DEPARTMENT OF COMMERCE

CIRCULAR
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, Director

No. 27

THE PROPERTIES AND TESTING OF OPTICAL INSTRUMENTS

[2d Edition]

ISSUED AUGUST 9, 1918

PRICE, 10 CENTS
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CONTENTS

I. CLASSIFICATION OF OPTICAL INSTRUMENTS

Optical instruments, in general, may be divided, roughly, into three classes, as follows:

1. Instruments for observation: Spectacle lenses, reading glasses, opera glasses, field glasses, telescopes, periscopes, microscopes, etc.

2. Instruments for reproduction: Camera lenses, photographic telescopes, photomicrographic cameras, condenser lenses, projection lenses, etc.

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[1] **This edition is to be considered as an emergency edition, pending a more complete revision in a few months. The first edition of this circular was issued Dec. 15, 1910.**

3. Instruments for measurement: (*a*) of directions—sextants, theodolites, spectrometers, etc.; (*b*) of light intensities—photometers; (*c*) of polarized light—polarimeters, saccharimeters; (*d*) of distances—range finders, interferometers, etc.

II. PROPERTIES OF OPTICAL INSTRUMENTS

The properties of all optical instruments depend partly upon their general design and partly upon the degree of correction and workmanship attained in the lenses and other parts used in their construction.

It is impossible to make optical instruments which are perfect in every way, so the design of all optical instruments is, therefore, a compromise giving the maximum of those properties most desirable for the use to which the particular instrument is to be put. In some instruments (e. g., spectacles, hand magnifiers, condensers, low-power telescopes) the requirements are not severe and are easily met, while in others (e. g., photographic lenses, microscopes, periscopes) the severest demands are made on both designer and constructor. In this circular the important properties of different types of optical instruments are discussed in an elementary way, and methods are described for testing the performance of each type of instrument.

In recent years, many types of optical instruments have been developed and have come into more or less common use. At the same time, a great deal has been written in the English language on optical subjects, but there is no general discussion of optical instruments in nontechnical language for the benefit of the average person who owns, for example, opera glasses or a camera. The primary purpose of this circular is to correct this deficiency by giving a simple description of the principal features of optical instruments, to explain the causes and corrections of various imperfections, and to indicate methods of testing for the presence of imperfections which mar the ideal performance of optical instruments. This information, for the most part, can be found in various textbooks and treatises on optical subjects, but the fact that it is inaccessible to many people, because it is widely scattered and generally couched in mathematical language, is the reason for this presentation. This circular should not be mistaken for a complete treatise on optical instruments. It is intended first of all to serve the public who use optical instruments but who have had little opportunity to study the physical theory of such instruments.

1. INSTRUMENTS FOR OBSERVATION

The instruments for observation are of the less specialized types and the considerations involved in making them good optically hold quite generally. The most important characteristics of the instruments which are used to aid vision are the following: (1) Definition and resolving power; (2) magnification; (3) brightness of image; and (4) field of view.

(1) DEFINITION AND RESOLVING POWER

It is obviously of prime importance to have an observing instrument reveal a group of points as such, instead of fusing them into a blotched area. Aside from its power of revealing faint stars, the value of an astronomical telescope is rated by its effectiveness in separating double stars. The same quality is needed in terrestrial telescopes—in fact, all observing instruments—in that it makes them effective in distinguishing individual objects and their form.

When a beam of light from a point passes through a lens an image is formed which resembles the point, more or less, depending on whether the lens is well or poorly designed and constructed. In general, the image of a luminous point, for example a star, will be a circular disk of light surrounded by a set of concentric dark and bright rings. The maximun brightness of the first bright ring is only about 1/60 of the brightness at the center. The others are still less bright, so that more than 9/10 of the whole light is concentrated within the area of the second ring. This diffraction pattern in image formation is a natural consequence of the wave nature of light and the finite character of these waves imposes a limit on the separating or resolving power of an optical instrument. The image of two bright points that are close together will consist of two correspondingly close disks, and each will be surrounded by diffraction rings. The points are said to be resolved when the image disks are far enough apart to be distinguished as two. This is generally considered to occur when the central disk in the image of one point coincides with the first dark ring in the image of the other. The brightness midway between the two central disks is then 81 per cent of that at the centers of the disks themselves. Under certain conditions duplicity may be detected when the disks are still closer. For a circular aperture, the diameter of the first dark ring is $d=\frac{2.4\lambda F}{D}$ where F is the focal length of the lens, D is the diameter of the lens, and λ is the wave length of light (in effect about 0.00055 mm for white light). The angle in radians subtended at the center of the lens by two disks which are just resolved, is therefore $\phi = \frac{1.2\,\lambda}{D}$. This angular distance between the closest objects which can be resolved (e. g., double stars or diatoms) is called the *theoretical limit of resolution* of the lens.

The angular *resolving power* of a lens is thus directly proportional to its diameter because the diffraction patterns of the image decrease in diameter as the aperture of the lens increases. To resolve stars at an angular distance of one second of arc requires a linear aperture (lens diameter) of about 12 cm and the Yerkes telescope with an aperture of 100 cm should, therefore, be able to resolve two stars which are only 0.12 of a second of arc apart.

Whereas in the telescope we generally use a linear measure of aperture and an angular limit of resolution, it is more convenient in the case of the microscope to use a linear limit of resolution expressed in terms of angular aperture. In the latter case, the refractive index, n, of the medium intervening between the object and the lens is also taken into account and the limit of resolution is expressed as

$$\delta = \frac{\lambda}{2n \sin A}$$

in which δ is the linear distance between resolved points and A is the angular semiaperture of the lens, as seen from the object. The quantity $n \sin A$ is well known as the "numerical aperture." The extreme value possible for A is a right angle so that for the microscopic limit we have

$$\delta = \frac{\lambda}{2n}$$

Thus, this limit can be reduced only by a diminution in λ, as in the ultra-violet microscope, or by an increase in n, as in the immersion microscope.

The eye is a compound-lens system which forms a disk image (diffraction pattern) of a point in the same way as any other lens system. The results of this can readily be seen if one draws two fine lines close together on a piece of paper and then tries to distinguish them at different distances. The two lines will be visible as two up to a certain distance, beyond which they will fuse and become unresolved. For this reason, in viewing oil paintings, one should be at a sufficient distance for the brush marks to fuse and thus allow the observer to see the picture as a whole.

Likewise in halftones and in certain color reproductions the dots are placed just close enough together so that the unaided eye can not distinguish them separately. Consistent with the above presentation, the limit of resolution of the eye varies with changes in the size of the pupillary aperture (2 to 8 mm), but it is generally agreed that under ordinary conditions two point sources of light can be distinguished if they are separated by an angular distance of 1 minute of arc. The unaided eye can see stars which subtend no sensible angle, but this visibility is a question of brightness only, and has nothing to do with resolving power. When stars closer than 1 minute of arc are to be recognized as separate, telescopic assistance is required. Similarly, with a microscope a bright object, however small, can be seen, but if its dimensions be less than one-half a wave length of the light its separate parts can not be distinguished.

In addition to the limits imposed upon the definition and resolving power of optical instruments by the undulatory nature of light, other restrictions are imposed on the one hand by the granular structure of the retina, and on the other by residual aberrations of various kinds in the instrument as well as in the eye. In the retina, the distance between centers of adjacent "rods," the nerve filaments responding to light stimulus is nearly 4/1000 of a millimeter. This is the size of the image produced on the retina by about 7/100 of a millimeter at 25 cm (distance of most distinct vision for the normal eye). The angle subtended by this distance corresponds roughly to the limit of resolution of the eye, i. e., one minute of arc.

"Aberrations" in the image refer to imperfections whose causes are inherent in all lens construction. By combining properly designed and constructed lenses, these aberrations may be neutralized to some extent. In general, those of a lens system are least troublesome in the neighborhood of the optical axis, and this region, therefore, shows the maximum definition or resolution.

Definition is the most important single characteristic which an observing instrument possesses, for it determines the assistance which the instrument may give to the eye in distinguishing details that could not be distinguished by the unaided eye. Telescopes enable the eye to see details which could not otherwise be distinguished from the given point of observation, and microscopes permit the recognition of details which are entirely outside the limit of the eye under any circumstances. The aberrations of such instruments are usually corrected carefully for objects within a few degrees of the axis, and in this region such an instrument should give definition equal to its theoretical resolving power at full aperture. A more detailed discussion of aberrations is given under "Instruments for reproduction."

When an instrument for observation gives as good definition as the eye is capable of perceiving, the definition may be regarded as satisfactory. The apparent limit of resolution of such an instrument, which is the product of the true limit of resolution by the magnification, should therefore be at least one minute of arc.

(2) MAGNIFICATION

An optical instrument may form either a real or a virtual image of the object under inspection. Instruments for reproduction, such as the camera and the projection lantern, form *real images*, images which actually exist in space and are formed on a photographic plate or a screen. Instruments for observation, on the other hand, form *virtual* images, images from which rays only apparently proceed, but which to the eye are as real as if they did. In all cases of vision, however, a real image is actually formed on the retina of the eye. To the eye the virtual image appears to be, and can always be treated as, a real image located at a definite point in space. Therefore, no distinction between the two kinds needs to be made.

The law governing magnification by a so-called thin lens is very simple, and sufficiently accurate for practical purposes to serve in the present discussion. Consider the extent of the object and image in a direction perpendicular to the optic axis of the lens. Calling these the diameters, respectively, of object and image, the law of magnification states that the ratio of these diameters is simply equal to the ratio of their respective distances from the lens. It follows that the *area of an image is to the area of the corresponding part of the object* as the *square of the distance of the image from* the lens is to the *square of the distance of the object.*

Some difficulty arises in directly applying this law of magnification when observing instruments, such as telescopes, are used, because of the uncertain distance of both object and image. In such cases the visual angle (=angle subtended by object or image at the eye) may be used. In case of the telescope, the magnification is given simply by the ratio of the visual angles subtended at the eye by image and object. With telescopes the magnification thus obtained is, except for very near objects, practically constant for all distances of object and image, provided the latter is clearly focused and is numerically equal to the ratio between the focal lengths of objective and ocular. In applying this law, precaution must be observed, of course, to view object and image from the same position.

Another important ratio which expresses the magnification produced by the telescope is that of the diameters of *entrance* and *exit pupils*. The *entrance pupil* is the image of the stop which limits the width of the beam that may pass through the instrument when this stop is viewed through the objective end; the *exit pupil* is the image of the same stop as viewed through the ocular. With the telescope the mounting of the objective is often the limiting stop, hence it is the entrance pupil. The image of this seen through the ocular (often a real image) is the exit pupil

The image produced by any optical instrument is almost universally at a place and of a size different from the object. Thus, the camera gives a reduced image upon the photographic plate; the projection lantern forms an enlarged image on the screen. Both the position and the size of the image depend upon the power of the instrument and the position of the object. With both of these instruments, the farther the object is from the lens the nearer and smaller the image. The greater the "principal focal length" of the lens (distance to the "burning point") the larger the image distance for the same distance of object. As the object distance increases, the image approaches a fixed position—the "principal focus"—so that all very distant objects are imaged on the principal focal plane. Thus for lenses of short focal length all objects beyond the first few feet are practically in focus, the principle upon which the universal focus lens operates.

As an optical instrument the eye is similar to a photographic camera with a real image formed on the retina. Instead of varying the distance between the lens and the retina, the eye largely "accommodates" to different distances by changing the focal length of the "crystalline" lens. This is effected by increasing the convexity of both front and back surfaces.

The so-called normal eye can acommodate to rays ranging from divergence from points as near as 25 cm in front to convergence toward points 100 cm behind the eye; hence the separate pencils of light emerging from an observing instrument must be at least within this range of accommodation for the normal eye. Parallel pencils or pencils diverging from the 25 cm point are customarily assumed.

The size of the image formed on the retina is obviously a measure of apparent size of the object. Consider a thin lens camera which is the exact equivalent of the normal eye and call it a model eye. Calling 25 mm, though more nearly 20, the distance of the retina behind the equivalent lens, and accepting the experience that the normal eye can resolve at 25 cm, 1/10 mm but not 1/20 mm, it follows directly from the law of magnification that the "normal eye can resolve retinal images 1/250 mm in diameter but not 1/500. Take the myopic man, with the same retinal resolving power, but who places the object at 12.5 cm for distinct vision. His eye will separate retinal images of the same diameter as the other, but since the object is half the distance from the eye he will resolve points between 1/20 and 1/40 mm apart, other factors being equal.

The performance of the eye as an optical instrument is commonly impaired by errors of refraction which may be compensated by the use of spectacle lenses. The condition of short sight, or myopia, is due to the elongation of the eye so that the retina lies behind the principal focus. All objects, therefore, which lie beyond a certain point are indistinctly seen; rays from them do not have the necessary divergence to be focused on the retina. This divergence may be obtained and myopia corrected by the use of suitable negative lenses. In hypermetropia, the retina is in front of the principal focus of the eye. In repose such an eye does not focus parallel rays from a distant object and, still less, divergent rays from a near object. This defect is corrected by convex (positive) lenses. Some eyes have differences in the refractive power of various meridians and a defect of vision known as astigmatism results. This condition is improved by the use of lenses whose surfaces are segments of cylinders. In ophthalmic practice the unit of refractive power is that of a lens whose focal length is 1 m. This unit is called the diopter. A lens of two diopters has a focal length of half a meter.

The magnification desirable in a given instrument depends upon several factors. If the exit pupil is greater than the pupil of the eye, the full aperture of the objective is not utilized. Hence, to obtain full resolving power, the magnifying power of a telescope should not be less than D/p where D is the diameter of the objective and p is the diameter of the eye pupil. This may be called *normal magnification.*

If the full resolving power of the objective is to be used, the magnification of the ocular should be sufficient to enable the eye to distinguish all the detail that exists in the image formed by the objective.

For the telescope the focal length of the eyepiece required for normal magnification may be readily calculated if the focal length and diameter of the objective be given and if a value be assumed for the diameter of the pupil of the eye. Inasmuch as the magnification may be expressed either in terms of focal lengths of objective and eyepiece or of entrance and exit pupils, one may write

$$M=\frac{F}{f}=\frac{D}{p}$$

There results immediately

$$f=F\frac{p}{D}$$

Oculars of shorter focal length than this are frequently employed, because it is very fatiguing to work just at the limit of resolution. The eye can better stand the slight diminution in brightness that necessarily accompanies this higher magnification rather than the continued strain that would otherwise be involved in close observation. On the other hand, it is sometimes desirable to allow the eye a little freedom of motion without sacrificing brightness, and consequently, with low-power instruments, the exit pupil is sometimes made appreciably larger than the normal eye pupil, and the magnification is consequently less than that given as the normal magnification.

The considerations of the last paragraph likewise apply to the microscope. From the expression,

$$\delta=\frac{\lambda}{2n}$$

for the limit of resolution of the microscope, it is easy to show that magnifications above about 400 are not necessary to enable an observer to distinguish the finest detail that the instrument is capable of. Magnifications of 1000 and higher are, however, common with high-power microscopes, for the reason referred to above.

(3) BRIGHTNESS OF IMAGE

This depends on four factors: (1) The brightness, obviously, of the source of light; (2) the angular aperture of the optical instrument, which determines the fractional part of the radiated light entering the instrument; (3) the magnification, which determines over what area the transmitted light is spread; and (4) the transmission of the instrument, which determines the fractional part of the light lost in passing through the instrument.

In treating the subject of brightness of an object, it is both customary and sufficiently consistent with experience to assume that a surface radiating light appears equally bright at all angles of view. Such surfaces radiate according to the so-called "cosine law." Further, neglecting loss of light in the transmitting medium, a radiant surface appears equally bright at all distances from the eye, so long as the area of its image on the retina does not approach too close to that which the eye is capable of resolving.

The physical quantity which usually determines the brightness of a source may be designated as the intensity per unit area b ("intrinsic brightness"). Consider the quantity of light radiated in unit time from each point through a small solid angle ω in a direction perpendicular to the surface. The quantity so radiated from all points in unit area (assuming uniform brightness) is $b\omega$. A surface which appears equally bright in all directions radiates the same quantity from each unit of projected area perpendicular to the direction of the radiation. In all cases of image formation it is this projected area alone which enters into consideration, not the actual radiant area.

When viewing a distant luminous area, ω becomes the solid angle subtended by the pupillary aperture of the eye, so from each unit component area the quantity of light received per second by the eye is equal to $b\omega$. With retinal sensitiveness constant, $b\omega$ should obviously measure the subjective brightness, for it represents the quantity of light falling in unit time on the given retinal area. With the pupillary aperture ω constant, b alone serves to measure the brightness; but it must be kept in mind that a reduction in the pupil reduces the subjective brightness even though b remains constant. Such a condition occurs with optical instruments whose light cones do not fill the pupillary aperture of the eye.

The brightness of an image is dealt with exactly as if it were an object. We speak of its brightness b' and when viewed directly by the eye the subjective brightness is measured by b' if the pencil of rays from a point of the image fills the pupillary aperture of the eye, otherwise the subjective brightness is proportionally smaller.

Every optical instrument possesses diaphragms or stops which limit the cone of rays passing through the instrument. This stop in the eye is called the iris and is just in front of the crystalline lens. In the photographic lens it is also called the iris, and usually situated between the two components of the lens. In both the eye and the photographic lens the diameter of the iris is variable. In the telescope and microscope the limiting stop is usually the rim of the objective, and of fixed diameter. With the reading glass and with eyeglasses, used in conjunction with the eye, the cone of rays proceeding from an object point to the lens is very much wider than the cone which is admitted to the eye, so that the limiting stop is the iris of the eye. This should also be the case when night glasses are used, in order to obtain maximum brightness of field. It often happens that an instrument has several stops located at different points. How is one to determine, in general, which one is the limiting stop?

Imagine the eye placed at some point of the object and looking back through the instrument under consideration. Of the possible stops thus seen, the one whose image subtends the smallest angle to the eye is called the limiting stop. This image, in location as well as size, is called the *entrance pupil* of the instrument; because, like the pupillary aperture of the eye, it limits the cone of light which the given point of the object sends through the instrument. The visual angle under which the entrance pupil is seen from the given point of the object is the *angular aperture* (α) of the instrument for that object point. Thus, while the entrance pupil is fixed in size and position, the angular aperture decreases as the distance from the object increases.

The image, real or virtual, of this same limiting stop seen on viewing through the instrument in the opposite direction is called the *exit pupil* of the instrument. When, in using the instrument, this exit pupil is located at the observer's eye, the entire field of view of the instrument is swept by simply rotating the eye; otherwise a lateral shift of the eye is necessary to cover the field, which makes the instrument less satisfactory. This is an impediment when using the Galilean telescope, so desirable from other standpoints.

The cone of rays of angular aperture α (solid angle ω) from the object point is converted by the instrument into a cone of aperture α' (solid angle ω') forming the image point. Equal cones from all the points of a small projected area σ form the corresponding small image area σ'. To determine the relative brightness of these two areas it is simpler to first assume that these cones lose no light in passing through the instrument. Then the quantity of light emerging per second ($b'\omega'\sigma'$) is equal to the quantity entering per second ($b\omega\sigma$). It is a fundamental law of geometrical optics that $\omega'\sigma'$ is always equal to $\omega\sigma$, irrsepective of distance; hence it follows that the brightness b' of the image is *equal* to the brightness b of the object if there is no loss in the instrument. Consequently, b' must be *less* than b if, as always, the instrument does not show perfect transmission. It is to be concluded that *the brightness of the image formed by any optical instrument whatsoever* (including the eye) *can never be greater than that of the object.* Moreover, the observed brightness of the image is always reduced below that of the object by loss of light in the instrument and may be still further reduced by the pupillary aperture of the eye not being entirely filled by the cone of light emerging from the instrument.

When the ratio of the diameter of the objective to the magnification is less than the diameter of the eye pupil, the subjective brightness varies proportionately to the area of the exit pupil, and hence, for the same objective is inversely proportional to the square of the magnification. To avoid this decreased brightness, an eye pupil of 3 mm diameter permits a magnification of not more than 3⅓ for each centimeter objective diameter. A somewhat higher magnification is sometimes advisable since the outer portions of the crystalline lens impair definition on account of aberration. In many instruments, such as opera glasses or field glasses for use in dim light ("night glasses"), the exit pupil is made much larger than 3 mm, because the eye pupil dilates considerably in dim light. For such glasses, to avoid loss of brightness, the objectives must be increased in diameter proportionately.

The brightness of the visual image is of great importance in all instruments used for observational purposes. If an extended surface, such as the moon, be viewed through a telescope having a magnifying power such that the exit pupil of the instrument just fills the pupillary aperture of the eye, the brightness of the image, neglecting loss in transmission, is as shown above, equal to that of the moon as seen by the unaided eye. Suppose now the magnification is increased by using another ocular with the same objective. Since the magnification by the telescope is equal to D/p, the ratio of the diameter D of the objective to the diameter p of the exit pupil, this ocular, to increase the magnification, must necessarily reduce p. In this case, however, the exit pupil will not fill the pupillary aperture of the eye, hence increased magnification, by changing the ocular, is obtained at the expense of brightness. The dilation of the eye pupil and the increased retinal sensitivity in dim light justifies the use of eye shields on optical instruments where maximum brightness is desired.

Any excess in size of the exit pupil over the eye pupil adds nothing to the brightness of the image; but it sometimes facilitates observation in allowing some freedom of motion to the eye.

If, instead of an extended image such as the moon, a star (subtending less than one minute of arc with the unaided eye) be used, the "grain" of the retina determines the apparent size of the image. In such a case the light received by the retina is effectively spread over a constant area. In that case the total flux of light from the star through the instrument determines the brightness if the magnifying power is equal to or greater than the ratio D/p of the diameter of the objective to that of the eye pupil. This flux is proportional to the area of the objective, i e., to the square of its diameter. Hence the brightness of the image of a star is proportional to the square of the diameter of the telescope objective so long as the magnifying power is kept equal to D/p. This accounts for the greater number of stars visible through telescopes. Even an opera glass having an aperture of an inch should increase the brightness from 10 to 30 times, depending on the pupillary aperture of the eye.

In the case of a photographic lens the lack in brightness of the image may be compensated, to a certain extent, by the time of exposure. Within those limits in which the photographic effect may be assumed proportional to both the quantity of light per second ($b'\omega'$) falling on unit area of the plate and to the time of exposure (t), the quantity $b'\omega't$ may be used to measure this effect. Neglecting loss of light by transmission the brightness b' of the image is equal, as shown above, to that of the object b; so that in comparing two photographic lenses on the same object, $\omega't$ becomes a measure of the photographic effect. Thus, for the same photographic effect, the time of exposure varies inversely as ω'; or, the "speed" of the lens is proportional to ω', the solid angle subtended at the photographic plate by the exit pupil of the lens. For distant objects ω' is proportional to $\left(\frac{F}{D}\right)^2$, the square of the ratio of the principal focal distance of the lens to its diameter. So $\left(\frac{F}{D}\right)$, called the "aperture ratio," is commonly used to designate the speed of a photographic lens. An "F/6" lens is one which has, with maximum aperture of its stop, a focal length six times its diameter. So an F/8 lens has four times the speed of an F/16 lens. This is the so-called F system for designating the speed of a lens. The United States system uses the solid angle ω' as a basis instead of the angular aperture (F/D). A lens designated U. S. No. 4 is the same as an F/8 lens and has twice the speed of a U. S. No. 8 lens. A U. S. No. 16 is the same as F/16, and has one-fourth the speed of a No. 4.

When the photographic camera is used in conjunction with another optical instrument, such as a telescope or microscope, the illumination of the image on the photographic plate is not necessarily determined by the aperture ratio of the photographic lens, unless the iris of the lens is the limiting stop of the entire system. If the photographic objective is placed so that its entrance pupil coincides with the exit pupil of the optical instrument with which it is used, they will not, in general, be of equal diameter. If the entrance pupil of the lens is larger than the exit pupil of the instrument, some of the speed of the lens is wasted. If it is smaller, as already stated, the full resolving power of the instrument is not obtained.

The eye is an optical instrument very similar in design to the photographic camera. Just as in the camera, the illumination of the retina is directly proportional to the area of the entrance pupil. So long as the pupil remains constant, the sensation of brightness (neglecting retinal fatigue) is found to be proportional to the intensity of the retinal image. The retinal sensitivity changes more or less slowly with the intensity of the light, becoming far greater in dim light. This change is enormous compared with the changes in brightness due to pupillary aperture, as is evident from the blinding sensation on emerging from a dark room into a brilliantly lighted one. The iris is merely an adjustment for small and rapid variations.

The quantity of light lost by reflection and absorption, but not by the stops, reduces the intensity of the image below that of the object. From an area σ of the object the quantity radiated per second through the solid angle ω subtended by the entrance pupil is $b\omega\sigma$. The quantity received in the same time by the corresponding area σ' of the image is $b'\omega'\sigma'$. The fractional quantity transmitted is

$$\tau = \frac{b'\omega'\sigma'}{b\omega\sigma}$$

Since, however, a fundamental law requires $\omega'\sigma'$ and $\omega\sigma$ to be equal, $\tau = \frac{b'}{b}$ gives what is called the "transmission" of the instrument.

A sufficiently accurate value for b can generally be obtained by calculation, when the refractive index and absorption coefficient of the glass parts are known. The total transmission is the *product* of the transmission of the separate parts because each transmits a definite fraction of that which reaches it. The transmission of each part is likewise the product of that through each surface together with that of the glass between. At each surface a fractional part ρ is lost by reflection, hence a part $(1-\rho)$ transmitted. The quantity ρ increases very slightly with the angle of incidence of light up to 30°, but more rapidly at larger angles (at 50° ρ is double that at 0°). At normal incidence

$$\rho = \left(\frac{n - n'}{n + n}\right)^2$$

where n and n' are the refractive indices, respectively, of the two boundary media. For cemented surfaces, ρ is negligible. For air-glass surfaces where $n' = 1$ and n varies from 1.5 to 1.65, the value of ρ varies from 4 to 6 per cent; hence $\tau = 96$ to 94 per cent. The absorption of visible light in glass suitable for optical instruments is usually small, may be less than 1 per cent per cm but usually more nearly 2 per cent. Its transmission is, therefore, at least 98 per cent. Apparently clear glass which has been artificially decolorized may actually absorb more than slightly colored glass. An instrument which has k reflecting surfaces (neglecting total reflecting surfaces of prisms) and a total glass thickness of x cm should have a transmission $\tau = (0.96)^k\,(0.98)^x$. Thus, a single lens ½ cm thick should transmit 91 per cent; two such lenses 83 per cent. Galilean telescopes (binoculars and opera glasses) usually have 4 or 6 reflecting surfaces and a glass thickness of about 2 cm, giving transmissions of, respectively, 82 and 75 per cent. Prism binoculars usually have 10 or 12 surfaces, and a glass thickness of about 10 cm, giving transmissions of 54 and 50 per cent.

High transmission is of extreme importance in binoculars for use at night, when it is desired to distinguish objects in very dim light. This is the reason why Galilean binoculars with a high transmission and with diaphragms so large that the eye pupil is the exit pupil of the system are used instead of the low transmission prism binoculars.

In general, however, the retina becomes accustomed to the brightness of the image upon it and is able to detect differences in brightness of less than 1 per cent of the actual value over a very considerable range of illumination. Vision thus depends less upon illumination than upon contrast in the details of the image. No instrument can accentuate contrast except as already discussed, in the case of brightness of stars and of fine luminous lines against a darker background.

In general, conditions combine to decrease the contrast. The aberrations surround each point of the image with blur circles, the diffraction disks are superposed upon these, and magnification spreads out the image so that the rate of brightness change is less and edges are less sharp.

The contrast in the image is also diminished by diffuse stray light in the field. This is in part due to multiple reflections from the refracting surfaces, frequently called "ghosts," and is very troublesome when the reflected images are almost, but not quite, in focus. Diffuse light reflected from the interior walls of the instrument will often cause trouble, but its presence is inexcusable since the walls may be easily blackened or the light excluded from the field of view by extra diaphragms.

Scattered light from bubbles in the glass, imperfectly polished surfaces, scratches, and dust on the surfaces may be present. Omitting exceptional cases, these factors contribute insignificant amounts of stray light, however, unless they are in focus. Some of the most desirable types of glass can not be made free from bubbles, and they are frequently found in highly corrected lenses.

Other faults in the glass may be very harmful. Striæ impair definition seriously in some cases, but a few fine-thread striæ probably do little harm except in instruments of high precision. Mechanical strain arising from imperfect annealing makes it impossible to grind and polish a true surface.

Finally, it is of much more importance than is frequently realized to shield the eye from light from other sources than the object viewed. This is especially the case when small details must be carefully examined and the image is dim compared with other light falling on the retina. The sensitiveness of the retina is determined by the brightest images upon it. This sensitiveness is only slowly reached, but it will be noticed after looking contինously through a pair of prism binoculars for some time.

In observing distant objects the intervening haze introduces diffuse light or glare, which diminishes the contrast in the image. When this haze is blue, yellow glass is often of considerable assistance.

(4) FIELD OF VIEW

The normal eye in a state of repose (accommodated for parallel rays) focuses objects distinctly which are as near as 50 feet. By almost insensibly rapid accommodation this depth of field extends from infinity to 25 cm distance. The width of field distinctly seen by the *fixed* eye is only about ½°, limited by the size of the "yellow spot" on the retina. Through rapid mobility and persistence of vision the practical field of distinct vision mounts to 30° horizontal and 20° vertical. Add to this the range of indistinct vision, the field of view of the single eye reaches 150° horizontal and 120° vertical. With a 3-power Galilean telescope (negative ocular) focused for infinity, objects at 20 feet distance are also in satisfactory focus. Near objects require focusing anew, which is accompanied by a decreased depth of field. With this telescope the apparent field of view (visual angle of the image field) is about 27°, while the true field is obviously only 9°. That is, magnification is accompanied by a corresponding reduction in the true field. A 6-power Galilean telescope has an apparent field of about 18° but a true field of only 3°. With a 3-power prism type of telescope (positive ocular) the apparent field is about 48°, the true 16°, while a 6-power binocular has the same apparent field, hence a true field of 8°. The larger field furnished by this type, as compared with the Galilean, is its chief advantage. The reason for the difference lies in the location of the exit pupil which, in the case of the Galilean type, is virtual and lies within the instrument. With the negative ocular, the exit pupil is necessarily virtual (on the object side of the ocular), while with the positive ocular it can be placed at the eye. So the fields of view in the two types are like that obtained by the small boy viewing a baseball game through a knot hole. With his eye at the knot hole the field may be of satisfactory width. As he withdraws his eye the visual angle subtended by the hole is rapidly diminished and the field of view correspondingly restricted. Where possible, the field which can be used is limited by a field of view stop in the image plane, or effectively so, else a detrimental shading off of the brightness near the image of the stop edge is observed. This effect is commonly noticed with Galilean binoculars. In case of the photographic lens it is usually the photographic plate which limits the field.

For simple lenses and lens combinations which can be treated as simple lenses, imperfect definition or illumination of the zones off the optical axis limit the serviceable field of view. By special combinations of lenses and stop, high-speed (large aperture) camera lenses are made which give a satisfactory field up to 60°; and lower speed ("wide angle") lenses up to 140°. The depth of field in case of a camera can only be increased by reducing its aperture. Highly corrected lens combinations are no better in this respect than simple lenses.

With a given type of instrument, the longer the optical system the greater the magnification. This requires either the diameter of the tube in the neighborhood of the real image to be increased proportionately to the magnification, or, the true field to be correspondingly reduced. Ordinarily the ocular of most instruments would have to be of a larger diameter, or the field of view smaller, were it not for the simple device of a collecting lens ("field lens") being placed at the image to bend all the cones of rays through the smaller ocular without, however, changing the aperture of the separate cones, or the location of their apices.[2]

(*a*) *Erecting Systems.*—The telescope in its simplest form consists merely of an objective and an ocular. The objective converts the diverging cone of rays from the object point into one converging toward the corresponding image point. The real image formed is both inverted and reversed (right and left), and so appears when viewed by the unaided eye or, as is the case with the "astronomical telescope," through a positive ocular placed slightly beyond where it is formed. For terrestrial observations it is often necessary to have the field "erect." With the Galilean telescope this is accomplished by inserting a negative ocular in the converging cones shortly before they come to a focus, by which they are converted into parallel rays giving a virtual erect image of the object, (i. e., without inversion or reversal). A positive ocular, by virtually placing the field of view stop at the eye, gives a larger field of view, hence other devices may be advantageously employed to erect the image.

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[2] To avoid the disturbing effect of having dust particles on its surface in focus, the field lens is shifted slightly out of the focal plane, but not enough to materially affect the location of the image.

One device makes use of a positive "erecting" lens which simply forms a second real image of the first image, thereby righting the position of the observed image. It may make, however, a telecsope of inconvenient length. This inconvenient length is overcome by using plane mirrors (total reflecting prisms) for righting the image. These make the space distance between objective and ocular for the same power instrument even less than in the Galilean type, by reflecting the light around a sort of open-link path between objective and ocular. However, the increased number of optical parts required reduces the transmission of this type much below that of the Galilean, and also increases the cost, but otherwise they are much more desirable especially where the higher magnifying powers are demanded.

(*b*) *Stereoscopic Vision.*—Imagine two points at different distances brought into the same line of sight of, say, the left eye. The right eye resolves them if they subtend at it an angle of one minute. In this way the two eyes reveal depth where the single eye can not. Using ordinary three-power binoculars, these two points would be resolved if they subtended one-third of a minute of arc. Using prism binoculars which have a greater width between the objectives than between the oculars, the angular separation between the given points is proportionately enlarged, hence the appreciation of depth correspondingly magnified. The difference in depth which can just be resolved by the unaided eyes is proportional to the square of the distance to the points observed, and inversely proportional to distance between the eyes. Using binoculars this limit varies inversely as their magnifying power, and inversely as the ratio of separation of the objectives to that of the oculars. A 6-power prism binocular with the objectives at twice the distance of the eyes should be 12 times as effective, in estimating depth, as are the unaided eyes.

In addition to the fact that they require the same degree of optical perfection as single terrestrial telescopes, binoculars are subject to the additional and rather severe requirement that their two fields must coincide. To accomplish this the distance between the oculars must be adjustable to any pair of eyes, 58 to 72 mm, their axes must always be parallel within a very few minutes of arc; and the two must produce the same magnification (within 2 per cent). When the rough usage to which these instruments may be submitted is taken into account, the mechanical difficulties involved in producing such an instrument are readily appreciated.

While the primary purpose is to increase the resolving power of vision, necessarily accompanied by magnification, the need for width of field and sufficient brightness along with a convenient size of instrument is hardly less pronounced. In ordinary daylight prism binoculars fulfill the imposed conditions very satisfactorily, and have the added advantage of increased stereoscopic effect, in that their objectives are about double the distance apart of the oculars. Their serious weakness develops in dim light in that they give only half the brightness of image given by the naked eye. For dim light no other type equals the Galilean which gives four-fifths the brightness effect of the naked eye. With "night glasses," to avoid reducing the brightness below this magnitude the exit pupil of the instrument should be as large as 8 mm, the maximum pupillary opening of the eye. The small field of view given by the Galilean type is a weakness especially with the higher powers. Because of this some think that they are not very useful above four powers.

2. INSTRUMENTS FOR REPRODUCTION

Optical instruments for projecting correct and well-defined pictures make the severest demands on materials, design, and workmanship. In general, it is desired that the light from objects at different distances and directions within the field of view shall focus on a single *plane* surface (screen or sensitive photographic plate). This flattened picture should correspond in illumination to the object, and be sharply defined over a sufficiently wide angle. Image brightness or rapidity is important in photographic systems; the chemical and visual foci should coincide and straight or parallel lines should not be distorted. The attainment of these properties is complicated by the presence of various focal aberrations.

These aberrations may be classified as monochromatic aberrations, due to the form of refracting or reflecting surfaces, and chromatic aberrations, caused by the composite nature of white light which is dispersed by refraction. By combining two or more simple lenses of different forms and different refractive and dispersive powers these aberrations may be to some extent neutralized, thus giving a "corrected" optical system.

(1) MONOCHROMATIC ABERRATIONS

Mathematical theories of image formation which treat aberrations generally and analytically by means of infinite series show that the monochromatic aberrations giving rise to errors in reproduction are infinite in number. These analytical treatments develop the coordinates of points in the object plane and entrance pupil in a series of ascending odd powers of the coordinates of collinear points in the exit pupil and image planes. The perfection of the image depends on making the different terms of this series vanish. Thus, an image of the "third order" is produced if the coefficients of the first power terms are made zero, a fifth-order image requires making the coefficients of the powers of third degree zero, etc. Each of the terms of a certain power represent a particular kind of aberration. There are thus 5 aberrations of the third degree, 12 of the fifth degree, 20 of the seventh degree, etc. The elementary theory of optical systems permits the determination of a third-order image of an object only when the objects, images, lenses, and consequent ray angles are infinitely small. This means that fairly satisfactory images may be obtained with such systems only when the length of the object is small in comparison with its distance from the lens or lens system, and when the image is formed by narrow pencils of light. These conditions are satisfied in the case of spectacles and reading glasses, and also in the case of a few laboratory reading instruments, where low intensity of illumination is permissible in the image.

Where greater brightness, higher resolution, or a large field are required, such systems are not satisfactory and the next step is to obtain a fifth-order image. This can be done by making the coefficients of third-degree terms zero. The coefficients can be expressed in terms of the constants of the optical system (radii of curvature, thickness, refractive indices, and distances between lenses) and five equations satisfied. These five monochromatic aberrations of the third degree are (1) aberration of the axis point (spherical); (2) aberration of points removed from the axis (coma, sine condition); (3) astigmatism; (4) curvature of the field; (5) distortion. In practice, these five are of greatest importance because the higher degree aberrations are too small to be sensible to the eye or on a photographic plate except for large angles and apertures.

(*a*) *Aberration of the Axis Point* (*Spherical*).—Rays from a point on the optical axis passing through different zones of a lens come to a focus on the optical axis at different distances from the lens. Thus, in any optical system, rays leaving the axis point O at an angle a_1 will unite in the axis point O'_1; and a_2 rays will focus at the axis point O'_2. The distance O'_1 O'_2 is called the longitudinal aberration, and $O'_1 B'_1$, the lateral aberration. In a plane perpendicular to the axis at O'_1 is a circular "disk of confusion" of radius $O'_1 B'_1$, and in a parallel plane at O'_2 another one of radius $O'_2 B'_2$. Between these two is situated the "disk of least confusion" at which place the image is brightest. The largest angle which rays reproducing O can have is limited either by the aperture of one of the lenses or by an aperture stop or diaphragm placed in front of, between, or behind the lenses. Obviously, the aberration may be reduced by diminishing the effective aperture. Spherical aberration is a question of curvature and could be eliminated by using other than spherical surfaces, but since such surfaces would not assist in eliminating the remaining aberrations they are not used. In practice, spherical aberration is remedied by combining a collective or positive lens with a dispersive or negative lens of equal but opposite spherical error, so that the excess of peripheral refraction of the positive lens is neutralized by that of opposite sign given by the negative lens. This requires the proper choice of radii of curvature for the lens surfaces. Since the compensation of spherical aberrations depends on the divergence of the rays incident on the first lens, complete correction can be made for but a single pair of focal planes. The residual aberration must be specified in terms of the distance of the object and all zones of the lens.

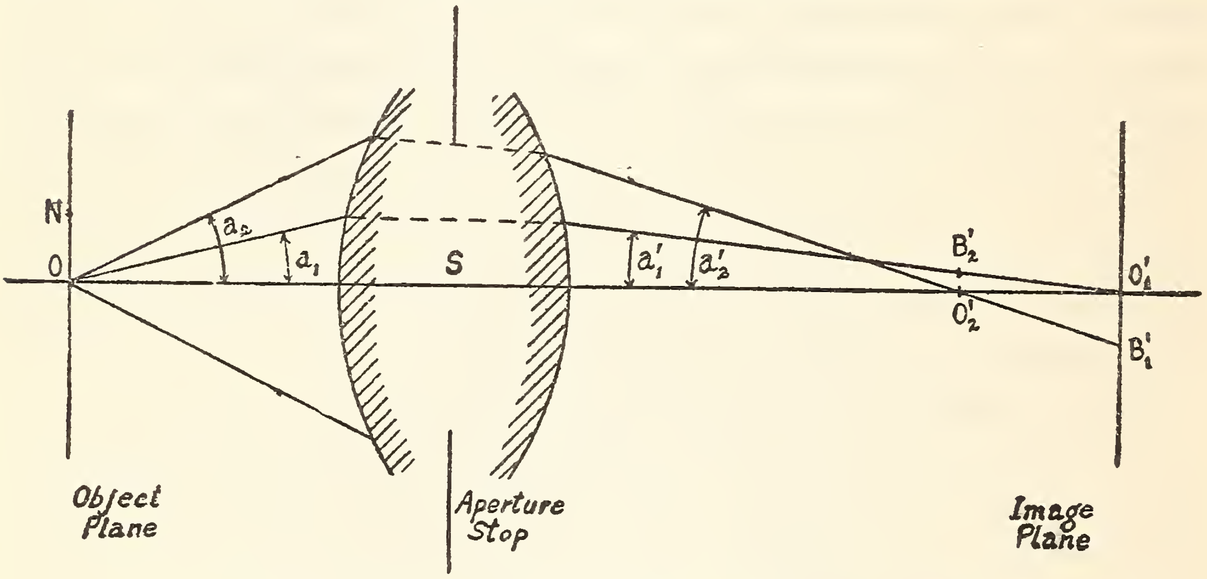

FIG. 1.—*Spherical aberration—Schematic representation*

(*b*) *Aberration of Points Removed from the Axis* (*Coma, Sine Condition*).—The aberrations in reproducing lateral points N near O are similar, but much more complicated than those belonging to axial points. All zones of the optical system may have the same focal length for a point O on the axis and still give lateral spherical aberration with oblique rays from points N off the axis. The confusion disk then becomes unsymmetrical about a point and usually has a bright nucleus of light somewhat resembling the head of a comet with the tail directed either toward or away from the axis, depending on the relative magnitudes of the focal lengths for the edge and central zones of the optical system. In a lens which is spherically corrected, this comatic aberration is avoided if all the rays reproducing the point O fulfill the so-called "sine condition," i. e., $\sin a'_1/\sin a_1 = \sin a'_2/\sin a_2$.

In uncorrected systems the deviation from the sine condition as well as the spherical aberration for axial points increases rapidly with the aperture. Coma is largely eliminated by making the lens system symmetrical, as is done in many photographic objectives. Since it depends upon the divergence of the light entering the lens and upon the distance from the axis it may be eliminated (for one zone of the lens) for objects at one distance from the lens and one angle from the axis, but not for all distances or all angles. Residual error is stated in terms of angle and object distance or aperture and object distance.

(*c*) *Astigmatism.*—This aberration always accompanies coma in single lenses and manifests itself by forming two sets of images of points off the axis. These two sets of images lie, with a simple lens, in two separate curved surfaces, one a radial line on the outer image surface and the other a tangential line at right angles to the first on the inner image surface. A circle of least confusion is formed at some point between these two focal lines or "astigmatic foci."

Astigmatism increases with the obliquity of the rays—i. e., with the field of view—and causes want of definition and difference of focus between horizontal and vertical lines in the edge of the field. This error in an optical system must not be confused in its origin with the similar defect sometimes present in the eye. In the former case, even though central definition be perfect and the lens surfaces absolutely spherical, astigmatism is produced by *oblique* rays originating at points off the optical axis. In the latter case astigmatism is produced by unequal curvatures in different meridians of the crystalline lens of the eye, so that a beam of light near the principal axis is brought to a focal line instead of to a focal point. The defect in the eye can be corrected by placing a cylindrical lens at the front focal plane of the eye, but a spherical lens can never be corrected in this way.

Only since the discovery of new types of optical glasses which make it possible to bring the two image surfaces together, has it become possible to eliminate astigmatism. Before the development of the so-called "new glasses," glasses possessing higher indices possessed also higher dispersion. Crown glasses are now made which still have smaller dispersive power, but larger refractive index than certain flint glasses with which they can be combined to reduce this and other aberrations.

(*d*) *Curvature of the Field.*—The focal surface after the removal of astigmatism may still be curved, and hence a flat photographic plate used with such a lens could not record the entire image in sharp focus. In most cases the image surface is concave toward the lens system, and in order to flatten this surface one must increase the focal distance for oblique rays. This can be accomplished by the use of a front stop placed at a proper distance before the lens or by separating the lenses, but this procedure brings in reduction of field and illumination and increases distortion. Such artificial flattening of the image is brought about not by any appropriate change in the ray path of each pencil, but only by exercising a suitable selection among its many partial pencils. A true correction compatible with the use of the full aperture is made possible by the choice of suitable kinds of glass. This condition for flatness of field is expressed by the "Petzval equation," $\Sigma \frac{1}{r} (n' - n) = 0$, where r is the radius of any refracting surface, n and n' the refractive indices of the boundary media, and Σ the sign of summation for all the refracting surfaces, traversed by the rays. If the Petzval condition is fulfilled, the field may be flattened by correctly placing the stop.

(*e*) *Distortion.*—If all of the above aberrations be corrected, a plane object will be reproduced in a sharply defined plane image, but the possibility remains that the image will be distorted in this plane. This error is caused by the different zones of the object being imaged with different magnifications. If the inner parts have greater magnification than the outer the distortion is said to be "barrrel shaped," if the converse is true distortion is "cushion shaped." Two different definitions of distortion commonly used are (1) the ratio of tangential to radial magnification, and (2) the magnification of a radial length relative to magnification near the axis.

This aberration is corrected in all systems which are symmetrical with respect to their aperature stop, or which consist of two like but different sized components placed from the stop at distances in the ratio of their size, and presenting the homologous curvature to it. As in the case of all the other aberrations, the complete elimination of this last one is limited since distortion can be corrected for but two distances of object and two angles from the axis.

(2) CHROMATIC OR COLOR ABERRATIONS

In optical systems composed of lenses the position, magnitude, and errors of the image depend upon the refractive indices of the glass. Since the index of refraction varies with the color or wave length of the light, it follows that a system of uncorrected lenses projects images of different colors in somewhat different places and sizes and with different aberrations. If white light is used, an infinite number of images are formed and when they are received upon the retina of the eye or upon a focusing screen of a camera, they cause errors named chromatic aberrations. The most important color aberrations are of two kinds: (1) Axial chromatism and (2) lateral chromatism.

(*a*) *Axial Chromatism.*—This aberration gives a variation in the image distance with wave length or color. All transparent media deviate violet light (short waves) more than they do the red light (longer waves), and a simple lens will therefore focus white light from a given axial point into a continuous spectrum along the axis. The amount of this axial chromatic aberration, or length of spectrum, is dependent upon the focal length of the lens and the dispersive power of the glass. By combining two lenses of proper focal lengths and glasses of suitable refractive indices and dispersions, this axial chromatism may be minimized. Thus, by combining a positive lens of crown and negative lens of flint glass two colors, such as red and blue, proceeding from a given object on the axis may be brought to a focus at the same image point on the axis. By using the new glasses referred to above the deviation of a third color may be eliminated by two lenses if separated; or by three lenses in contact, if they do not all consist of the optical glasses in which a large index of refraction is accompanied by large dispersion. The residual color errors of a lens system which is achromatic for two colors are called the "secondary spectrum." In uniting three colors still better achromatism is obtained; there still remains a "tertiary spectrum" but it can always be neglected.

(*b*) *Lateral Chromatism.*—This aberration is due to a variation of the size of the image with wave length. If the light of all wave lengths from a point on the axis is focused at the same point, the magnification given by red light may be different from that of blue light. If this error is present, it is readily seen that the image of an extended object will be of different size in each color. Toward the edge of the field this will cause colored fringes. This error in an objective may be reduced by combining with the objective a compensating ocular which gives chromatic differences of magnification of the same amount but in the opposite sense.

(3) CORRECTED LENSES

On account of the numerous aberrations, due to properties of lens materials and the composite nature of white light, it is quite impossible to make an optical system which gives a perfect image with finite pencils of white light; but practical systems solve this problem with an accuracy which is generally sufficient for the special purpose of each type of instrument. The analytical difficulties involved in the problem of finding a system which accurately reproduces a given object upon a given plane with given magnification are insuperable in most cases. Analytical approximations are sometimes used provisionally, but lens constructors almost always employ the inverse method; they devise a system from certain, often quite personal experiences, and test, by the trigonometrical calculation of the paths of several rays and different wave lengths, whether the system gives the desired reproduction. The radii, thicknesses, and separations of the lenses are continually altered until the errors of the image become sufficiently small. By this method certain or all of the above-mentioned errors of reproduction are investigated, and the final form of a practical system for a particular purpose is always a compromise in which residual errors of one kind or another are allowed to remain so that certain others may be more effectively corrected.

The necessary corrections for large-aperature systems are for the axis point and the sine condition, while errors of the field of view are almost disregarded. Such systems are called "aplanats," although this term is sometimes also applied to lenses which are free from chromatic and axial spherical aberration. High-power microscope objectives and certain rapid photographic objectives are made aplanatic.

To obtain a large field of view, astigmatism, field curvature, and distortion must be corrected, and other errors may be more or less neglected. The widest angle objectives and oculars have these properties, but frequently only one or two of these field errors are corrected. Thus, "stigmats" or "anastigmats" are lenses in which attention has been given chiefly to the elimination of astigmatism. "Orthoscopic" or "rectilinear" lenses imply freedom from distortion. Astigmatism and distortion are both corrected in "orthostigmats" and all three field errors are corrected in "collinear" lenses. The term "homocentric" has been used to describe a lens which is free from spherical aberration and which fulfils the Petzval condition for flatness of field. "Isostigmars" are anastigmats which ignore the Petzval condition but still give a practically flat field and are free from central and oblique spherical aberrations.

An optical system which has been corrected for achromatism is called an "achromat." A system is said to be chromatically undercorrected when it shows the same kind of chromatic error as a thin positive lens; otherwise it is said to be overcorrected. Achromats made of two cemented lenses constructed of the original types of optical glass are called "old" achromats in contradistinction to the "new" achromats which can only be constructed with modern glasses in which a large refractive index is associated with relatively small dispersion. An achromat which has a flat field was made possible only by the discovery of these glasses. Lenses which are aplanatic for several different colors are termed "apochromatic." When a soft image is desired in portrait and landscape photography it is sometimes produced by chromatic aberration and only spherical aberration, distortion, and astigmatism are corrected. Such systems of lenses are called "anachromatic."

3. INSTRUMENTS FOR MEASUREMENT

Rays of light and wave lengths of light are most powerful auxiliaries in the measurement of angles and distances. A ray of light in a homogeneous medium constitutes our most perfect realization of the geometrical notion of direction. Two directions determine an angle, and if the knowledge of a base length is combined with the measurement of angles, all the elements for evaluating distances are at hand. This is the principle of surveying and of range finding. Furthermore, a wave length of light on account of its immutability and reproducibility constitutes, in some respects, the most satisfactory standard for direct measurement of length. Accordingly, the wave length of the red radiation from cadmium has been measured to be equal to $6438.4696 \times 10^{-10}$ m, and the length of this wave is now the international standard of length in spectroscopy and interferometry. The interference phenomena of light waves serve very effectively, not only in the measurement of lengths but also for the measurement of extremely small angles, for the testing of plane surfaces, etc. The various types of optical interferometers represent a class of instruments for measurement which is quite unique in the accuracy attainable.

Optical instruments for purposes of measurement usually employ some kind of an optical system in connection with a base line, accurate screws, or either linear or circular scales which are carefully graduated. The telescopes included in this class of instruments resemble the instruments for observation and particular emphasis is placed upon good definition in the center of of the field. Telescopes used for sighting upon objects for the purpose of measuring angles or distances are provided with cross hairs or a reticle, and require means of placing the cross hairs simultaneously in the focus of the eyepiece and in the plane of the image formed by the objective. These requirements are met by freedom of motion of the cross hairs and eyepiece (or objective) along the axis of the telescope. When the cross hairs are in the image plane of the objective, parallax recognized by shifting the eye laterally is eliminated, and they may thus be accurately set on any point of the object. Examples of the instruments for direction and angle measurements are the astronomer's transits, meridian circles, altitude and azimuth instruments, heliometers, etc.; the surveyor's transits and theodolites; goniometers used for the measurement of angles of crystals; refractometers and spectrometers for the measurement of prism angles and the deviations of different light rays passing through refracting media; the telescope and scale arrangement by which deflections of sensitive galvanometers are read; etc.

Reading or micrometer microscopes consist essentially of a compound microscope furnished with a pair of parallel spider lines or a cross wire movable in the plane of the image by a micrometer screw of known pitch and provided with a graduated head. Here as well as in all other precision instruments a reasonable balance should exist between the different elements of the instrument, and the possible precision of the instrument as a whole should be determined by the use to which it is to be put.

Photometers of many different types are used in practice and the optical parts involved may consist of telescopes, mirrors, or prisms which are either of single or double refracting materials. The light intensity of any source is measured by observing the change necessary in a standard source to bring its intensity to equality with that under investigation. These changes in the standard source are brought about by changing its distance from the test object which it illuminates, by interposing diaphragms or sector disks, or by introducing other optical devices such as screens or polarizing apparatus.

The intensities of two sources of different color can not be compared directly very satisfactorily. Such cases require the use of a spectro-photometer which, in general, combines a photometer with a dispersing apparatus such as a prism, and permits the comparison of luminosities of any particular wave length or color.

Several different types of polarimeters are used in practice for measurement of rotary polarization and ellipticity of vibration, but the essential optical features consist of simple telescopes and prisms or plates of doubly refracting substances such as calcite, quartz, mica, etc.

Range finders or telemeters may be described, in general, as optical instruments which automatically solve a triangle. They are essentially two transits mounted respectively at the ends of a fairly short base line. When directed toward the same object, their angular positions with respect to the base line serve to give, by various calibrated devices, the distance of the object. The range is read directly in meters or yards without calculation. High-grade telescopes and prisms are required in these instruments.

The most essential optical features of the various types of interferometers in laboratory use are those of *absolutely* plane reflecting surfaces and sometimes of plane parallel surfaces. These requirements severely tax the ability of even the best opticians.

III. TESTING OF OPTICAL INSTRUMENTS

The properties of an optical instrument depend largely upon the particular use to which the instrument is to be put, and preliminary tests or measurements of these properties will show how a given instrument will perform when in use. As was previously stated, all optical instruments used to aid vision may be rated on their definition or resolving power, magnification, brightness of image, and angular field of view.

In addition to a sufficiently large aperture and a satisfactory elimination of aberrations, good definition requires good mechanical construction such as careful surfacing, centering, and mounting of the lenses. The general freedom from aberrations is easily determined qualitatively by means of an artificial star test. A satisfactory artificial star is obtained by placing a bright light source, such as a Nernst filament or nitrogen filled tungsten lamp, behind a very small pinhole. Another method is to silver a small glass bead and illuminate it with an arc. This stellar imitation is viewed through the instrument from a large distance (about 100 to 200 times the focal length of the lens or objective), and if an image is obtained which is sharp and uncolored at the center of the field, then the instrument is reasonably free from spherical aberration and axial chromatic aberration. The appearance of the star image in and out of focus will give an experienced observer a very good indication of the kind and amount of axial aberrations present. If axial astigmatism is present, line foci will be obtained both inside and outside of the principal focus. Instruments for reproduction generally require more elaborate tests for aberrations so that the magnitudes of the residual errors can be determined.

In examining the image of an artificial star or of a test plate used to determine resolving power, an auxiliary telescope is very frequently used. This telescope should have a power of five or six and the aperture ratio should be at least as great as that of the telescope which is being tested. The use of this auxiliary telescope allows one to work quickly and without eye strain. It has sometimes been objected that the use of this telescope causes the instrument under test to show its defects too strongly. It is said that there is no use in magnifying the image more than in the case of actual use; a moment's consideration will show the fallacy of these objections.

In testing an instrument we are interested in finding what it will do under the very best observing conditions when used by the very best observer. Such conditions are not usually to be obtained when the instrument is being tested, and if they were the observer would have to take the greatest pains to make sure he is getting the very best results possible from the instrument. By using the auxiliary telescope a person whose eyes may be poor can test the instrument for definition and resolving power as well as for aberrations. Experience has shown what resolving power is necessary and the amount of aberrations permissible in order that the instrument shall perform perfectly under service conditions.

The effect of aberrations is to decrease the resolving power; hence resolving power forms a general criterion of the kind and size of the aberrations allowable in an optical instrument. The limit of resolution may be measured in the following manner: A silvered plate upon which groups of lines are ruled is illuminated from the rear and forms the object. The lines in each group are equally spaced, but each succeeding group has closer spacing than the preceding one. A good order of spacing is as follows: First group, 1.0 mm; 2nd group, 0.9 mm; 3rd group, 0.8 mm; 11th group, 0.1 mm. This test screen is placed at a known distance and viewed through the instrument, an auxiliary telescope being employed, if necessary, just as in the star test. The groups of lines with coarse spacing will be plainly resolved, but lines which are too close for the instrument to resolve will appear indistinct or blurred. The angular resolving power can then be computed from the spacing of the last resolved group and the distance of the test screen from the instrument. A good telescope of 12 cm aperture will resolve lines whose centers are one second of arc apart. This angle of separation varies inversely as the diameter of the objective, so the actual performance of any telescope may be readily compared with that of the ideal instrument. For this determination of the limits of resolution of small telescopes a test screen due to Bigourdan[3] is sometimes convenient and is easily reproduced. The screen consists of a number of groups of black lines ruled on white paper with the distance between the lines changing as above from group to group by regular steps. The screen should be used in the same way as the ruled plate referred to above and has the advantage of requiring no illumination other than that usually present in a well-lighted room. The definition of a high-power microscope is tested by resolving lines ruled very closely or by examining slides bearing diatom markings.

The magnifying power of a telescope may be tested roughly by focusing the telescope on an object which contains many equally spaced lines (e. g., a marked scale or a brick wall). Looking through the telescope with one eye and observing the object directly with the other eye it is possible to determine how

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[3] Comptes Rendus, 160; pp. 18–21, 1915.

many divisions as seen by the unaided eye correspond to one division as seen through the telescope. This is the magnification at this observing distance. For more accurate determinations of the magnifying power, measurements should be made on angles or on pupil diameters. A spectrometer may be used for the angular measurements. For this purpose it is convenient to introduce at the focus of the collimator a scale which is marked on silvered glass. If the angle subtended by a division on this scale be measured, first through the spectrometer alone and then with the telescope under test interposed between the collimator and the observing telescope, the ratio of the two angles gives the angular magnification directly. The magnification may be determined almost as accurately and somewhat more rapidly by finding the ratio of the diameters of entrance and exit pupil, or, what amounts to the same thing, by measuring the size of an object placed at some distance before the telescope and the size of the image of this object formed by the telescope. A transparent scale may be placed just in front of the telescope objective, the telescope, of course, being focused for parallel light, and then the image of this scale formed near the exit pupil may be measured with a micrometer microscope. The ratio of the size of the object to the size of its image is the magnification. The diameter of the exit pupil is measured at the same time. If the diameter of the exit pupil multiplied by the magnification is less than the diameter of the objective, the effective aperture of the objective is being reduced by a stop.

The brightness of the image when the exit pupil has the same diameter as the eye pupil is limited only by the transparency of the optical system. For the measurement of light transmission, any brightness-measuring photometer may be used. The light from a suitable source is balanced against the light from a standard source; the instrument is then placed between the light source and photometer, and the brightness of the two sources again brought to equality. The light transmission may be read directly if the photometer is calibrated, or may be derived from simple computations which depend on the type of photometer used. Transmission measured in this way is proportional to the brightness of a visual image only in case the exit pupil of the telescope is equal to or larger than the pupil of the eye. In case this is not true, the brightness of the image is decreased in the ratio of the area of the exit pupil to the area of the eye pupil. Consequently, the transmission test does not give the brightness of the image of instruments which are used under conditions in which the eye pupil is larger than the exit pupil, as may be the case with binoculars used in dim light.

The illumination of the image may be sufficiently great, but if the contrast in detail is lacking, the image will look "flat," and faintly illuminated objects will not be seen in weak light. This effect may be caused by internal reflection or flare, and by light scattered over the image. The former cause is generally absent if the instrument tubes and lens cells are covered with a dull-black paint. Stray light may be caused by imperfections in the glasses or in their surfaces, and is usually detected in the examination of the artificial star. If the stray light is due to striae in the optical glass, these can be discovered by a careful examination, through the glass, of a point source of light in the focal plane of the objective. It is necessary, especially in high-power instruments, that the optical glass be well surfaced and free from striae.

The actual field of view may be roughly measured by observing the length of a scale which can be seen through the instrument at a given distance away. A result which is more accurate may be obtained by using a spectrometer. With the exit pupil of the telescope over the center of rotation of the spectrometer, the angle between the two extreme edges of the emergent beam is measured, and this is the apparent field of view. The actual field will then be obtained by dividing this apparent field by the magnification. With a Galilean type of telescope the field of view is by no means so definite, since in some cases it is dependent upon the diameter of the pupil of the observer's eye, and at low magnifications it is generally limited by the objective or by some stop within the instrument. In case the field of view is limited by the pupil of the eye, it is obvious that the field will increase whenever the pupil of the eye expands. To make the field of view of such a telescope definite, the field should be measured with some arbitrary artificial pupil placed where the eye will be when the instrument is used. This arbitrary pupil should range from 3 to 7 or 8 mm in diameter, according to the probable size of the eye pupil under the condition of light in which the instrument is to be used.

Both telescopes of a binocular should have the same field of view, and this means that their magnifications should be identical and their optical axes parallel. Binoculars may be tested for parallelism of axes by placing them to receive the light from the objectives of two collimators which have been adjusted to parallelism. The cross hairs which are in the focal planes of the two collimators are then viewed through the binoculars by aid of two parallel adjusted observing telescopes. If the collimator cross hair images are coincident with the telescope cross hairs before inserting the binoculars, they will also be coincident afterwards if the axes of the binoculars' telescopes are parallel. Any lack of parallelism of the two halves of the binocular can be read from suitable scales placed in the observing telescopes.

Instead of the second pair of collimators, a single long-focus lens may be used to project the image of suitable reticles placed in the focal planes of the first two collimators. The images of the reticles are projected on a screen and made to coincide. Now, when a binocular is introduced between the collimators and the projection lens, the coincidence of the images of the reticles is disturbed by any lack of parallelism in the axes of the binoculars.

Generally speaking, the perfection of instruments for reproduction may be judged from tests for the aberrations, both monochromatic and chromatic. As explained above, no optical systems in practice are entirely free from errors; an effort is made to remove or reduce only those errors which are most detrimental in the particular use for which the instrument is designed. The following simple tests on the actual performance of such corrected lenses will show, more or less qualitatively, which of the aberrations have been eliminated and which remain.

Axial spherical aberration may be tested by blocking off the peripheral half of the area of the lens and focusing sharply any bright object in the line of the axis. If the image formed by the edge area is not focused at the same point when the central area of the lens is blocked out, spherical aberration is present and the distance between the positions of the focusing screen in the two cases gives a rough idea of the size of the longitudinal error of focus.

Aberrations off the axis may be examined in the following manner: A chart is made up of various types of markings which contains small and large dots and circles and figures of different shapes. It also contains parallel lines ruled both horizontally and vertically and at different distances apart. This chart may be a foot square and can be copied by photography. Then similar charts are put side by side so as to extend over as large a field as desired at the distance at which it is to be used. This distance will vary from 50 to 200 feet, according to the focal length of the instrument under test. In case of a photographic lens, this chart may be photographed at different camera settings, and a study of the resulting plates will give a fairly good idea of the field curvature and astigmatism, also definition in each part of the field. In the case of a telescope or binocular, the test plates are examined visually. The image of the test plates may be projected onto a screen and examined or the aerial image may be examined, using a microscope. If the image of a group of horizontal lines is back of or in front of the image of a group of vertical lines, the lens shows astigmatism. If the image of a small point is comet-shaped, then coma is present. If the focal length of the lens varies when its aperture is reduced, spherical aberration is present. If the lines toward the edge of the field show colored fringes, this is due to lateral chromatism. Distortion may be discovered in a similar manner by observing long straight lines in the edge of the field. If the image of a straight line is curved, then distortion is present. The distortion is barrel shaped if the image is concave toward the center of the field, and pincushion shaped if it is convex.

The axial achromatism of an optical system may be roughly tested by carefully focusing some small test objects and then examining the image when different colored screens are successively placed in front of the lens. Colored glasses which have fairly narrow transmission bands for different parts (e. g., deep blue, yellow-green, and ruby red) of the spectrum may be used. The distances through which the focusing screen must be moved between sharp images in different colors give an idea of the axial aberration of color.

For a more accurate determination of the aberrations, it is advisable to use an optical bench. The lens is mounted on this bench in such a way that it may be rotated around the "Gauss point." If this adjustment be made, the image of a distant point may be observed directly by means of a microscope which contains a finely divided scale or some other means of measuring small displacements. After the axis of the reading microscope is directed toward the center of rotation of the lens system, the lens is rotated several degrees, being moved (or moving the microscope) until the image of the distant point source is again in sharp focus in the plane of the cross hairs. If the ways along which the lens (or the microscope) is moved are true and if the movement be directly toward the source, any distortion in the image will produce a displacement of the star image. The reading may be checked by rotating the lens an equal angle in the opposite direction. A number of such readings with successively increasing angles will give values from which the distortion may be calculated, and, if the distances moved by the lens or microscope in bringing the star image into focus be measured at the same time, the curvature of the field may be obtained and also the astigmatism.

If a lens is to be used for copying work, the distortion may be measured with object and image distances about those used in the copying. The procedure above is not applicable here, but by carefully measuring the image lengths of a good scale, the amount of distortion in the image may be obtained directly.

The field curvature and astigmatism may also be conveniently measured if a large room is at the disposal of the observer. In this case fine *vertical* lines (which are illuminated by a source of light so placed as to send rays through the entire lens) are placed in the principal focal plane of the lens under test. The image of these lines is thrown upon a distant screen placed at right angles to the optical axis of the lens. The lines with the illuminating source are then moved by known amounts in the focal plane of the lens. Curvature of the field necessitates refocusing the lens in order to make the image on the fixed screen as sharp as possible. The amount of this change in focus gives the field curvature directly. The field curvature is then measured, using *horizontal* lines. The difference between the two field curvatures at various angular distances off the optical axis gives the astigmatism.

IV. TESTS BY THE BUREAU OF STANDARDS

1. DESCRIPTION OF TESTS

The Bureau of Standards, on demand, will undertake to test the following optical instruments: Spectacle lenses, magnifiers, opera glasses, field glasses, laboratory telescopes, field and astronomical telescopes, range finders, telemeters, periscopes, microscopes, projection lenses, condenser lenses, and telephoto lenses. The testing of other special optical instruments may be arranged for by special request.

The instruments, as a whole, will be subjected to any or all of the following tests as desired: Magnifying power; maximum field angle; resolving power; star test for spherical aberration, coma, and astigmatism; distortion; transmission; exit pupil; and parallelism of axes in case of binoculars.

Special single lenses, including the unmounted components of compound lenses, will be given any or all of the following tests as desired: Equivalent focal length; back focus; radii of curvature of surfaces; centering of surfaces; striæ and strain in the glass; indices of refraction and dispersion of the glass; and transmission.

Compound lenses, such as photographic objectives, and objectives and compound eyepieces of observing instruments, will be subjected to any or all of the following tests as demanded:

(*a*) General tests: Equivalent focal length; back focus; effective aperture and numerical aperature; transmission.

(*b*) Residual aberrations in actual value: Spherical aberration; coma, sine condition; astigmatism; curvature of field; distortion; axial chromatism (4 wave lengths); lateral chromatism (4 wave lengths).

Miscellaneous tests related to optics also will be made upon request, e. g., strain and striæ, indices of refraction and transmission of crude optical glass, camera shutter speed, optical flats, plane parallel surfaces, etc.

Those desiring tests should state either the use which is to be made of the lens or optical instrument, or else the precision required, for the labor and expense of the tests increase greatly with the number and precision of the tests made. Further information may be had on request relating to the calculation of numerical aperture, speed, depth of focus, the various aberrations, specifications and limits of tolerance for the different lenses and instruments for different kinds of work, more detailed descriptions of various tests, etc.

2. REGULATIONS CONCERNING TESTS

(*a*) *Application for Test.*—The request for verification of any instrument should state explicitly the points at which test is to be made and the temperature or any other conditions which it is desired should be observed.

(*b*) *Identification Marks.*—Instruments and the packages in which they are shipped should both be plainly marked to facilitate identification, preferably with the name of the manufacturer or shipper, and a special reference number given to the article.

(*c*) *Shipping Directions.*—Instruments should be securely packed in cases or packages which may be used in returning them to the owner. Tops of cases should be *screwed* down whenever possible. Transportation charges are payable by the party desiring the test, and should be prepaid. Unless otherwise arranged, articles will be returned by express "collect."

(*d*) *Address.*—Articles should be addressed simply, "Bureau of Standards, Attention: Division IV–4, Department of Commerce, Washington, D. C." Delays incident to other forms of address will thus be avoided.

(*e*) *Remittances.*—Fees may be remitted by money orders or check drawn to the order of the "Bureau of Standards." Delays in forwarding fees will involve corresponding delays in the completion of tests, as the articles are not returned until all fees due thereon have been received.

(*f*) *Breakage.*—Since all possible care is taken in testing and packing, the Bureau can not assume any responsibility for breakage or other damage to instruments or materials submitted for test.

3. SCHEDULE OF FEES FOR OPTICAL TESTING

Optical instruments:	
Equivalent focal length and back focus	$1.00
Aperture ratio or numerical aperture	1.00
Field curvature and astigmatism	5.00
Distortion	5.00
Transmission	1.00
Telescope test complete (star test, resolving power, field of view, entrance and exit pupils magnification, transmission, astigmatism, striæ)	10.00
Binocular parellelism of axes	1.00
Single lenses:	
Equivalent focal length and back focus	1.00
Radii of curvature per surface	1.00
Striæ or strain in glass, each	1.00
Index of refraction to fourth decimal, for each wave length	1.50
Index of refraction to fifth decimal, for each wave length	2.00
Transmission	1.00
Compound lenses:	
(*a*) General tests—	
Equivalent focal length and back focus	1.00
Aperture ratio or numerical aperture	1.00
Transmission	1.00
(*b*) Residual aberrations, complete, four colors	25.00
Spherical aberration, approximate, one color	2.00
Coma and sine condition, one color	5.00
Field curvature and astigmatism	5.00
Distortion	5.00
Axial chromatism, lateral chromatism, sine condition, and coma	15.00

Miscellaneous:

Striæ and strain in crude optical glass, each	$1.00
Transmission of glass	1.00
Preparation of glass sample for above tests	1.00
Refractive index of glass sample to fourth decimal, one color	1.00
Refractive index of glass sample to fifth decimal, one color	2.00
Refractive index of glass sample to fifth decimal, four colors	4.00
Preparation of prism for index measurement	2.00

The above fees are suggested as reasonable charges for the specific optical tests named. Fees for special tests and tests of the same kind in large number will be given upon request.

All fees are waived in the cases of tests for Government departments, Federal or State institutions, and other branches of public service. Tests without charge will also be made for private individuals, manufacturers, and institutions engaged in research and development of the optical glass and optical instrument industries in this country.

S. W. STRATTON,
Director.

Approved:
E. F. SWEET,
Acting Secretary.

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