization.

4. NORMAL INDUCTION

The induction resulting from the application of a given magnetizing force depends upon the previous magnetic condition of the specimen and upon the mode of approach to the given magnetizing force; that is, the magnetization curve obtained depends upon the initial magnetic condition of the material and the way in which the test is made. In order to obtain consistent and reproducible results, therefore, a standard procedure must be followed. The effect of previous magnetic history can be removed by demagnetization. This is accomplished by subjecting the specimen to a succession of reversals of magnetizing force gradually decreasing from a certain maximum value to one somewhat lower than the lowest at which a determination is to be made.

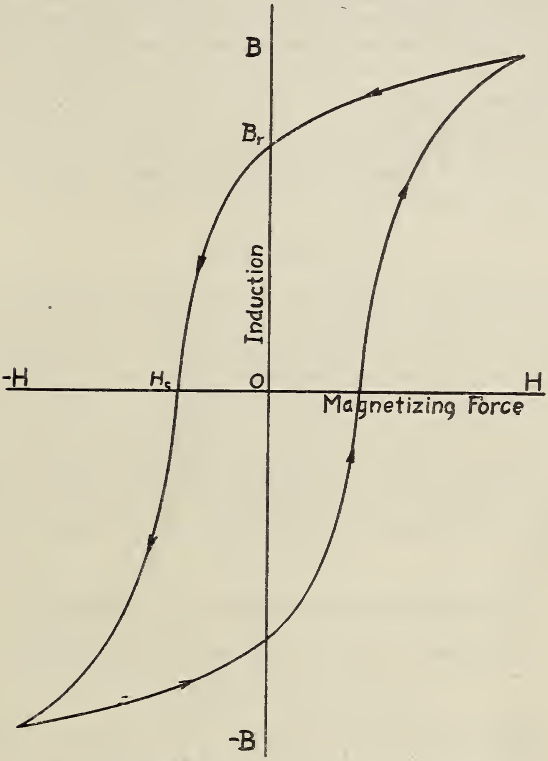

Figure 2.—*Typical magnetic hysteresis loop.*

It has been shown by experiment [1] [2] that, although demagnetization should be started from an initial value of magnetizing force well into the third stage of magnetization, it is not necessary to start from a magnetizing force higher than any previously experienced by the specimen. After demagnetization, points on the induction curve are obtained by observing the induction resulting from the application of a given magnetizing force after a sufficient number of reversals to bring the material to a cyclic condition, thus closing up the hysteresis loop. After a determination has been made, further measurements can be made at higher values of magnetizing force without demagnetizing again, but if a lower point is desired the demagnetizing process

[2] The numbers in brackets here and throughout the text refer to the references given in the last section of the Circular.

must be repeated. Values of induction obtained in this manner are called "normal induction" and are reproducible for a given specimen within the limits of experimental error. The normal induction curve is the locus of the tips of a succession of hysteresis loops, as illustrated in figure 3.

5. NORMAL HYSTERESIS

When points on a normal hysteresis loop are to be determined, cyclic condition is first obtained by a number of reversals of the magnetizing force corresponding to the desired tip, and the induction is measured. Then the change in induction is noted when the magnetizing force is reduced to some lower value either in the same or opposite direction. The difference between the maximum induction and the observed change is the value of induction desired. Before each determination, the material is brought back to a cyclic condition by reversals of the maximum magnetizing force. By this procedure the cumulative errors possible in the "step-by-step" method are avoided.

6. CORE LOSS

When materials are subjected to alternating magnetic fields, as in the cores of transformers, a certain amount of energy is expended which cannot be recovered but is dissipated in the form of heat. This loss of energy is called core loss. Core loss is made up of two components, hysteresis and eddy currents. The hysteresis loss depends upon the area of the hysteresis loop and the frequency of alternations. The eddy currents are induced in the core by the alternating magnetic flux and depend not only upon the frequency and maximum induction but also upon the electrical resistivity of the material, the thickness of the laminations, and the insulation between them.

IV. TESTS WITH DIRECT CURRENT

1. NORMAL INDUCTION AND HYSTERESIS

(a) GENERAL PRINCIPLES

Ballistic methods, so called because they employ a ballistic galvanometer in the measurements, are most commonly used in ordinary magnetic testing. Other methods which have been employed in the past, such as the Koepsel permeameter [2], the Esterline permeameter [3], and the Du Bois balance [4], have been superseded almost entirely by the more accurate ballistic methods.

When the magnetic flux linked with a test coil connected to a ballistic galvanometer is suddenly changed in magnitude a momentary current is induced in the circuit. The quantity of electricity which flows during the impulse is proportional to the change in flux and is measured in terms of the deflection or "kick" of the galvanometer. A moving-coil type of galvanometer is usually employed.

The mathematical theory of the behavior of a ballistic galvanometer when subjected to an impulsive current through its circuit is given in various textbooks on electrical measurements [5]. Briefly, the behavior of an individual galvanometer depends primarily on the resistance of the galvanometer circuit. According to the value of this resistance, the galvanometer may be critically damped, underdamped, or overdamped. If the galvanometer is critically damped, the coil returns to zero after a deflection in the minimum time without oscillation. In other words, the action is dead beat. The value of external resistance for critical damping is one of the principal characteristics of a ballistic galvanometer. If the resistance is greater than the critical value, the galvanometer is underdamped and the return to zero after a deflection will be by a series of oscillations through the zero position. If the external resistance is less than the critical damping resistance, the galvanometer is overdamped and the return to zero is slower than

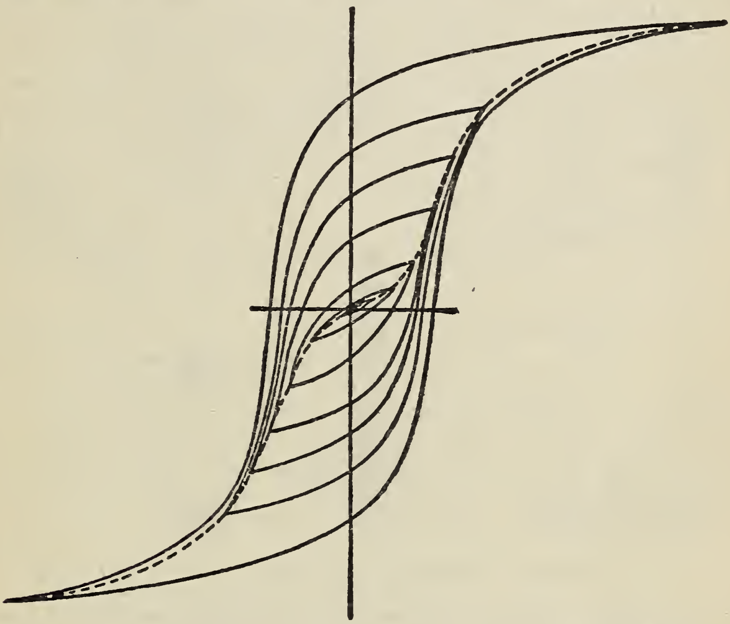

FIGURE 3.—*Normal induction curve shown as locus of tips of a family of hysteresis loops.*

it would be with critical damping. In magnetic testing, the ballistic galvanometer is usually either critically damped or overdamped.

In using a critically damped ballistic galvanometer in magnetic testing, it is important that the period of the galvanometer be long enough so that the coil does not have any appreciable deflection until after the impulse is over. Otherwise, a low reading will be obtained. Under some conditions, the impulse may be sufficiently long to require the use of a galvanometer having an inconveniently long period. Ordinarily, it is preferable to use a very heavily overdamped galvanometer.

If it were possible to reduce the torsional control of a galvanometer to zero, the coil would remain in its deflected position indefinitely after an impulse and the amount of the deflection would be independent of the duration of the impulse. An instrument in which this condition is very closely approximated is called a fluxmeter [5]. In practice, it has been found that a ballistic galvanometer having a high external resistance for critical damping but operated with a low resistance in the external circuit approximates the performance of a fluxmeter sufficiently well and is sensitive enough for most purposes. Under this condition, the torsional control of the suspension is negligible with respect to the electromagnetic damping.

The principles involved in all ballistic methods are the same, the differences being in the type of magnetic circuit and the arrangement of the magnetizing and test coils. A typical diagram of connections is shown in figure 4. Current from the storage battery *B* is controlled

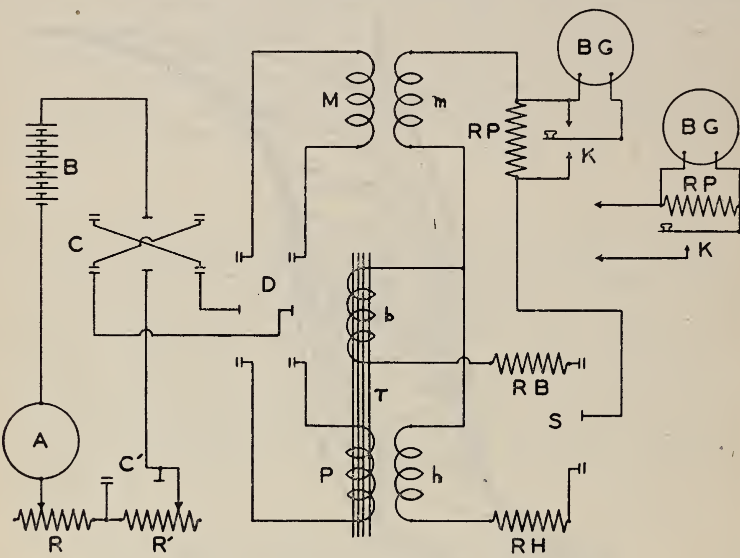

Figure 4.—*Diagram of connections for ballistic test.*

by the rheostats *R* and *R′* and measured by ammeter *A*, or, if desired, by a standard shunt and a potentiometer. *C* is a reversing switch, and switch *C′* serves to insert the auxiliary resistance *R′* into the circuit. The current flows in primary *M* of a mutual inductor when switch *D* is closed upward and in the magnetizing windings of the apparatus *P*, when *D* is closed downward.

The secondary *m* of the mutual inductor, or an equivalent noninductive resistance, is always included in the galvanometer circuit. By means of selector switch *S*, the galvanometer can be connected either to test coil *b*, which surrounds test specimen *T* or to coil *h*, which does not surround the specimen but is located in such a position as to give an indication of the magnetizing force. The sensitivity and damping of the galvanometer are adjusted by means of the parallel resistance *RP* and the series resistances *RB* and *RH*. In some types of apparatus, magnetizing force is measured in terms of the magnetizing current, in which case coil *h* and resistance *RH* are omitted. Two alternative methods of galvanometer control are shown. The choice between these two methods is mainly a matter of individual preference.

As shown in the main diagram, the galvanometer can be short-circuited, open-circuited, or connected to the test circuit by closing the key upward, opening it, or closing it downward, respectively. By the alternative connection shown at the right the parallel resistance, usually of such a value as to give critical damping, is always connected to the galvanometer, which in turn is connected to the test circuit or not according as the key is closed or open. If desired, a second parallel resistance may be connected between the lower contact of the key and the left-hand terminal of the galvanometer. With this connection, the galvanometer will be overdamped when connected to the test circuit.

The galvanometer is calibrated by means of a standard mutual inductor. It is usually convenient to adjust the sensitivity so that the scale is direct reading in terms either of induction or of magnetizing force, thus avoiding the necessity of multiplying the scale reading by odd factors. It is customary to make a deflection of 1 cm correspond to the reversal of an induction of 1,000 gausses or of a magnetizing force of 1, 10, or 100 oersteds as required. When calibrating for induction, the current to be reversed in the primary of the mutual inductor depends upon the value of the mutual inductance, the number of turns in the test coil, and the cross-sectional area of the specimen, and is calculated from the formula:

$$I_c = BAN/M \times 10^8,$$

in which

I_c = calibrating current in amperes
B = induction in gausses
A = cross-sectional area of specimen in cm^2
N = number of turns in the test coil
M = mutual inductance in henrys.

The corresponding formula for calibrating for magnetizing force is

$$I_c = HAN/M \times 10^8,$$

in which

H = magnetizing force in oersteds
AN = product of the number of turns by the average area of the test coil

and the other quantities are the same as in the preceding formula.

Since the calibration is made by reversal of the calibrating current, care must be taken when measuring changes in induction or magnetizing force as in the determination of points on the hysteresis loop to multiply the readings by 2.

Unless the test coil for the measurement of induction is wound directly on the specimen, a correction must be made to the observed value of induction. This is to take into account the flux in the space between the specimen and the test coil. The amount to be subtracted from the observed value of induction is proportional to the magnetizing force and depends upon the difference between the areas of the test coil and the specimen. The correction is equal to kH. where

$k = (a - A)/A$
a = area of test coil
A = area of specimen.

In making a normal-induction test, the specimen is first demagnetized. The magnetizing current is then set to the value corresponding to the lowest point desired and reversed several times until successive readings of the induction are in agreement. The specimen is then in a cyclic condition and the observed induction is the normal induction. The magnetizing force is then determined either by taking a ballistic deflection or by measuring the magnetizing current according to the type of apparatus being used. Additional points on the normal-induction curve are determined in the same way, except that repeated demagnetization is not required if each point so determined is higher than any preceding one. It is the practice of some observers to start with the highest point to be determined and demagnetize from each point to the next lower one. This is sometimes the preferable procedure, especially if otherwise, the specimen is likely to be heated unduly.

If points on a hysteresis loop are to be determined, cyclic condition is first obtained by reversals of the magnetizing current corresponding to the tip of the loop and corresponding values of B and H observed. The current is then suddenly reduced in value by opening the switch C', thus inserting into the circuit the rheostst R'. For points on the negative side of the H-axis switches C and C' are operated simultaneously, thus reversing as well as reducing the magnetizing force. The observed values of the corresponding changes in B and H are subtracted from the values at the tip, and the results thus obtained are taken as the coordinates of the required point on the hysteresis loop. The process is repeated for other points, care being taken to reestablish cyclic condition at the tip before each reading.

In setting up apparatus for ballistic tests, it is important to twist the conductors in both primary and secondaiy circuits and so to locate the mutual inductor with reference to the rest of the apparatus that errors due to stray fields will be negligible.

Of the considerable number of ballistic-test methods which have been developed both in this country and abroad, our attention will be confined to those which are commonly used in this country. Among the other methods might be mentioned the Ewing double-bar method [6], the Picou permeameter [7], the Iliovici permeameter [8], the Hughes Magnetic Bridge [9], and the Niwa permeameter [10].

(b) STRAIGHT BAR AND SOLENOID

Although the use of a straight bar magnetized in a simple solenoid without yokes does not yield results of high accuracy, this method is sometimes the most convenient. The best results can be obtained with specimens in the form of ellipsoids of revolution, but as this is a difficult form to prepare, straight bars of uniform cross section, either circular or rectangular, are generally used. Values of induction are obtained in the usual way by means of a test coil surrounding the middle part of the bar. The magnetizing force is calculated in terms of the current and dimensions of the magnetizing coil. Unless the coil is very long with respect to its mean diameter, a correction must be made to account for its finite length. The equation $H=0.4\pi nI$ is strictly true only for a solenoid of infinite length. For relatively short coils, the following formula gives the approximate value of the magnetizing force at the middle of the coil.

$$H_0=0.4\pi nI\frac{l}{\sqrt{l^2+d^2}},$$

in which

H_0=magnetizing force at middle of coil
n=turns per centimeter
I=current in amperes
l=length of coil
d=mean diameter of winding.

For multilayer coils, a more exact value can be calculated by summing the values obtained for all the layers taken separately.

The value thus obtained must be corrected to account for the self-demagnetizing effect of the specimen. The correction is generally assumed to be proportional to the magnetization and applied by means of the formula.

$$H=H'-NJ,$$

in which

H=effective magnetizing force
H'=applied magnetizing force
N=demagnetizing factor
J=intensity of magnetization.

If the magnetization is expressed in terms of intrinsic induction, the correction is KB_i instead of NJ, and the equation becomes

$$H=H'-KB_i$$

Since $B_i=4\pi J$, it is obvious that

$$K=N/4\pi.$$

The demagnetizing factor depends upon the ratio of length to diameter of the specimen. For rectangular bars, the diameter of the circle that would give the same area is taken. The factor can be calculated for an ellipsoid, but for bars of uniform section it must be determined experimentally.

The principal investigations of demagnetizing factors have been made by Mann [11], DuBois [12], and Shuddemagen [13]. Some of the values obtained by these investigators are given in table 1, page 38. The factor is a function of the actual dimensions and magnetic properties of the specimens, as well as of the dimensional ratio. Also, as Shuddemagen points out, the demagnetizing factor is approximately constant only for values of B less than about 10 kilogausses and decreases in value as the induction is increased above that point. In the last two columns of the table, values are given calculated from the equations

$$\log_{10}N=1.15-1.75\ \log_{10}D$$
$$\log_{10}K=0.05-1.75\ \log_{10}D,$$

in which D=ratio of length to diameter. These equations give values intermediate between the extremes obtained experimentally

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and have the advantage that factors for odd values of D can be calculated directly without the necessity of plotting a curve.

(c) RING METHOD

It is generally considered that the highest accuracy possible by a ballistic test is to be obtained by the use of ring specimens. In order to obtain high accuracy, however, it is essential that certain conditions be fulfilled. The principal advantage of the ring is absence of end effects and of errors due to magnetic leakage. The rings should be continuous, having no joints or welds. The bending of bars into the form of rings and welding the ends together, as sometimes advocated, is not good practice. It is not possible, even by subsequent annealing, to eliminate the effect of the joint which gives rise to magnetic leakage.

The magnetizing winding should be uniform. Bunched or irregular winding gives a nonuniform distribution of magnetomotive force around the magnetic circuit, thus causing leakage and errors of uncertain magnitude.

Since the inner circumference of a ring is shorter than the outer circumference, the magnetizing force and induction are not uniform over the cross section. The error involved depends upon the ratio of the mean diameter to the radial width and the magnetic properties of the material. It is customary to calculate the magnetizing force in terms of the current and the mean diameter. The ratio of the average magnetizing force to the value calculated in terms of the mean diameter for rings of rectangular and circular cross section, with various ratios of mean diameter to radial width, have been calculated by Lloyd [14] and are given in table 2, page 39. It is usually specified that the ratio shall be not less than 10.

The obvious disadvantages of the ring specimen are the labor involved in winding each individual ring and the limited value of magnetizing force to be obtained without excessive heating. It is principally used in the testing of small quantities of material of very high permeability.

(d) BURROWS PERMEAMETER

The Burrows permeameter [15] employs a compensated double yoke on which the magnetizing windings are arranged so as to bring all parts of the magnetic circuit as nearly as possible to the same magnetic potential.[3] If this condition could be completely realized, there would be no magnetic leakage from one part of the circuit to another, and the magnetizing force at any point could be calculated in terms of the current and number of turns per centimeter of the winding at that point.

The magnetic circuit consists of two bars of the material to be tested, one of which is the test bar. The other bar serves as an auxiliary. The bars are connected magnetically at their ends by soft-iron yokes as shown in figure 5. Two uniformly wound coils surround the two bars. Over the ends of these coils are short compensating windings whose function is to furnish the extra magnetomotive force required for the joints and yokes. These four similar coils are connected together in series, and the current in them is

[3] The difference of magnetic potential between two points in a magnetic circuit is the line integral of magnetizing force between the points.

adjusted independently. The currents in the two main coils are also adjusted independently.

In order to test the compensation, there are three test coils t, a, and j, all having the same number of turns. Coil t is wound over the middle of the test bar. Coil a is wound over the middle of the auxiliary bar and the third coil, j, is wound half over one end and half over the other end of the test bar far enough from the ends to avoid disturbances from the joints and yokes. When, upon reversal of the current in the magnetizing windings, the ballistic galvanometer shows no residual deflection whether connected to t and a or to t and j in series opposition, the magnetizing currents are properly adjusted. There is then equal flux at the middle of the two bars and no leakage from the part of the test bar between the two parts of test coil j. The value of induction is determined from the deflection of the galvanometer connected to t alone when the currents are reversed. The magnetizing force, H, is proportional to the current in the main winding surrounding the test specimen. It is usual to wind this coil

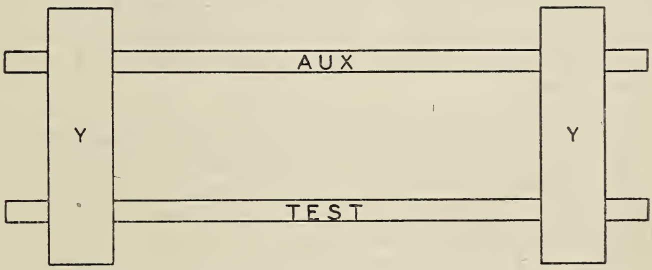

FIGURE 5.—*Magnetic circuit of Burrows permeameter.*

so that H is 100 times the current in amperes. The switching for the determination of points on the hysteresis loop is somewhat more complicated, but the principle is the same.

In a modification of this method devised by Gokhale [16], the j coil is replaced by a test coil located close to the test bar at its middle but not surrounding it. The magnetizing currents are so adjusted as to make the field as measured by this test coil equal to the value calculated from the current in the main magnetizing coil. This is determined conveniently in practice by opposing the test coil to the secondary of a mutual inductor of proper value whose primary is connected in series with the main magnetizing coil. This method is particularly suitable where the area of the test specimen is small with respect to that of the magnetizing coil.

A simplified form of the compensated double-yoke apparatus has been devised by A. W. Smith [17]. By making the bars long and providing joint contacts of large area, the joint and yoke reluctance can be made small with respect to that of the bars. The extra reluctance in the magnetic circuit is then regarded as equivalent to an extra length of the bars and the compensating coils and the two main magnetizing windings are all connected in series. The test coil t is divided and wound half over one test bar and half over the other bar. The procedure for making a test is the same as for the ring test.

The Burrows permeameter in either its original or modified form is subject to one serious limitation. It is easy to see that with the arrangement of magnetizing coils employed it is not possible to take into account the effect of magnetic inhomogeneity of the specimen along its length. Variations in magnetic permeability along the length of the test bar give rise to magnetic leakage which cannot be calculated or neutralized and lead to errors which may be large. For this reason the method can be depended upon to give satisfactory accuracy only if the test specimens are known to be magnetically uniform along their length. The maximum magnetizing force obtainable with this apparatus is of the order of 300 oersteds.

(e) FAHY SIMPLEX PERMEAMETER

This permeameter operates on a somewhat different principle [18] from the Burrows permeameter and requires but a single specimen. The magnetic circuit is shown in figure 6. The magnetizing force is applied by means of an electromagnet across the poles of which the specimen is clamped. A uniformly wound test coil extends over the whole length of the specimen. A ballistic galvanometer connected to this coil indicates the induction in the specimen when the magnetizing current is reversed.

The magnetizing force is measured by means of a test coil uniformly wound on a nonmagnetic form and extending between the tops of two iron blocks which are clamped to the ends of the specimen. The deflection of a ballistic galvanometer connected to such a coil when the magnetizing force is changed in value is proportional to the change in magnetic potential between the ends of the coil. The function of the iron blocks is in effect to transfer the ends of the test coil to the ends of the specimen. When the magnetic circuit is properly constructed of suitable materials, this method is capable of giving very satisfactory results [19]. It does not require an auxiliary specimen and no compensation is necessary. It is also less sensitive than the Burrows permeameter to the effect of magnetic inhomogeneity along the length of the specimen. Measurements can be made with this permeameter up to about 300 oersteds.

A motor-driven flip-coil attachment has been developed for this permeameter at the National Bureau of Standards [20]. When this device is used, measurements of magnetizing force are made by rotating the *H*-coil through 180 degrees about a vertical axis. Thus, it is possible to obtain more precise values of points on the hysteresis loop than can be done with the stationary *H*-coil.

(f) MAGNETIC COMPARATOR

For measurements on materials of which only a small quantity is available the magnetic comparator may be used. As originally described [21], it consists of an electromagnet having holes 6 mm in diameter so located in its pole pieces that the sample under test and a reference bar can be clamped parallel to each other and about 12 mm apart. Test coils mounted on brass forms encircle each bar. With this arrangement, if there were no magnetic leakage, an equal magnetomotive force would be impressed on each bar. As a matter of fact, however, there is magnetic leakage and the magnetizing forces are not exactly equal on the two bars. In order to take this inequality into account, the apparatus is calibrated by using a standard having properties as near as possible like those of the sample to be tested. Corresponding values of induction in the reference bar and the standard, together with the magnetization curve for the standard, furnish the data for determining the magnetizing force in terms of the induction in the reference bar. The test specimen is then inserted in place of the standard and another set of corresponding values of induction observed. From these values together with the previous calibration of the reference bar, the normal induction curve for the test specimen can be obtained. A similar procedure is carried out for obtaining hysteresis data. The accuracy obtainable with this method depends primarily upon the degree of similarity between the specimen and the

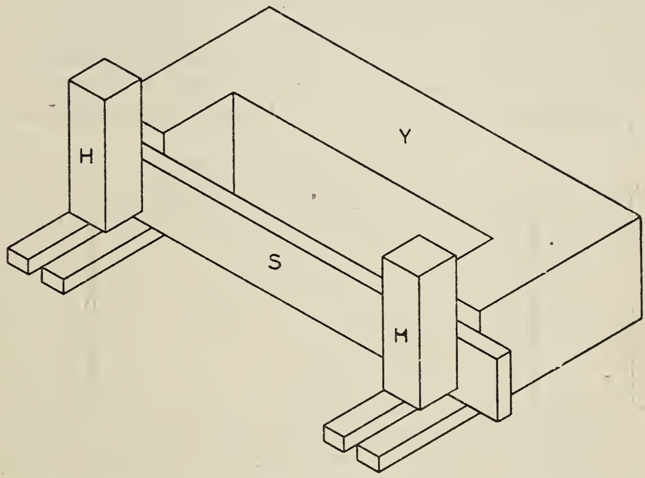

FIGURE 6.—*Magnetic circuit of Fahy Simplex permeameter.*

standard used for calibration. Measurements can be made with this apparatus at magnetizing forces up to 500 oersteds provided that suitable standards are available.

(g) FULL-RANGE PERMEAMETER

This instrument is a compensated single yoke [22]. The magnetic circuit is indicated in figure 7. The specimen is clamped between the poles of a laminated U-shaped yoke, and the magnetizing coil surrounds the specimen. An auxiliary coil on the yoke serves to provide the magnetomotive force required for the joints and yoke. The test for compensation is somewhat similar to that employed in the Gokhale modification of the Burrows permeameter. An *H*-coil mounted within the main magnetizing coil adjacent to the specimen is connected in series and opposing the secondary of a mutual inductor whose primary is in series with the magnetizing coil. The current in the winding on the yoke is adjusted to such a value that a ballistic galvanometer connected to the test coils shows no residual deflection upon reversal of the currents. The magnetizing force can then be determined either from the magnetizing current or from the deflection of a ballistic galvanometer connected to the *H*-coil alone. The intrinsic induction is determined by means of a test coil surrounding the middle part of the specimen in series with another *H*-coil having the same value of area turns. The total induction is calculated by adding to the value of intrinsic induction thus obtained the value of the corresponding magnetizing force. The range of this instrument is from 0.01 to 1,000 oersteds.

(h) BABBITT PERMEAMETER

The magnetic circuit of the Babbitt permeameter [23] is similar to that of the Full-Range permeameter, except that the laminated yoke is made up of a number of different materials so chosen that the effective permeability is practically constant in the range of inductions required in the yoke. The yoke coil of the right number of turns is then simply connected in series with the main magnetizing coil which surrounds the specimen and the compensation is automatic. Measurements of magnetizing force and intrinsic induction are made in the same way as with the Full-Range permeameter. The range is from 40 to 1,000 oersteds.

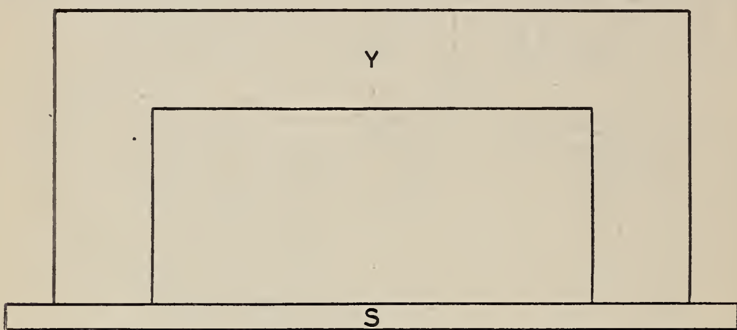

FIGURE 7.—*Magnetic circuit of Full-range, Babbitt and Saturation permeameters.*

(i) SATURATION PERMEAMETER

As its name indicates, the Saturation permeameter [24] was originally developed for the determination of saturation values. Its magnetic circuit is essentially the same as that of the Full-Range and Babbitt permeameters, but there is no compensating winding on the yoke since the compensation is less important at the higher magnetizing forces. The test-coil arrangement is the same as in the Babbitt and Full-Range permeameters. The magnetizing winding is so designed as to permit of the application of magnetizing forces as high as 4,000 oersteds, and there is provision for cooling the apparatus to prevent overheating.

(j) HIGH-*H* PERMEAMETER

Some of the methods employed for measurements at magnetizing forces much higher than 300 oersteds are based upon the classical isthmus method of Ewing and Low [25]. In this method a short specimen of the material to be tested is magnetized in the gap between the poles of a powerful electromagnet. The magnetizing force is measured in terms of the magnetic field in the space close to the surface of the specimen, and the induction is determined by means of a test coil closely surrounding the specimen. In the original method, the measurements were made by suddenly turning the specimen end for end between the poles of the electromagnet. For general testing it is not convenient to follow this procedure, as test specimens of a special form are required. Furthermore, it is not feasible to determine hysteresis data in this way.

The principal modifications necessary to adapt the isthmus method to general testing consist in making the core of the electromagnet of suitable laminated material so that the required changes in magnetization can be brought about by means of switches and rheostats in

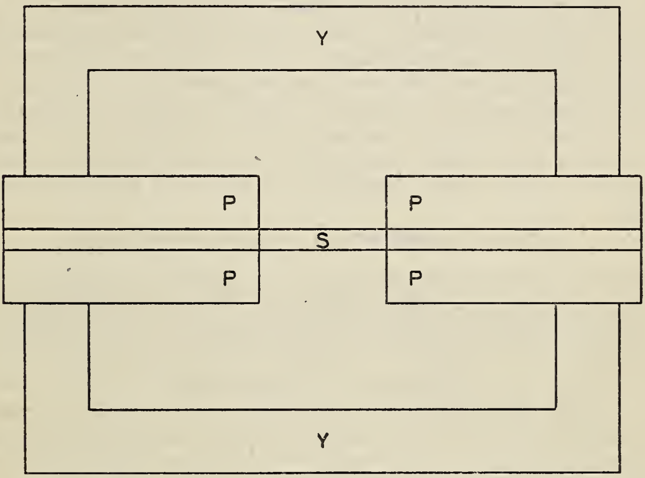

FIGURE 8.—*Magnetic circuit of High-H permeameter.*

the usual way, and so designing the pole pieces as to permit the insertion of test specimens of greater length than the gap between them. In order to obtain satisfactory accuracy, the induction must be uniform across the section and along the length of the part of the specimen covered by the *B*-coil and the field in the space occupied by the *H*-coil must be uniform and equal to that at the surface of the specimen. These conditions are brought about by proper proportioning of the magnetic circuit and distribution of the magnetizing coils.

The magnetic circuit of the High-*H* permeameter which was developed at the National Bureau of Standards [26] is shown diagrammatically in figure 8. This form of magnetic circuit is more symmetrical than one in which a single yoke is employed and produces a more uniform induction throughout the cross-sectional area of the specimen. The main magnetizing coils surround the pole pieces and auxiliary magnetizing coils are wound on the yokes. The pole pieces are adjustable so that the gap length can be chosen with reference to the size and properties of the specimen so as to obtain the maximum degree of uniformity in the magnetic field immediately adjacent to the specimen. The degree of uniformity of this field and its value, which is equal to the magnetizing force acting on the specimen, are determined by means of a small double *H*-coil which is rotated 180 degrees by a motor. The induction in the specimen is measured by means of a test coil wound on a thin brass form which surrounds the specimen. In order to keep the air-flux correction small, *B*-coil forms are made to fit specimens of different sizes. With this apparatus, measurements can be made at magnetizing forces up to 5,000 oersteds.

(k) FAHY SUPER-H ADAPTER

The Fahy Super-H adapter is designed to be used as an auxiliary to the Fahy Simplex permeameter. It consists of specially shaped pole pieces between which the specimen is clamped, thus confining the test to a short length of the specimen. The pole pieces are clamped edge-on to the permeameter in order to equalize the magnetic potential on the two sides of the specimen. The usual *B*-coil and *H*-coil are mounted in the gap, the faces of which are cut at an angle so as to make the field between them as nearly uniform as possible. A magnetizing coil surrounds the pole pieces and gap, and is used in conjunction with the main winding on the permeameter. With this arrangement magnetizing forces up to about 2,500 oersteds can be obtained.

(l) TESTS OF MATERIALS OF LOW PERMEABILITY

Materials having a permeability only slightly in excess of unity are generally tested in a straight solenoid. Since the intrinsic induction for such materials is very low, the correction for self-demagnetization is small and can be neglected without appreciable error in the value of magnetizing force calculated in terms of the current. In order to obtain a satisfactory degree of precision in the determination of the intrinsic induction it is necessary to balance out the direct effect of the magnetizing field. This can be done by connecting the secondary of a mutual inductor of variable value in series-opposition with the test coil which usually has several hundred turns. The primary of the inductor is connected in series with the solenoid. The mutual inductor is adjusted so that, with no specimen in the test coil, there is no deflection of the ballistic galvanometer upon reversal of the magnetizing current. The galvanometer can then be used at its maximum sensitivity. When a specimen is inserted within the test coil and the magnetizing current is reversed, the galvanometer deflection is proportional to the intrinsic induction. The galvanometer is calibrated by means of a standard mutual inductor in the usual way.

In the Fahy Low-Mu permeameter [27] a somewhat different arrangement is employed. Two similar test coils are used. The specimen is inserted in one coil and the other constitutes a compensating coil to balance out the effect of the magnetizing force on the main test coil. The compensating coil has a value of area-turns somewhat greater than that of the other coil. In order to balance the system with no specimen present, the compensating coil is shunted with a resistor whose value is adjusted so that, with no specimen in the test coil, no deflection of the galvanometer results from a reversal of the magnetizing current. Then when a specimen is inserted in the test coil and the magnetizing current is reversed, the resulting deflection is proportional to the intrinsic induction. The galvanometer is calibrated in the usual way by means of a standard mutual inductor.

A null method is described in the specifications of the American Society for Testing Materials [28]. In this method, the primary windings of two variable mutual inductors are connected in series with the magnetizing solenoid. The secondary circuits of the mutual inductors are connected in series with the test coil mounted within the solenoid in such a way that they both oppose the test coil. One mutual inductor is calibrated in microhenrys. The other serves to compensate for the effect of the test coil before the specimen is inserted. The preliminary adjustment is made by connecting a galvanometer to the secondary circuit and adjusting the compensating inductor so that the galvanometer shows zero deflection upon reversal of the magnetizing current. The calibrated inductor is set at zero for this adjustment. The specimen is then inserted in the test coil and the calibrated inductor adjusted for balance upon reversal of the primary current. The permeability of the specimen is calculated from the equation.

$$\mu=1+\frac{M_1\times10^3}{4\pi NnA_s},$$

in which

M_1=Mutual inductance of the calibrated mutual inductor in microhenrys
N=Number of turns in the test coil
n=Number of turns per cm of length of the solenoid
A_s=Cross-sectional area of the test specimen in cm^2.

2. MAGNETIC STANDARDS

In view of the nature of the magnetic units, it is obviously not feasible to realize them in concrete form. The calibration of magnetic testing apparatus is carried out in terms of the electrical units by the use of standard shunts, potentiometers (or ammeters), and mutual inductors. The use of mutual inductors so designed that their values can be calculated from their dimensions has for the most part been abandoned in favor of standards of more convenient form whose values are experimentally determined. For the intercomparison and standardization of permeameters it is necessary to make use of carefully selected and prepared test specimens whose magnetic properties are accurately determined by some standard method. It may, perhaps, be proper to refer to such test specimens as magnetic standards.

(a) LIMITS OF ACCURACY

The problem of magnetic testing consists in determining simultaneous values of magnetic induction and magnetizing force. It is relatively easy to obtain fairly accurate values of induction, but the accurate determination of the corresponding magnetizing force is more difficult. It is only by the exercise of great care in the selection of test specimens and manipulation of the testing apparatus that an accuracy of 1 percent can be attained. The influence of the quality and condition of the test specimen is of great importance, especially in the standardization of permeameters, because inaccuracies really arising from the condition of the specimen itself should not be charged to the testing apparatus.

(b) REQUIREMENTS OF STANDARD SPECIMENS

Specimens to be used as standards for the calibration or intercomparison of permeameters should be chosen and prepared with the following points in view: (1) magnetic uniformity along the length, (2) metallurgical stability, and (3) uniformity of section.

If the specimen varies in permeability along its length, errors are introduced in the measurements which cannot be calculated or eliminated by compensation, and which may be of considerable magnitude. It is not impossible to have errors due to this cause alone which amount to 25 percent or more. Moreover, various methods are sensitive to this influence in varying degrees. It is obvious, therefore, that such specimens should not be used for the intercomparison or standardization of testing apparatus. Various methods for the determination of the degree of uniformity of magnetic-test specimens have been proposed [29], but probably the most satisfactory one is to prepare a specimen much longer than is required for the final form and to make measurements at suitable intervals along its length. If the results of these measurements are in agreement, then the specimen is, from this point of view, satisfactory to use as a standard.

It is well known that specimens of steel freshly heat-treated are not metallurgically stable; that is, changes in internal structure or condition may go on for some time. These changes are accompanied by corresponding changes in magnetic properties. It is necessary, therefore, to make sure that specimens to be used as magnetic standards are metallurgically stable. This can be accomplished by either natural or artifical aging.

It is quite obvious that irregularity in cross section along the length of a specimen would have an effect similar to that of a variation in magnetic permeability. For this reason, it is important that care should be used in preparing the specimen to maintain a uniform cross section.

(c) GENERAL PRECAUTIONS

In the calibration and use of magnetic standards it is necessary to avoid (1) mechanical strain, (2) variations in temperature, and (3) mechanical vibration.

Mechanical strain influences the magnetic properties of materials to a marked degree. It is important, therefore, in the calibration and use of magnetic standards that they be clamped so as to be free from bending. The effect of bending is particularly noticeable in materials of high permeability and in the steep part of the magnetization curve.

The effect of variations in temperature is not negligible [30], and care should be taken that standards be not heated during the course of a test. The temperature coefficient is not constant and varies for different materials or even for the same material with different heat treatments.

Mechanical vibration should be avoided in magnetic testing. It has a tendency to increase the apparent permeability and to decrease the hysteresis. This is generally not a serious factor, but for work of high accuracy it should be considered and the specimens protected from excessive vibration.

(d) INTERCOMPARISON OF MUTUAL INDUCTORS

Mutual inductors used for the calibration of ballistic galvanometers are ordinarily calibrated by comparison with standard condensers, using alternating currents. It is often desirable to intercompare mutual inductors by a direct-current method. A method essentially similar to one originally proposed by Maxwell [31] has been found to be very convenient and satisfactory. A diagram of connections is shown in figure 9. M_1 and M_2 are the mutual inductors to be compared of which M_1 must have the greater value. The primary coils are connected in series to the battery through a reversing switch, rheostat, and ammeter. The secondary coils are connected in opposition as shown. R_1 and R_2 are adjustable precision resistors. It is

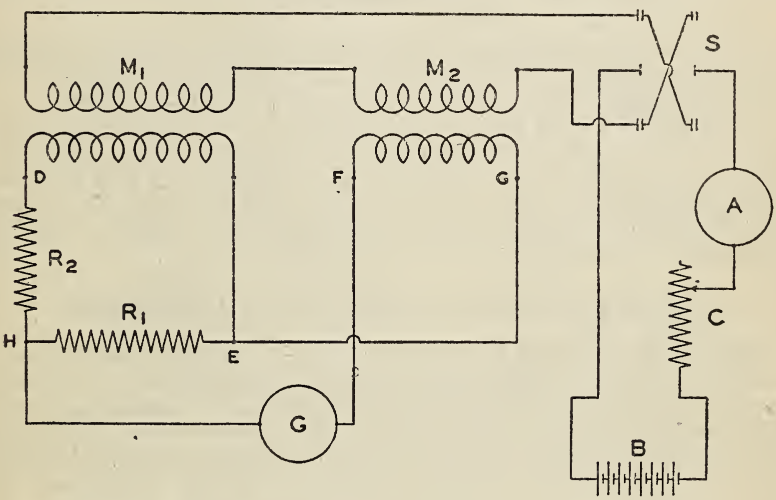

FIGURE 9.—*Diagram of connections for comparing mutual inductors.*

important that the galvanometer be connected as indicated because if it is connected between points E and G, leakage currents will give trouble unless there is practically perfect insulation between the coils of the inductors. Resistors R_1 and R_2 are adjusted to such values that there is no residual deflection of the galvanometer upon reversal of the primary current. If the self-inductances of the two secondary coils are not equal there may be a slight double kick of the galvanometer, but this is generally not troublesome. When balance is obtained, the value of M_2 in terms of M_1 is

$$M_2 = M_1 \frac{R_1}{R_1 + R_2}.$$

The resistance of M_1 must be included in the value of R_2. The usual precautions against stray fields should be taken. It is advisable to have the two inductors separated by some distance and so oriented as to avoid interaction between the primary of one inductor and the secondary of the other.

Errors due to capacitance which may exist between the primary and secondary circuits are reduced by repeating the observations after reversing the connections to the primary coil of one inductor and the secondary coil of the other and averaging the results.

(e) CALIBRATION OF TEST COILS

It is usually difficult to determine the value of area-turns of a test coil by measurements of its dimensions. It is much more convenient to do this experimentally by placing the coil in a long solenoid and measuring the mutual inductance between them. This can be done by the method described above. The value of area-turns is then calculated from the equation

$$AN = (M_2/K) \times 10^8,$$

in which AN is the product of the number of turns by the average area in square centimeters, M_2 is the mutual inductance between the test coil and solenoid, and K is the constant, H/I, of the solenoid.

In addition to the precautions necessary in the determination of mutual inductance, it is important that the axis of the test coil be aligned with that of the solenoid. The error due to lack of alignment is proportional to $1-\cos\theta$, where θ is the angle between the two axes. If the angle between the two axes is less the 2.5 degrees, the error from this source will not exceed 0.1 percent.

V. TESTS WITH ALTERNATING CURRENTS

Magnetic tests with alternating currents for the measurement of core loss, a-c permeability, and incremental permeability are ordinarily made in accordance with the specifications of the American Society for Testing Materials which should be consulted for details [28]. Such tests are made on flat-rolled materials cut into strips of appropriate size. The strips are assembled in four bundles in the form of a square within a set of coils generally known as a test frame. The dimensions of the strips and of the test frame and the form of joint at the corners of the magnetic circuit depend upon the type of test. Three different types of test are specified: (1) the 50-cm Epstein test for core loss, (2) the 25-cm Epstein test for core loss and a-c permeability, and (3) the 25-cm Epstein test for normal and incremental permeability and core loss at low inductions.

1. 50-CM EPSTEIN TEST FOR CORE LOSS

The test specimen for the 50-cm Epstein core-loss test consists of strips 50 cm (19 11/16 in.) long and 3 cm (1 3/16 in.) wide. The standard sample is composed of 10 kg (22 lb) of strips but 5 kg (11 lb) may be used. For the usual type of material, half of the strips are cut parallel and half at right angles to the direction of rolling. Under certain conditions, however, material manufactured in strip form or materials having pronounced directional properties may be tested with strips all cut in the same direction. The strips are assembled in four equal bundles and inserted in the test frame so as to form a square with butt joints. The bundles are ordinarily arranged so that opposite sides of the square consist of material cut in the same direction. The strips are held in place at the corners by clamps.

The four coil forms of the test frame are uniformly wound with two sets of coils connected in series to form primary and secondary windings of 600 turns each. The connections are as shown in figure 10, except that for the core-loss test the mutual inductor is not used and need not be connected.

An electromotive force of approximately sinusoidal wave form is applied to the primary winding and a voltmeter and the potential coil of a wattmeter are connected to the secondary winding. The wattmeter, whose current coil is connected in series with the primary winding, should be especially designed for low-power factors. The impressed voltage is adjusted, preferably by means of a suitable

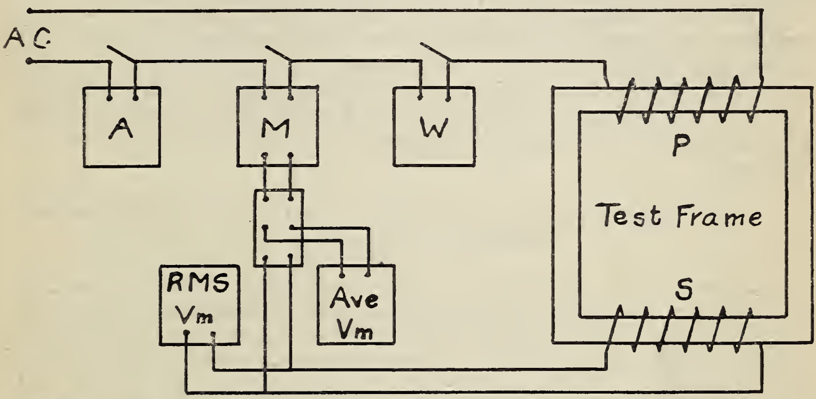

FIGURE 10.—*Diagram of connections for core loss and a-c permeability tests.*

autotransformer, until the voltage of the secondary is that given by the equation

$$E=4\text{ff}\ NfBM/4lD10^2,$$

in which

ff $=$form factor [4] of primary emf$=$1.11 for sine wave
$N$$=$number of secondary turns$=$600
f $=$frequency in number of cycles per second$=$60
B $=$maximum induction in kilogausses
$M$$=$total mass in kilograms
l $=$length of strips in centimeters$=$50
D $=$density in grams per cubic centimeter.

Standard tests are made at specified values of maximum induction. For tests at 15 kilogausses or higher, or whenever the form factor of the applied electromotive force departs from the value 1.11 by more than 1 percent, a voltmeter reading average volts is used in parallel with the rms voltmeter. The scale of such an instrument is conveniently marked in terms of the average volts times 1.11, in which case the voltage, as calculated for a sine wave, is held on the average voltmeter.

[4] Form factor is the ratio of the effective (rms) value of the emf to its half-period average value.

The density of silicon steel is usually assumed, the value depending upon the silicon content, and ranging from 7.55 to 7.85 [28]. For nickel-iron alloys the density is assumed in terms of the nickel content from the straight lines joining points having coordinates as follows:

Nickel	Density
Percent	*g/cm*^{3}
0	7.85
30	8.00
100	8.90

For other alloys the density is usually determined experimentally.

When the voltage and frequency have been adjusted to the proper values, the wattmeter indicates the total loss, including the loss in the secondary circuit. The loss in the secondary circuit can be calculated in terms of the rms voltage and the resistance. Subtracting this correction from the total loss gives the net loss in the steel, and dividing this value by the mass in kilograms gives the core loss in watts per kilogram.

2. 25-CM EPSTEIN TEST FOR CORE LOSS AND ALTERNATING-CURRENT PERMEABILITY

In recent years, the manufacturers of electrical sheet have been able to improve the uniformity of the magnetic properties of the various commercial materials to such an extent that it is now possible to obtain a sufficiently representative sample of a given heat or "lift" of steel by selecting not over 2 kg of strips instead of the 10 kg previously required. The 25-cm Epstein test is designed to test specimens weighing from 0.5 to 2 kg, but for general commercial testing the 2-kg sample is the standard. On account of the shorter length of the magnetic circuit, a better joint at the corners is required than the butt joint employed in the 50-cm test [32]. A double lap joint has been found to give best results and consequently a minimum length of 28 cm is needed. However, longer strips can be used up to a maximum length of 50 cm. The width of the strips is 3 cm.

The connections and testing procedure for measuring core loss are essentially the same as for the 50-cm test (Fig. 10) but, on account of the smaller cross-sectional area the number of turns in the test frame windings is increased to 700 and somewhat more sensitive instruments are required.[5] Also, in calculating the core loss it is necessary to take into account the additional material at the corners. This is done by using the "active weight" calculated on the assumption that the effective length of the magnetic circuit is 94 cm. This value was determined by experiment.

When testing specimens of very small cross-sectional area relative to the area of the secondary winding or when the magnetizing force is high enough to make the air-flux voltage more than 0.33 percent of

[5] By using a suitable power supply and appropriate instruments, the 25-cm test frame can be used for testing at frequencies up to 4,000 cycles per second or even higher, if proper precautions are taken.

the total voltage it is necessary to make a correction for the air-flux voltage. This can be done by calculation or, preferably, by means of a compensating mutual inductor. The compensating mutual inductor is adjusted to have the same value of mutual inductance as that between the primary and secondary windings of the test frame with no specimen inserted. Its primary is connected in series with the primary winding of the test frame, and its secondary is connected in series-opposition with the secondary test-frame winding. Under this condition, the measured voltage is proportional to the intrinsic induction B_i. If the form factor as indicated by the ratio of the rms voltage to the average voltage departs from 1.11 (the value for a sine wave) by more than 1 percent, it is necessary to make a correction to the observed value of core loss to account for the fact that whereas the hysteresis component of the total loss is a function of the average voltage, the eddy current component is a function of the rms voltage.

For the measurement of magnetizing force when determining a–c permeability the mutual inductor M (Fig. 10) is used. The secondary voltage measured with the average voltmeter is proportional to the crest value of the magnetizing current. The magnetizing force is calculated from the equation

$$H=\frac{0.4\pi NI_c}{l_2}=10I_c,$$

where

H=magnetizing force in oersteds
N=number of turns=700
I_c=crest current in amperes
l_2=assumed length of specimen=88 cm.[6]

The value of B in gausses is determined from the secondary voltage as for the core-loss test and the permeability, μ, is $\mu=B/H$.

3. NORMAL AND INCREMENTAL ALTERNATING-CURRENT PERMEABILITY AND CORE LOSS AT LOW INDUCTIONS

The methods described above are not sufficiently sensitive for testing laminated core materials at the low inductions usually employed in apparatus used in many types of communications equipment. Also, these methods do not provide for testing under the condition in which a unidirectional magnetizing force is applied in addition to the alternating magnetizing force. For testing at low alternating inductions, either with or without a superimposed unidirectional magnetizing force, two different methods are specified by the American Society for Testing Materials [28]: (a) an a-c bridge method or (b) an a-c potentiometer method. The tests are made at frequencies of either 60 or 1,000 cycles per second. The standard induction with no superimposed unidirectional magnetizing force is either 10 or 1,000 gausses at 60 cycles or 10 gausses at 1,000 cycles. With unidirectional magnetizing force applied, the standard induction is 10 gausses at either frequency. Values of unidirectional magnetizing force up to 2 oersteds are employed. The test frame is similar to that for the 25-cm Epstein test. Three windings are provided, an inner winding of 100 turns, an intermediate winding of 1,000 turns, and an outer

[6] Note that this value is different from that used in the calculations for core loss. See reference [32].

winding of 100 turns, which is used for the application of the d-c component of magnetizing force. The specimens are cut and assembled in the same way as for the 25-cm Epstein test described above. The test frame and specimen are the same for either the bridge or potentiometer method.

Figure 11 is a diagram of connections for the bridge method. The a-c supply should be of approximately a sine wave form having not more than 10 percent of total harmonics and should be effectively insulated from the bridge circuit by a suitable coupling transformer as shown. A storage battery provides a steady source of direct current. As shown in the diagram, one winding of the test frame

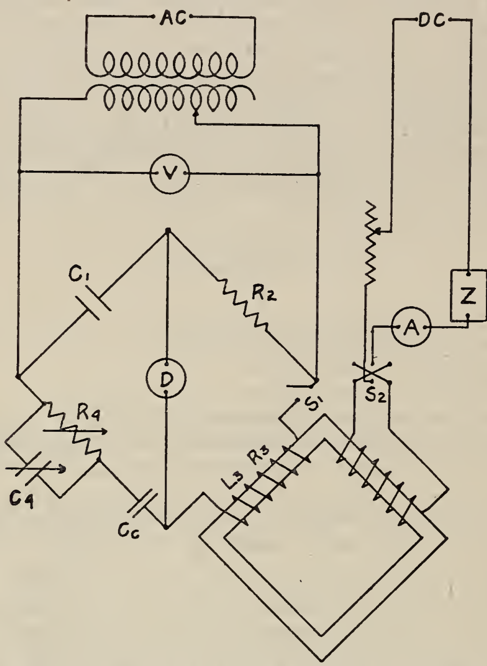

FIGURE 11.—*Bridge method for a-c tests at low inductions.*

constitutes one arm of the bridge. The opposite arm is a fixed capacitor, C_1. The other two arms consist of a fixed resistor, R_2, and a network in which a fixed capacitor, C_c, is in series with an adjustable capacitor, C_4, in parallel with which is the adjustable resistor R_4. The a-c voltage applied to the bridge is measured by the voltmeter, V. The detector, D, is a null-point indicator. For the 60-cycle test a vibration galvanometer is recommended; for the 1,000-cycle test a telephone receiver may be used. The detector may be preceded by a suitable amplifier.

Before making the test, the specimen is thoroughly demagnetized. In view of the fact that the permeability of most materials at low inductions drifts to an appreciable extent with time after demagnetization [33], this should be done at least 24 hours previous to making the test. During the interval between demagnetization and testing, the specimen should be protected against stray magnetic fields or mechanical vibration.

The constants of the bridge circuit depend upon the kind of material to be tested. For the 60-cycle test on materials that have exceptionally high permeability and low loss, such as cold-reduced silicon steel or nickel-iron alloys, the inner 100-turn winding of the test frame is used and R_2 is 10 ohms. For the 60-cycle test on ordinary silicon steel, or materials having similar magnetic properties, the 1,000-turn coil is used and R_2 is 100 ohms. In either case C_1 is 1 microfarad. Capacitor C_c balances the ohmic resistance of the test-frame coil and has a value calculated from the equation

$$C_c = \frac{R_2 C_1}{R_3},$$

where R_3 is the ohmic resistance of the test-frame winding. The adjustable resistor, R_4, is nonreactive and has a range of 10,000 ohms in steps of 1 ohm. C_4 is an adjustable capacitor having a range up to 2 microfarads in steps of 0.001 microfarad.

Measurements of incremental permeability or core loss are always made in the order of increasing values of unidirectional magnetizing force. Direct current is supplied to the outer 100-turn winding through the reactor Z, which should have an inductance of 1 henry or more when carrying current equivalent to a magnetizing force of 2 oersteds. This reactor is for the purpose of limiting the a-c current in the d-c circuit to a negligible value. The d-c current is adjusted to the required value with switch S_1 open and reversed by means of reversing switch S_2 several times to establish a cyclic condition in the specimen. Switch S_1 is then closed and the test made. Values of normal permeability or core loss are made with the d-c circuit open.

The d-c magnetizing force is calculated from the equation

$$H_{dc} = \frac{0.4\pi N I_{dc}}{l},$$

in which

H_{dc} = unidirectional magnetizing force in oersteds
N = number of turns
I_{dc} = Direct current in amperes
l = assumed length of the magnetic circuit.

When $N=100$ and $l=94$, this reduces to

$H_{dc}=1.34 I_{dc}$, or
$I_{dc}=0.748 H_{dc}$.

In making a test either with or without unidirectional magnetizing force, the voltage across the bridge is set according to the equation[7]

$$E=0.0707AB,$$

in which

A = the cross-sectional area of the specimen in square centimeters
B = the maximum induction in gausses.

[7] Under certain conditions it may be necessary to make a correction to the voltage thus calculated. The ASTM specifications [28] should be consulted for details.

The resistance R_4 and capacitance C_4 are adjusted to balance the bridge. When the bridge is balanced, the permeability is calculated from the equation

$$\mu(\text{or } \mu_\Delta)=\frac{0.748R_4}{A}\times\frac{R_2}{N^2}\times 10^4,$$

in which

μ=normal permeability
μ_Δ=incremental permeability
R_4=resistance in ohms
A=cross section in square centimeters
R_2=resistance in ohms
N=number of turns.

The loss is calculated from the equation

$$P_c(\text{or } P_\Delta)=E^2C_4\omega^2C_1R_2,$$

in which

P_c=total loss in watts without unidirectional magnetizing force
P_Δ=total loss in watts with unidirectional magnetizing force
E=electromotive force in volts
C_4=capacity in farads
ω=2π times the frequency in cycles per second
C_1=capacitance in farads
R_2=resistance in ohms.

The total core loss in watts divided by 84 percent of the mass of the specimen in kilograms is the value of core loss in watts per kilogram.

Figure 12 is a diagram of connections for the a-c potentiometer method. The a-c potentiometer is of the coordinate type, which indicates voltage in terms of two components having a quadrature phase relation. A phase shifting device (not shown) is used to adjust the phase of either the potentiometer current or the magnetizing current. The d-c circuit and procedure for applying a unidirectional magnetizing force are the same as for the bridge method.

In making a test, the 100-turn coil is used as the primary winding and the 1,000-turn coil is the secondary unless the secondary voltage exceeds the range of the potentiometer, in which case the two windings must be interchanged. The potentiometer is connected to the secondary coil, and the magnetizing current and phase relations are adjusted so that the total voltage is read on the in-phase dial of the potentiometer, the other dial being set at zero. The voltage corresponding to a given induction is calculated from the equation

$$E=\sqrt{2}\pi fN_2AB\times 10^{-8},$$

in which

E=electromotive force in volts
f=frequency in cycles per second
N_2=number of turns in the secondary winding
A=cross-sectional area in square centimeters
B=induction in gausses.

With the secondary voltage set at the proper value, the two components of the magnetizing current are measured by observing the drop across the noninductive resistor R, which is connected in series with the magnetizing winding. The power component is in phase with the voltage and the magnetizing component is in quadrature. Permeability and core loss are calculated in terms of secondary voltage and the quadrature and in-phase components of the current respectively, by using the equations

$$\mu(\text{or } \mu_\Delta)=52.9\frac{B}{N_1 I_q}$$

$$P_c(\text{or } P_\Delta)=\frac{N_1}{N_2}EI_p,$$

in which

μ = normal permeability
μ_Δ = incremental permeability
B = induction in gausses
N_1 = number of primary turns
I_q = quadrature component of the current in amperes
P_c = total loss in watts without unidirectional magnetizing force
P_Δ = total loss in watts with unidirectional magnetizing force
N_2 = number of secondary turns
E = secondary electromotive force in volts
I_p = in-phase component of the current in amperes.

The loss in watts per kilogram is obtained by dividing the total loss by 84 percent of the mass of the specimen in kilograms.

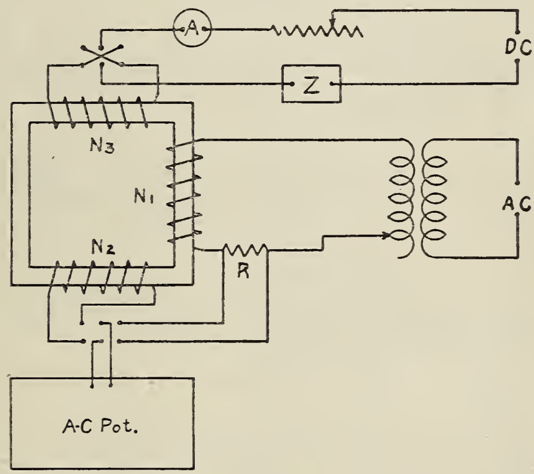

FIGURE 12.—*Alternating-current potentiometer method for tests at low inductions.*

VI. MAGNETIC PROPERTIES OF TYPICAL MATERIALS

Magnetic materials are used under a great variety of conditions and therefore must meet a great variety of requirements. In general the alloys employed in practice are mainly composed of one or more of the three ferromagnetic metals, iron, nickel, and cobalt. Other elements are always present, either as undesired impurities or as alloying elements added for the purpose of producing certain desired characteristics. In figure 13 are shown typical normal induction curves of annealed samples of iron, nickel, and cobalt of comparatively high purity. These curves are given only for the purpose of general comparison and should not be considered as representing critical values. Small variations in the degree of purity or in annealing procedure lead to substantial differences in normal induction.

Magnetic materials employed in commercial practice may be considered under the following five classifications: (1) Solid core materials, (2) electrical sheet or strip, (3) special-purpose alloys, (4) permanent-magnet alloys, and (5) feebly magnetic materials. In view of the rapid and continued progress in the development of magnetic materials, it is only feasible to give data characteristic of several of the materials commercially available at present. These data indicate merely the range of properties obtainable in the various classes of material and should not be considered as critical, since considerable variation from these values will be found in practice.

1. SOLID CORE MATERIALS

These materials are used for the cores of direct-current electromagnets, relays, field frames of d-c machines, etc. The principal requirement is high permeability particularly at relatively high induc-

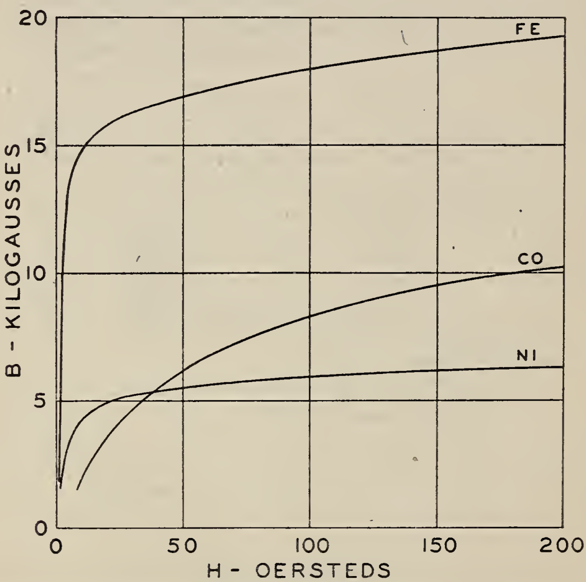

FIGURE 13.—*Typical normal induction curves of annealed samples of iron, nickel and cobalt.*

tions. For the majority of uses it is also desirable that the hysteresis be low. The principal materials employed are soft iron, relay steel, cast steel, cast iron, and an alloy of approximately 35 percent of cobalt and 65 percent of iron known as ferrocobalt. Ferrocobalt is characterized by very high permeability in the upper part of the normal induction range and a saturation induction approximately 10 percent greater than that of pure iron. Its cost is relatively high, however, and its use is limited in general to pole pieces in which a very high induction is required. Cast iron has a relatively low permeability and is used principally in field frames when cost is of primary importance and extra weight is not objectionable. Several varieties of soft iron are available, such as Norway iron, Armco iron, and Swedish charcoal iron. Relay steels contain from 1 to 3.25 percent of silicon to reduce aging. Electrolytic iron may also be used. All these materials are relatively pure iron, low in carbon and other impurities. Cast steel should be low in carbon, not over 0.1 to 0.2 percent, and contain only the usual small amounts of the ordinary impurities. Cast iron is high in

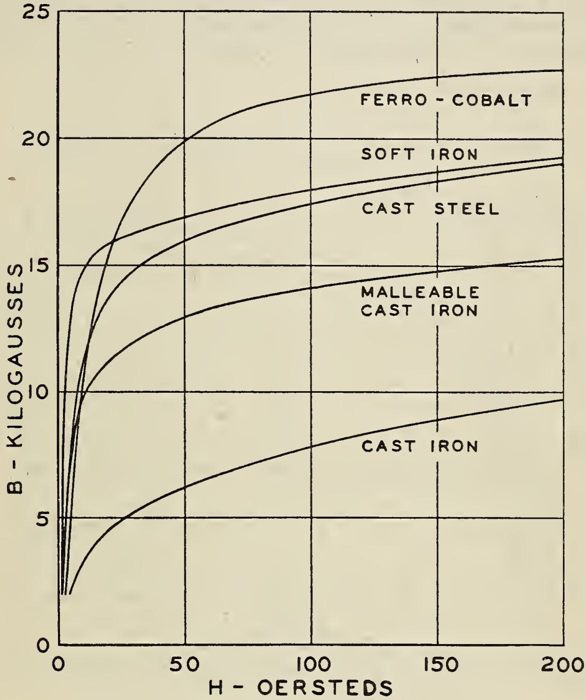

FIGURE 14.—*Typical normal induction curves for solid core materials.*

carbon, about 3 percent, and also contains about 3 percent of silicon, and varying percentages of phosphorus, manganese, and sulfur.

The best magnetic properties are obtained by a suitable annealing treatment. The properties of cast iron can be greatly improved by malleablizing, a process that converts a large part of the carbon to the amorphous form.

Typical normal induction curves for solid core materials are given in figure 14.

2. ELECTRICAL SHEET AND STRIP

The terms *electrical sheet* and *electrical strip* are commonly used to designate silicon-iron alloys produced in sheet or strip form and used as core materials in alternating-current apparatus, such as transformers, generators, motors, electromagnets, or relays. The principal requirements are high permeability, low hysteresis, and high resistivity. The several grades differ mainly with respect to their silicon content, which ranges from about 0.5 percent to approximately 4.5 percent. Alloys containing the higher percentages of silicon are practically nonaging; that is, the permeability and losses do not change with time. The required magnetic properties are produced by annealing.

By a suitable combination of cold-rolling and heat treatment, materials are produced in which the crystal axes are given a definite orientation. Such material has considerably better properties, when magnetized in the preferred direction, than the ordinary grades. Figure 15 shows typical normal induction curves for two grades of electrical sheet and orientated-grain material. The improvement in

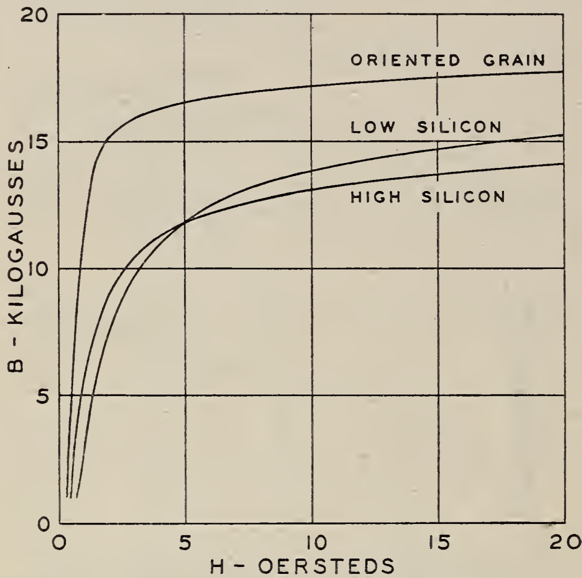

Figure 15.—*Typical normal induction curves for electrical sheet.*

the orientated-grain material is particularly conspicuous in the upper part of the normal induction curve.

The different grades of electrical sheet and strip are usually sold on the basis of guaranteed maximum values of total core loss, as determined in accordance with the specifications of the American Society for Testing Materials [28]. The common designations of the various grades are armature, electrical, motor, dynamo, and transformer. The transformer grades are further subdivided into classes denoted by numerals corresponding to the core lose under standard conditions. Armature, electrical, and motor grades are used principally in small motors, a-c magnets, and starting transformers. The dynamo grade is used in high-efficiency rotating machines and small transformers. The transformer grades are used in power and radio transformers.

3. SPECIAL-PURPOSE ALLOYS

For certain applications, special alloys have been developed which, after proper heat treatment, have superior properties in certain ranges of magnetization. For instance, alloys of nickel and iron with possible small percentages of molybdenum or chromium have very high values of initial and maximum permeability. Alloys of this class, which may have from 70 to 80 percent of nickel, are called permalloys. An alloy of 50 percent of nickel and 50 percent of iron is called hipernic. Another alloy having a small percentage of copper in its composition is called mumetal. The characteristics of these alloys differ in detail,

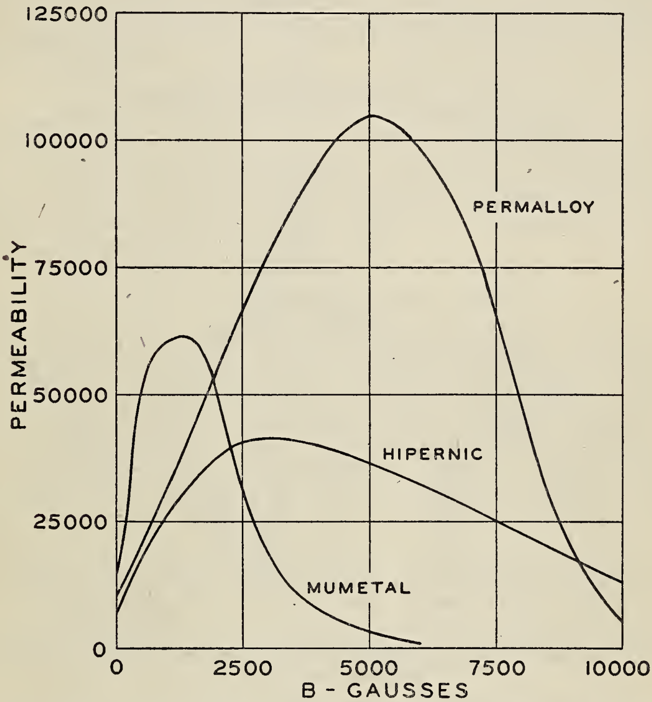

FIGURE 16.—*Typical permeability curves for special-purpose alloys.*

but in general they have high initial and maximum permeability, low hysteresis, and low saturation values. They are particularly applicable for use at low inductions. Typical permeability curves are given in figure 16.

A certain alloy of nickel, cobalt, and iron after suitable heat treatment has very nearly constant permeability for inductions below 1,000 gausses and is called perminvar. The 50–50 nickel-iron alloy can also be heat-treated so as to have similar characteristics.

An alloy of equal proportions of iron and cobalt has high permeability which persists at higher values of induction than the nickel-iron alloys and is called permendur.

Another series of magnetic alloys of copper, nickel, and iron is temperature-sensitive, having an approximately linear relation between permeability and temperature. These are called thermalloys. The principal use is in the compensation of watt-hour meters for temperature variations. They are also used in certain types of thermal relays.

4. PERMANENT-MAGNET ALLOYS

For permanent magnets, high values of residual induction and coercive force are required. Ordinary high-carbon steels and some

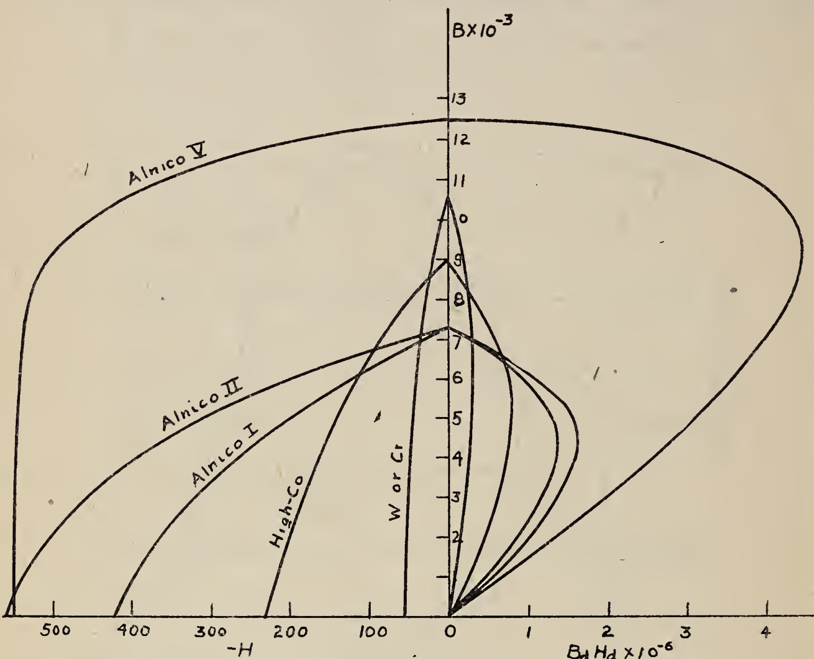

FIGURE 17.—*Typical demagnetization and energy-product curves for permanent-magnetic materials.*

special tool steels make fair magnets when properly hardened, but better results are obtained by the use of alloys specially made for the purpose. Development along this line has been rapid during the past few years and is still going on. There are at present three general classes of permanent-magnet materials: those in which the principal alloying elements are tungsten or chromium or both; those in which cobalt is present in substantial amounts; and those in which aluminum and nickel play an important part.

Tungsten, chromium, or cobalt magnet steels are hardened by quenching. The aluminum-nickel alloys are among the so-called dispersion-hardening alloys, the final treatment for which is prolonged heating at a temperature of the order of 660° to 700° C. The alloys that are hardened by quenching can be forged and machined, but most of the dispersion-hardening alloys cannot be forged or machined and must be cast to shape and can be cut only by grinding.

The important magnetic characteristics of permanent-magnet alloys are indicated by the part of the hysteresis loop lying between the residual induction and the coercive force and a curve plotted between B and the products of corresponding values of B and H for the same part of the loop. These products are proportional to the magnetic energy stored at the given values of induction. The maximum value of the energy-product may be taken as an index of magnetic quality. Typical curves for the three classes of permanent-magnet material are shown in figure 17 [34].

The choice of material for a given application depends upon several factors and, consequently, no one material can be considered as best under all conditions.

5. FEEBLY MAGNETIC MATERIALS

It is occasionally desirable to use ferrous materials for parts which should be practically nonmagnetic. Certain types of corrosion-resistant steel, as well as the well-known nonmagnetic manganese steel, meet this condition fairly well. The permeability of such materials may range from 1.01 to 1.10.

VII. MAGNETIC ANALYSIS

The magnetic properties of materials are very closely related to their other physical properties. It is possible, therefore, under proper conditions, to test materials by magnetic methods to determine whether or not they have certain requisite mechanical properties. The use of magnetic tests to obtain information about properties other than magnetic has been termed magnetic analysis. Magnetic analysis involves not only the carrying out of magnetic tests but also the interpretation of the results in terms of other characteristics of the material.

The magnetic and mechanical properties of materials appear to have no direct relationship by which one set of properties can be predicted from data on the other set of properties alone. It is necessary, therefore, to have a knowledge of the composition and nominal treatment in order to apply magnetic analysis in a practical way. For this reason most of the practical applications of magnetic analysis are based upon a comparison of some magnetic property of the test specimen with that of another similar specimen of the same composition known to possess the requisite mechanical properties and which is taken as a standard of quality. As a rule, the form of pieces which are to be tested is such that it is not feasible to determine the magnetic properties quantitatively in terms of the magnetic units and the usual types of magnetic permeameters cannot be used. It is necessary, in general, to devise special apparatus for each kind of product to be tested.

Magnetic analysis is also applied for the detection of hidden defects in ferromagnetic materials. This is done either by exploration or by the magnetic-powder method. For exploration, the material is magnetized and the distribution of magnetic flux within the material, or of magnetic leakage from the material, is determined by suitable test coils and instruments. Deviations from the normal distribution indicate variations in properties or the presence of defects. In the powder method, magnetic powder, either dry or suspended in a suitable liquid, is deposited on the surface of the magnetized piece. Cracks or other flaws are revealed by characteristic patterns on the surface. The principal advantage of magnetic analysis is that the tests are nondestructive, thus permitting a hundred percent inspection, if desired.

Another application of magnetic testing is known as theremomagnetic analysis. By following the changes in magnetic properties during the heating or cooling of a specimen, structural transformations can be detected. In this way a great deal can be learned of the phenomena associated with the heat treatment of materials.

Although many practical applications of the methods of magnetic analysis are in daily use, relatively little is known of the fundamental relationships involved, and so this constitutes a fruitful field for research. In carrying out such investigations, it is very important to have complete and accurate data with regard to the composition and history of the materials investigated. This precaution has too often been neglected.

VIII. TABLES AND REFERENCES

TABLE 1.—*Demagnetizing factors*

D [1]	$H=H'-NI$			$H=H'-KB_i$				Equation [2]	
	Ellipsoid	Mann [10]		Du Boise [11]		Shuddemagen [12]			
	N	N	K	N	K	N	K	N	K
10	0.2549	0.2550	0.0203	0.2160	0.0172	0.2000	0.0159	0.2512	0.0200
15	.1350	.1400	.0111	.1206	.0096	.1040	.0083	.1235	.0098
20	.0848	.0898	.00714	.0775	.0062	.0655	.0052	.0747	.0059
30	.0432	.0460	.00366	.0393	.00313	.0335	.00267	.0367	.00292
40	.0266	.0274	.00218	.0238	.00189	.0206	.00164	.0222	.00176
50	.0181	.0183	.00146	.0162	.00129	.0139	.00110	.0150	.00119
60	.0132	.0131	.00104	.0118	.00094	.0101	.00080	.0109	.00087
70	.0101	.0099	.00080	.0089	.00071	.0077	.00061	.0083	.00066
80	.0080	.0078	.00062	.0069	.00055	.0061	.00049	.0066	.00052
100	.0054	.0052	.00041	.0045	.00036	.0041	.00033	.0045	.00035
150	.0026	.0025	.00020	.0020	.00016	.0020	.00016	.0022	.00017
200	.0016	.0015	.00012	.0011	.00009	.0012	.00009	.0013	.00011

[1] D=ratio of length to diameter.
[2] Equations:

$$\log_{10}N=1.15-1.75\log_{10}D.$$
$$\log_{10}K=0.05-1.75\log_{10}D.$$

TABLE 2.—*Ratio of average value of H to value at mean radius in rings of rectangular and circular sections* [13]

Ratio of diameter to radial thickness	H_a/H_R	
	Rectangular	Circular
2	1.0986	1.0718
3	1.0397	1.0294
4	1.0216	1.0163
5	1.0137	1.0102
6	1.0094	1.0070
7	1.0069	1.0052
8	1.0052	1.0040
10	1.0033	1.0025
19	1.0009	1.0007

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