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BUREAU OF STANDARDS
George K. Burgess, Director

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# TESTING OF MEASURING TAPES AT THE BUREAU OF STANDARDS

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# TESTING OF MEASURING TAPES AT THE BUREAU OF STANDARDS[1]

ABSTRACT

The testing of the steel tapes used by the engineer and surveyor is done at the Bureau of Standards by comparison with a steel bench standard described in this paper. For the test of base-line tapes (usually made of invar with a length of 50 m) a much more elaborate apparatus is employed. This geodetic comparator is also described in detail. Results have shown that the length of 50 m invar base-line tapes can be determined with this apparatus to an accuracy such that the absolute errors of the certified results do not exceed 1 part in 1,000,000.

CONTENTS



## I. TAPE-TESTING LABORATORY

Two distinct classes of tape testing are carried out at the Bureau of Standards—the testing of steel tapes used in ordinary surveying, and verification of the more precise tapes used in geodesy.

All of the tape testing at the Bureau of Standards is done in a special underground laboratory, a view of which is shown in Figure 1. This is a room about 170 feet long and 7 feet wide, extending between two of the bureau buildings. This room, or "tape tunnel," as it is usually called, is provided both with brine coils and with steam pipes, so that a range in temperature from about 8 to 35° C. may be obtained for the purpose of determining the coefficients of expansion of tapes.

## II. BENCH STANDARD FOR TEST OF STEEL TAPES

In testing the ordinary steel tapes used by surveyors use is made of the steel bench, or wall, standard. This consists of two parallel bars of steel 2 inches wide and one-half inch thick, 164 feet long, each consisting of a number of bars united by autogenous welding. One of these bars is so graduated, both in feet and meters, that a

[1] By Lewis V. Judson, associate physicist.

tape can be compared with it when supported throughout; the other bar, which is the one farther from the wall, is so graduated, with the lines extending to the edge, that a tape suspended on pins at specified points and along the edge of the bar will be in position for direct comparison with the graduated bench. It is thus seen that comparisons of a tape when suspended at points can only be made at those points which are points of suspension of the tape.

The construction of the bench is shown in detail in Figure 2. It will be noted that the bench standard is supported on rollers so as to be free to expand or contract.

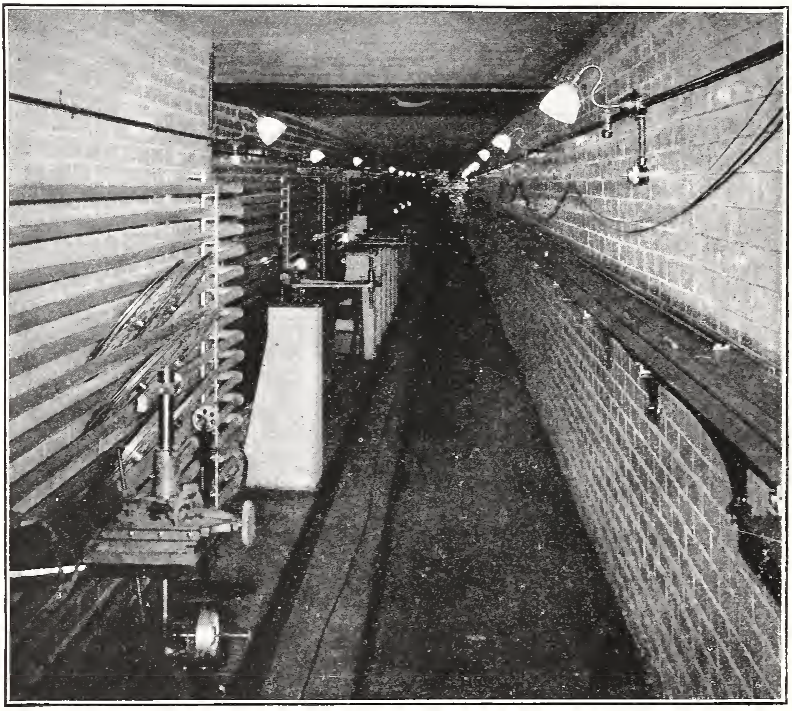

Fig. 1.—*Tape-testing laboratory at the National Bureau of Standards*

The graduations of the bench are on platinum plugs permanently set in the steel bars. The graduations on the bench are as follows: On Bench I, which is the bar on which tapes are compared when supported throughout, graduations are at each foot point from 0 to 10, at each 5-foot point from 10 to 100, at each 10-foot point from 100 to 150, and at the 33, 66, and 99 foot points. There are also graduations at each 1 m point from 0 to 50 m. On Bench II, which is the one on which tapes are tested when suspended at points, graduations are at each 10-foot point from 0 to 100, and at the 25, 75, and 150 foot points. There are also graduations at each 5 m point from 0 to 50 m. The zero graduations are all at the left end as one faces the bench.

In the comparison of a steel tape with this mural standard the tape is stretched by means of a calibrated spring balance. A special pulley-and-weight arrangement, such as is used for geodetic tapes and described later in this paper, might be designed for this purpose, but the calibrated spring balance is sufficiently accurate for the purpose. Comparisons of the lines of the tape with those on the bench are made with a thin steel scale having very fine lines especially graduated for this work. Coincidence at the zero mark and the variation indicated by the use of this steel scale is made precise by the use of a low-power microscope having a magnification of about 10 diameters.

These benches have been calibrated a number of times by means of specially graduated tapes, the total lengths of which have been determined on the geodetic comparator.

## III. STANDARD TAPES

As a basis for issuing a certificate and for placing the bureau's seal on a tape the Bureau of Standards has adopted the following specifications:

### 1. SPECIFICATIONS FOR STANDARD STEEL TAPES

A steel tape is standard when it conforms to the following specifications: It shall be made of a single piece of metal ribbon, and none of the graduations shall be on pieces of solder or on sleeves attached to the tape or on wire loops, spring balances, tension handles, or other attachments liable to be detached or changed in shape. The error in the total length of the tape, when supported horizontally throughout its length at the standard temperature of 68° F. (20° C.) and at standard tension, shall not be more than 0.1 inch per 100 feet (2 mm per 25 m). The standard tension is 10 pounds (4.5 kg) for tapes 25 to 100 feet or from 10 to 30 m in length and 20 pounds (9 kg) for tapes longer than 100 feet or 30 m.

### 2. CERTIFICATION OF TAPES

Tapes conforming to the above specifications will be certified by the Bureau of Standards, and a precision seal showing year of test will be placed on each tape certified. For tapes not conforming to the specifications a report will be issued, but the tapes will not be sealed. The bureau's serial number is placed on each tape tested, and this number on a tape simply signifies that it has been tested by the bureau and either a certificate or a report issued.

These specifications are intended to cover steel tapes of the highest precision and should not be taken to imply that only tapes conforming to them are suitable for use. For instance, tapes having the terminal graduation at the end of the ring or on plugs are for certain purposes very desirable, and the bureau will test and report on such tapes if satisfactory in other respects.

Certain points as to the details of testing of steel tapes should be noted:

Unless otherwise stated, the comparison of a tape with the bench standard is made at the center of the lines on the edge to which the shortest graduations are ruled. If all the graduations extend entirely across the tape, the ends farthest from the observer when the zero of the tape is at his left hand are used. On tapes which have been cut off at the zero mark, the extreme end of the steel ribbon is taken as the zero point and not the center of any line that maybe at that point. On tapes which have the zero point on a loop attached to the steel ribbon at the end, the zero is taken at the outside of this loop unless noted to the contrary.

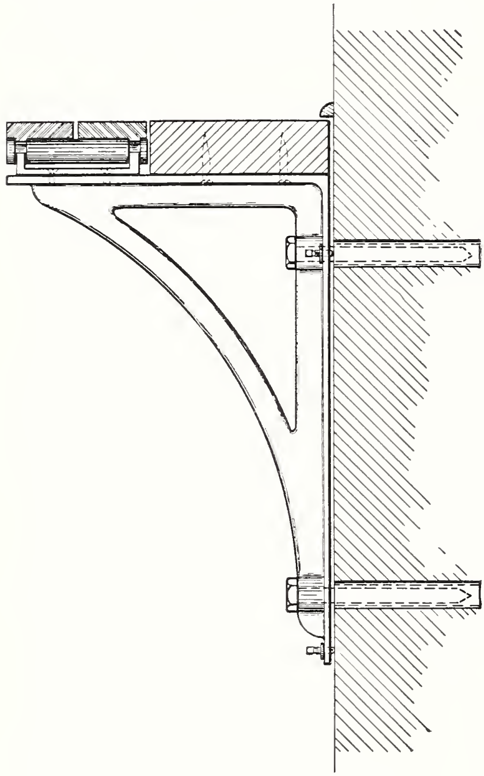

Fig. 2.—*Steel bench for testing steel tapes*

In the use of tapes for precision work, attention should be paid to the temperature and to the tension and corrections made for variation from the conditions given in the certificate or report. The accuracy of the balance used with the tape should be checked by comparison with a calibrated balance, or with calibrated weights using a suitable pulley wheel.

A plus correction in a tape certificate or report means that the tape is too long; that is, longer than the nominal length of the interval; a minus correction, that it is shorter by the amount indicated.

The observations are usually made at a temperature higher than 68° F., and in reducing to 68° F. (20° C.) the coefficient of expansion of the tape is assumed to be 0.00000645 per °F. (0.0000116 per °C.). The temperature at which the observations were made is stated in the certificate.

## IV. GENERAL DESCRIPTION OF THE GEODETIC COMPARATOR FOR BASE-LINE TAPES

The geodetic comparator is designed especially for 50 m tapes of high precision, although tapes having a length of an integral number of meters up to 25, also 30, 35, 40, or 45 m, can be compared. Should a demand arise, modifications could be made permitting 100-foot tapes properly graduated for geodetic work to be tested on this apparatus.

This comparator is shown at the left and central portion of the view of the tape tunnel given in Figure 1. This apparatus is modeled after the field apparatus used on the Holton, Ind., base in 1891 and described in Appendix No. 8 of the Report of the United States Coast and Geodetic Survey for 1892. Indeed, some of the equipment used in the field, notably the Woodward iced bar No. $B_{17}$, is in use in the comparator at the Bureau of Standards.

Eleven concrete piers are set at equal intervals of 5 m apart. Each of these piers has attached to its upper surface an adjustable mounting for a micrometer microscope. The two end microscopes are the most important ones, the cross hairs of the intermediate ones serving only as reference points in laying off the 50 m interval by means of the 5 m standard. In building up the 50 m interval the cross hairs are set on the lines of the 5 m bar and thus play a part analogous to that of pencil marks used by a carpenter in connection with his 2-foot rule in laying off a distance on a board.

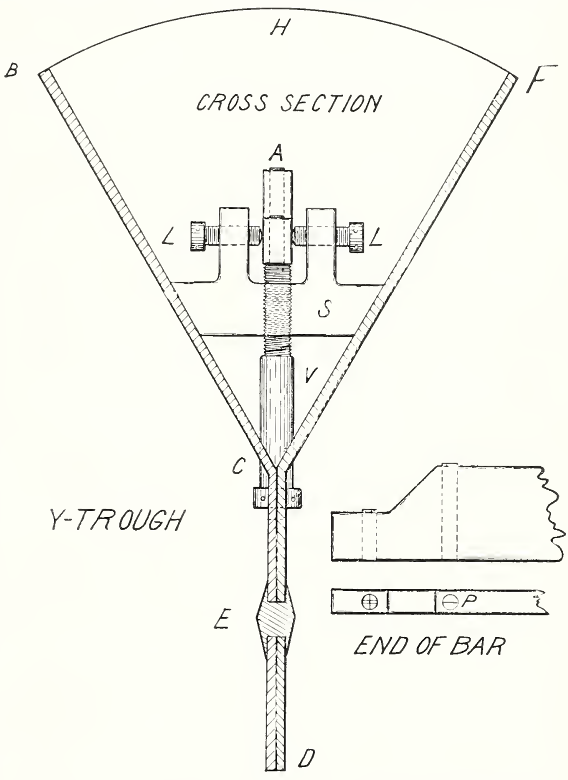


FIG. 3.—*Cross section of the 5 m bar and its trough*

The working standard in this geodetic tape comparator is the Woodward bar. This bar is mounted in a **Y**-shaped trough so that ice ground into small pieces can be packed around it. The bar in its trough is mounted on trucks, and the bar thus mounted can be moved by successive steps under the microscopes lengthwise of the tunnel. The trucks have the necessary focusing and lateral adjustments.

The procedure in testing a 50 m base-line tape consists in first laying off a distance of 50 m by means of the 5 m bar and the microscopes, and then comparing the interval on the tape with this distance. For this work the tension is applied to the tape by a weight, usually 15 kg, hanging vertically and fastened to the tape by a wire passing over a frictionless pulley. In order to compare tapes differing in length from 50 m by more than 1 or 2 mm a movement of one or both of the end microscopes is necessary. To measure the movement of the end microscopes, either intentional or accidental, the positions of the microscopes are determined with relation to two fiducial points at the floor level by means of a special device known as the cut-off, and described later.

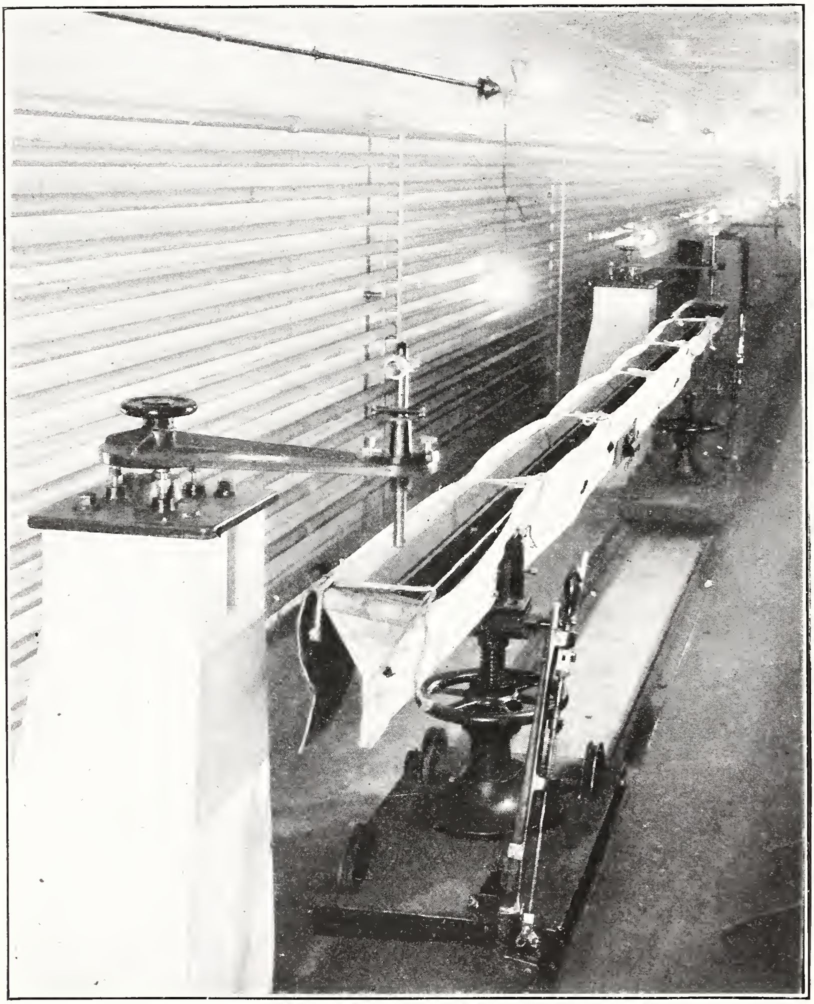

Fig. 4.—*Five-meter bar on the trucks*

## V. DETAILS OF THE GEODETIC COMPARATOR [2]

The Woodward 5 m bar is a rectangular bar of tire steel which was rolled in the steel works at Lancaster, Pa. It is 5.02 m long, 8 mm thick, and 32 mm deep. A cross section is shown at *A* in the accompanying illustration, Figure 3. The description by Doctor Woodward of this bar follows:

[2] Appendix No. 8, United States Coast and Geodetic Survey Report for 1892, p. 339.

The upper half of the bar is cut away for about 2 cm at either end to receive the graduation plugs of platinum-iridium, which are inserted so that their upper surfaces lie in the neutral surface of the bar. Three lines are ruled on each of these plugs, two in the direction of and one transverse to the length of the bar. These lines were ruled by Louis A. Fischer, adjuster in the Office of Standard Weights and Measures. The longitudinal lines, which serve to limit the parts of the transverse lines used, are 0.2 mm apart. Although great pains were taken to have these lines of the same width, the transverse lines differ widely, the narrow one being 16.3 $\mu$ and the broader one 36.2 $\mu$ wide.

This bar is known in the Survey records as No. 17. It is designated in all the work done with it thus far as $B_{17}$. The end having the narrower transverse graduation mark is called the *A* end.

To secure alignment of the bar, 11 German silver plugs of 5 mm diameter are inserted at intervals of 495 mm along the bar, so that they project about 1 mm above its top surface. The upper surfaces of these plugs are all the same distance, within a few hundredths of a millimeter, from the neutral surface of the bar. On the top of each plug is ruled a fine line in the direction of the bar, as shown at *P* in the illustration. The length of the bar as regards alignment is defined to be the distance between the transverse graduation marks when the upper surfaces of the alignment plugs are all in one plane and when the lines on these plugs are in one straight line. The means of securing these two adjustments are described below.[3]

The most important and distinctive part of this apparatus is the trough which supports the bar, keeps it aligned, and carries the ice load essential to control the bar's temperature. This trough is called the **Y** trough by reason of the resemblance of its cross section to the letter **Y**. The drawing shows a cross section of this trough. It is made of two steel plates 5.14 m long, 25.5 cm wide, and 3 mm thick. They are bent to angle *B C D* of the figure, and are riveted together as shown at *E*, thus making the angle of the trough $B\ C\ F = 60°$. The bar, shown in cross section at *A*, is supported at every half meter of its length by saddles, one of which [the end one] is shown in the figure. The vertical adjusting screws of the saddles project, as shown in the diagram, below the vertex of the trough, and their capstan heads are accessible through slots cut in the web of the trough. These slots serve also as drainage ways for the melted ice. To prevent circulation of air through them they are stuffed with cotton batting, through which the water percolates freely. The ends of the trough are closed with wooden **V**-shaped blocks.

The trough is very rigid in all directions, and especially so with respect to vertical stresses. It weighs 82 kg, exclusive of the bar and ice load. The whole trough is covered by a closely-fitting jacket of heavy white cotton felt, which protects the trough and ice load alike from direct radiation.

A correction, commonly termed the grade correction, must be applied because the foci of the intermediate microscopes are not absolutely in the plane passing through the foci of the two end microscopes. To obtain this correction the relative tilt of the 5 m bar, when it is in position for observation at the 10 successive positions, is read by means of a graduated sector reading by two opposite verniers to 10 seconds. This sector, with a sensitive spirit level attached to the vernier arm, is mounted to one side of the trough near its middle point. The total grade correction is length of the bar times the sum of the differences from unity of the cosines of the angles made by the bar with a line through its mean position.

[3] The form of bar described is evidently not the best form. Theory and experience indicate that a bar having a Y-shaped cross section with metric subdivisions on its neutral surface would best meet the requirements. However, the question which presented itself in planning the apparatus was not what is the best form of bar, but what is the most economical form possessing the requisite properties.

40557°—27——2

This bar has been compared with the platinum-iridium standard meter No. 21 a number of times, beginning in 1892. Special facilities for this comparison are incorporated in the equipment of the present geodetic tape apparatus. For the limited number of bars of this type requiring test in this country a very simple arrangement gives adequate results. In Europe, where 4 and 5 m bars are more frequently used in geodesy, rather elaborate instruments have been constructed. Between the eighth and ninth piers, counting from the north end of the bureau's tape tunnel, there are intermediate piers with an interval of 1 m between them. With microscopes mounted on these piers it is a relatively simple matter to build up a 5 m interval by the use of a 1 m bar packed in ice and mounted so as to be movable in a longitudinal direction and to have the requisite adjustments for focus and alignment. By substituting the 5 m bar under the microscopes its total length can be compared with a distance equal to five times the length of the 1 m bar. A repetition of this process 15 or 20 times generally results in a comparison of adequate precision.

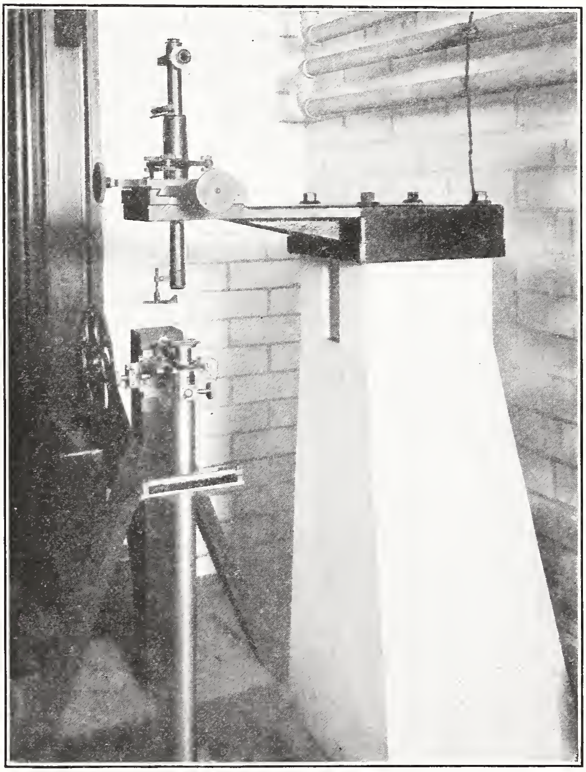

Fig. 5.—*Tension apparatus, one of the end microscopes, and a cut-off cylinder*

The general design of the trucks for supporting the 5 m bar is shown in Figure 4. The screws for lateral adjustment and for leveling are to be noted. A modification of the design has been proposed which will permit the trough to be lowered farther than is at present possible. At the same time it is hoped to be able to arrange the adjustments so that they can be handled more readily by the observer at the microscope.

Each of the two end microscopes has a magnification such that one division on the divided drum of the micrometer represents a measurement of 0.75 $\mu$. The intermediate micrometers have a magnification such that one division of the micrometer represents 1 $\mu$. All the microscopes have the prism type of vertical illuminator,

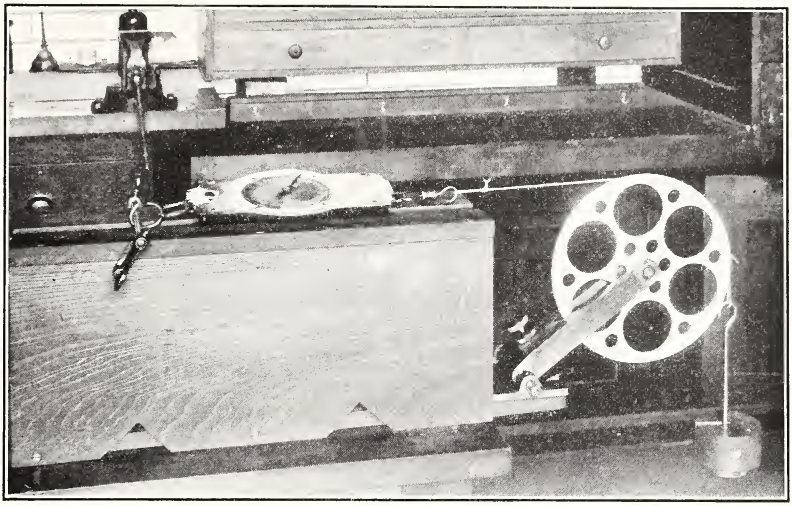

FIG. 6.—*Apparatus for testing spring balances*

the light being received through an opening in the upper part of each pier from electric bulbs placed behind it.

The tension apparatus is shown in Figure 5. This apparatus was designed after the author had seen the apparatus at the Topographical

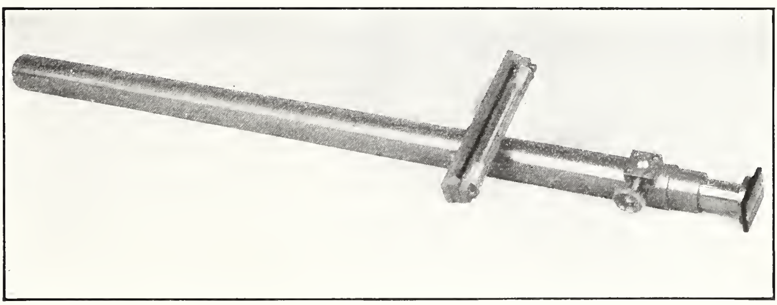

FIG. 7.—*Cut-off cylinder*

Surveys Laboratory in Ottawa, Canada, and has been found to be excellent for this work. As illustrated in Figure 6 it is shown as modified for testing spring balances. Blue prints of this apparatus can be furnished to anyone contemplating making a device of this type.

Secondary in importance as far as the general principles of the measurements of geodetic tapes are concerned, the cut-offs are of primary importance in securing a high degree of precision. The cut-off appears to have originated with the German instrument makers, Repsold, who designated it "Ablothungszylinder," whence the

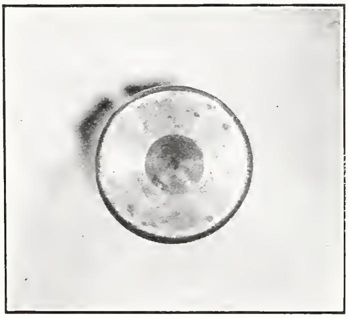

FIG. 8.—*Lower end of cut-off cylinder*

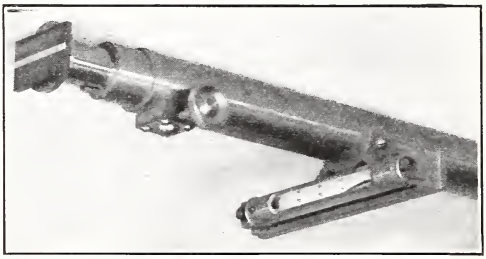

FIG. 9.—*Level vial and graduated scale on cut-off cylinder*

present designation in the English language. The cut-off cylinder as designed at the Bureau of Standards is shown in Figure 7. Its lower end, as shown in Figure 8, has a conical hole which is coaxial with the cylinder itself. A spirit level, shown more clearly in Figure 9, is mounted on the cylinder, and a scale divided to millimeters is attached to the top of the cylinder, which is movable with respect to the body of the cylinder by a rack and pinion. One cut-off is mounted at each end of the 50 m base, the cut-off being held in a frame as shown in Figures 10 and 11. This framework is in effect a collar. The lower end of each cut-off is set on a sphere specially mounted (see fig. 12) on a pier, the top of which is at the level of the floor. The appearance of one of the mounted cut-offs is shown in Figure 5. The function of this apparatus is to project a point in the focal plane of a microscope down to a horizontal plane passing through the center of the sphere at about floor level, or vice versa.

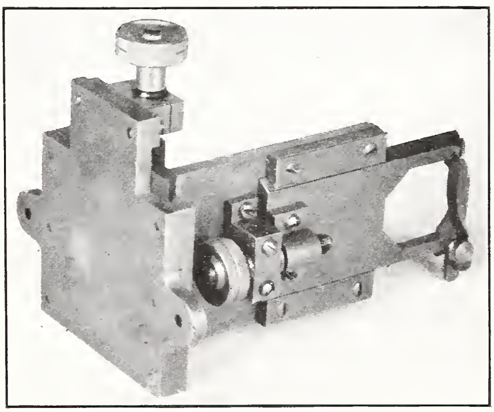

FIG. 10.—*Mounting for holding cut-off cylinder*

From the geometry of the sphere and the conical hole, the axis of the cone will always pass through the center of the sphere. If the two cut-offs are so adjusted that that plane through the axis of each cylinder which is parallel to the level vial is a common vertical plane through the centers of the two spheres, and the graduations on the upper surface of each cut-off are perpendicular to this plane, then a reading on each respective level vial will give that inclination of the cut-off, which, together with a reading of each micrometer microscope on some known line on the scale of its cut-off, will give the location of that line in space. By a simple calibration the relation of the tilt of one scale division of the level bubble can be determined with respect to the shifting of a line on the upper end of the cut-off. In this way a position, relatively unstable, in the focal plane of the microscope can be transferred to the stable pier below and referred back again at will. In order to increase the precision, four sets of measurements are made, two with the spirit level in front and two with it in back.

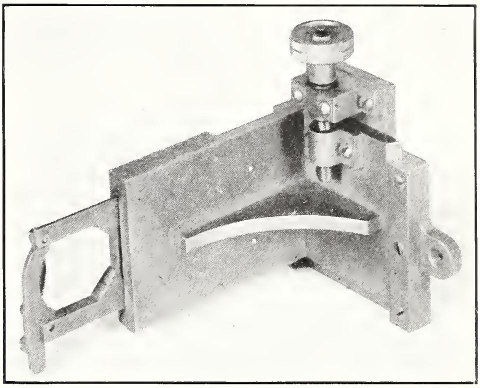

Fig. 11.—*Mounting for holding cut-off cylinder (second view)*

When a tape is being tested on the geodetic comparator it is supported on specially designed ball-bearing rollers, so that the amount of friction at the supports is negligible even in the most precise work.

## VI. PROCEDURE IN TESTING GEODETIC TAPES

### 1. ADJUSTMENT OF APPARATUS

The first adjustment necessary before any testing can be done on the geodetic comparator is to align the microscopes. This is done by sighting on a rather tightly stretched wire. The microscopes are also placed so that the distance from center to center is 5 m within a millimeter.

The alignment of the 5 m bar is made by means of the adjusting screws provided for the purpose, a fine silk thread being used for the guide. The 5 m bar is also approximately leveled by means of a striding level resting on successive pairs of alignment plugs, the final adjustment being made with the bar packed in ice.

All tape supports are adjusted for height so as to lie in the straight line passing through the focal points of the two end microscopes. This adjustment is usually made by the aid of a surveyor's level.

### 2. ESTABLISHMENT OF THE BASE

The 50 m base is laid off by successive 5 m steps, using the iced bar. The terminal points are projected down by means of the readings on the cut-off apparatus and the horizontal distance between the centers of the sphere calculated. The centers of these spheres are sometimes called the fiducial points, and the distance between them is generally designated as the length of the base. For a complete establishment of the base, two determinations of the base are made the first thing in the morning, one with the bar being moved to the right, the other with it being moved to the left, and two determinations in the afternoon after the day's comparisons of tapes is completed. This is the standard procedure for each day of actual testing. The grade correction is also calculated from readings on the sector attached to the trough of the 5 m bar.

### 3. COMPARISON OF TAPES

The tape suspended at designated points and under a specified tension is adjusted so that the lines are properly located in the field of the microscopes. If necessary to move one or both of the microscopes, cut-off readings give a precise value for the amount of this motion.

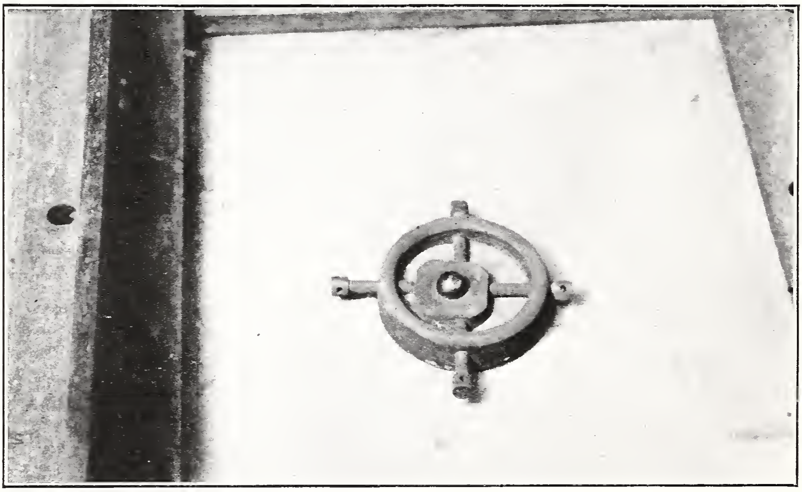

FIG. 12.—*Sphere in floor, for use with cut-off cylinder*

Standard conditions for United States Coast and Geodetic Survey invar base-line tapes are: Tension, 15 kg; suspension at 0, 25, and 50 m points, and also suspended at 0, 12.5, 25, 37.5, and 50 m points; thermometers weighing, with cases, 45 g each attached to tape at 1 and 49 m points. These invar tapes are usually about 6.3 mm wide and 0.5 mm thick and weigh about 25.5 g per m.

The length of a geodetic base-line tape when supported throughout is found more accurately by computation than by comparison on the steel bench. In this calculation the effect of the weight of the thermometers upon the length of the tape must be taken into consideration.[4]

[4] Judson, L. V., The Effect of Concentrated Loads on the Length of Measuring Tapes, B. S. Sci. Paper No. 534.

The temperature of the tape is found by means of the two thermometers on the tape and nine equally spaced thermometers hung in the air very near the tape.

Although steel tapes can be compared on this geodetic apparatus, such tapes are not usually suitable for precise geodetic work and

| Micrometer Readings |
|---|
| 21.96<br>4 |
| 23.02<br>2 |
| 28.25<br>7 |
| 19.61<br>2 |
| 19.27<br>5 |
| 20.31<br>5 |
| 21.48<br>50 |
| 23.10<br>0 |
| 23.95<br>3 |
| 26.38<br>42 |
| 23.92<br>2 |
| 23.17<br>8 |
| 21.57<br>61 |
| 20.45<br>1 |
| 19.34<br>9 |
| 19.76<br>5 |
| 28.28<br>8 |
| 23.07<br>5 |
| 21.90<br>4 |
| 21.23<br>6 |

| | Micrometer Readings | | | Level | | |
|---|---|---|---|---|---|---|
| | Bar | Cut-off | Diff. | L | R | Diff. |
| Left | | 20.55<br>22.12<br>22.12<br>20.57 | (C.O-Bar)<br>0.0 μ | 13.8<br>10.3<br>10.0<br>12.0 | 14.8<br>18.2<br>18.5<br>16.5 | (L-R)<br>-22.2 μ |
| | 21.34 | 21.34 | 0.00 | 11.5 | 17.0 | - 5.5 |
| Right | | 17.45<br>17.97<br>18.04<br>17.29 | (Bar-C.O)<br>+628.5 μ | 20.0<br>21.5<br>21.0<br>21.0 | 17.5<br>16.0<br>16.4<br>16.0 | (R-L)<br>- 22.7 μ |
| | 26.40 | 17.69 | + 8.71 | 20.9 | 16.5 | - 4.4 |
| Sum | | | +628.5 μ | Sum | | - 44.9 μ |
| Right | | | (Bar-C.O)<br>+628.5 μ | | | (R-L)<br>- 22.7 μ |
| | 26.40 | 17.69 | + 8.71 | 20.9 | 16.5 | - 4.4 |
| Left | | 21.18<br>19.85<br>19.79<br>21.00 | (C.O-Bar)<br>- 55.5 μ | 21.0<br>19.8<br>19.0<br>22.0 | 7.5<br>8.3<br>9.1<br>6.5 | (L-R)<br>+ 50.9 μ |
| | 21.24 | 20.46 | -0.78 | 20.4 | 7.8 | + 12.6 |
| Sum | | | +573.0 μ | Sum | | + 28.2 μ |

| | Microns | Microns |
|---|---|---|
| Micrometer Differences | + 628.5 | + 573.0 |
| Scale Corrections | 0.0 | 0.0 |
| Level Corrections | - 44.9 | + 28.2 |
| 10 × Correction to 5 Meter Bar | - 297.0 | - 297.0 |
| Grade Correction | - 39.7 | - 39.7 |
| Total Correction | + 246.9 | + 264.5 |

FIG. 13.—*Observation and computation sheet: Length of geodetic comparator*

should not be submitted to the Bureau of Standards for this test without previous correspondence with the bureau about them.

## 4. OBSERVATION AND COMPUTATION FORMS

The observations and computations of the length of the geodetic base are made as shown in Figure 13. Figure 14 illustrates the method of recording and computing a comparison of a tape. The temperature of the tape is also recorded. In Figure 15 is given the method of computing the length of a tape when supported throughout from observations on its length when suspended in loops.

| Readings on Cut-Off | | | | | | | |
|---|---|---|---|---|---|---|---|
| Left | | | Level Corrections | | Right | | |
| Micrometer Reading | Level L | Level R | (R-L) | (L-R) | Level L | Level R | Micrometer Reading |
| 19.92 | 10.7 | 7.7 | | | 15.3 | 14.6 | 17.65 |
| 21.34 | 10.9 | 7.7 | | | 14.2 | 16.8 | 16.52 |
| 21.35 | 10.8 | 7.8 | -12.1 μ | -2.6 μ | 14.0 | 16.9 | 16.50 |
| 19.97 | 10.9 | 7.8 | -3.0 | -0.5 | 17.0 | 14.0 | 17.37 |
| 20.64 | 10.8 | 7.8 | Sum | | 15.1 | 15.6 | 17.01 |
| -3000.0 μ Cut-off Intervals 0.0 μ | | | | | | | |

| Method of Support | Difference (Tape-C.O.) | Readings on Tape: Left Mic. | Readings on Tape: Right Mic. | Difference (C.O.-Tape) | C.O. Intervals / Level Corrections |
|---|---|---|---|---|---|
| 3 | | 18.50 | 30.59 | | C.O. Intervals -3000.0 |
| | | 61 | 63 | | Level Corrections |
| | -146.7 μ | 60 | 77 | -987.2 μ | - 14.7 |
| | - 2.06 | 63 | 77 | - 13.68 | |
| | L.C.O. 20.64 | 18.58 | 30.69 | R.C.O. 17.01 | - 1133.9 |
| | Correction to Length of Comparator | | | | + 255.7 |
| | Correction to Length of Tape | | | | - 3892.9 |
| 5 | | 37.83 | 8.54 | | C.O. Intervals -3000.0 |
| | | .88 | 56 | | Level Corrections |
| | +1226.6 μ | 86 | 55 | +611.2 μ | - 14.7 |
| | +17.23 | 91 | 52 | + 8.47 | |
| | L.C.O. 20.64 | 37.87 | 8.54 | R.C.O. 17.01 | + 1837.8 |
| | Correction to Length of Comparator | | | | + 255.7 |
| | Correction to Length of Tape | | | | - 921.2 |
| 5 | | 37.62 | 8.35 | | C.O. Intervals -3000.0 |
| | | 74 | 33 | | Level Corrections |
| | + 1213.0 μ | 71 | 37 | + 624.2 μ | - 14.7 |
| | + 17.04 | 69 | 38 | + 8.65 | |
| | L.C.O. 20.64 | 37.69 | 8.36 | R.C.O. 17.01 | + 1837.2 |
| | Correction to Length of Comparator | | | | + 255.7 |
| | Correction to Length of Tape | | | | - 921.8 |
| 3 | | 12.37 | 24.34 | | C.O. Intervals - 3000.0 |
| | | 31 | 37 | | Level Corrections |
| | - 586.6 μ | 42 | 35 | - 530.4 μ | - 14.7 |
| | - 8.24 | 37 | 30 | - 7.35 | |
| | L.C.O. 20.64 | 12.37 | 24.36 | R.C.O. 17.01 | - 1117.0 |
| | Correction to Length of Comparator | | | | + 255.7 |
| | Correction to Length of Tape | | | | - 3876.0 |

FIG. 14.—*Observation and computation sheet: Length of tape*

## 5. PRECISION ATTAINABLE; SOURCES OF ERROR

The United States Coast and Geodetic Survey, in requesting the test of base-line tapes, usually specifies that the probable error shall not exceed 1 part in 1,000,000 and that the absolute error shall not exceed 1 part in 300,000. It has, however, recently been considering the advisability of increasing the precision specified for the tape standardizations. It is believed that the absolute error of the determinations as certified at the present time to the Coast and Geodetic Survey does not exceed 1 part in 1,000,000. The consistency of the results is greater still, judging from data on hand. For instance, the length of a tape supported throughout as computed from observations made with the tape suspended in two loops usually does not differ by more than 20 $\mu$ from that computed from observations made on the tape suspended in four loops; very frequently the discrepancy is less than 10 $\mu$.

3 SUPPORTS

$N=2$ $\quad$ $L=\frac{N}{24}\left(\frac{W}{T}\right)^2 L^3$

$L=25$ m

$W=25.8$ g $\quad$ $=\frac{2}{24}\left(\frac{25.8}{15,000}\right)^2 25^3$

$T=15,000$ g

$=0.003851$ m

| | mm |
|---|---|
| Tape is longer when supported throughout by | +3. 851 |
| Correction to length with 3 supports at 24.0° | −3. 717 |
| Correction to length supported throughout at 24.0° | +0. 134 |

5 SUPPORTS

$N=4$ $\quad$ $L=\frac{4}{24}\left(\frac{25.8}{15,000}\right)^2 12.5^3$

$L=12.5$ m

$W=25.8$ g $\quad$ $=0.000963$ m

$T=15,000$ g

| | mm |
|---|---|
| Tape is longer when supported throughout by | +0. 963 |
| Correction to length with 5 supports at 24.0° | −. 823 |
| Correction to length supported throughout at 24.0° | −. 140 |

Mean correction to length supported throughout at 24.0° +0.137 mm

FIG. 15.—*Determination of the length of 50 m tape supported throughout from observations made on tape supported at intervals*

Certain potential sources of error are worthy of consideration. The absolute length of the base may be in error because of an error in the length of $B_{17}$. $B_{17}$, however, has been determined with an accuracy such that 10 $\mu$ in the 50 m will amply cover this. Errors in sighting on the lines of $B_{17}$ because of improper focus, or improper illumination, or other similar causes, besides being small, are not of the cumulative type. The total grade correction is less than 50 $\mu$. Its precision is indicated by the smallness of the changes which have occurred in it in the past three years. All determinations during that period lie between $-35$ and $-44$ $\mu$. As an indication of the smallness of the error caused by friction in the pulleys, it may be stated that no ordinary means suffice to detect the friction. The weights, too, have been carefully calibrated. An unevenness of the temperature distribution and an uncertainty as to radiation effects are recognized, but the low coefficient of expansion of invar makes these relatively unimportant.

In fact, so satisfactory has this apparatus proven that, if an opportunity presented itself to construct a new tape-testing laboratory and apparatus, the present apparatus would either be duplicated with minor changes or used again. The changes would be more noticeable in the laboratory itself. Twenty feet additional length, thermostatic control, adequate means for hanging up tapes when not actually being compared, and a suitable ventilating system—these are some improvements that might well be made.

WASHINGTON, March 4, 1927.