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Technical Note 83

ON THE SCATTERING OF γ RAYS BY NUCLEI

U. S. DEPARTMENT OF COMMERCE
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83

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U. Fano

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Preface

This report consists, in essence, of a number of exercises and of discussions of specific topics, which were developed in connection with the experimental program of E. G. Fuller and E. V. Hayward.

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On the Scattering of γ Rays by Nuclei

U. Fano

The theory of scattering by electric dipole interaction is developed by tensorial techniques, which permit an early separation of geometric and dynamic factors. The geometric relationships are formulated in terms of variables that represent arbitrary partial polarization of the incident and scattered γ rays. The relevant dynamic properties of a nucleus are represented by a scalar, a vector, and a quadrupole polarizability. These polarizabilities correspond respectively to the values 0, 1, and 2 of the quantum number j which indicates the angular momentum transfer in the scattering process. The analysis of scattering according to angular momentum transfer is compared to the ordinary theory of angular distributions. The nuclear polarizability is discussed from the standpoint of different models. The magnitudes of the three polarizabilities can be determined by experiments with unpolarized nuclei but with some degree of circular polarization of the γ rays; linear polarization contributes no additional information. Nuclear polarization is required to determine the phases of the polarizabilities.

1. INTRODUCTION

The theory of elastic or inelastic γ-ray scattering by nuclei has been treated by many authors.[1] It is equivalent, in essence, to the theory of the scattering of light by molecules, which has been presented in a comprehensive article by Placzek.[2] The geometrical aspects of this and of related phenomena can be treated in a compact form by means of techniques of tensorial algebra that have been described in recent

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[1] This process is a special case of the general theory of angular correlation. Particular aspects of the (γ,γ) process for deformed nuclei have been treated by A. M. Baldin, Zh. Exp. Theor. Phys. 37, 202 (1959), transl. JETP 10, 142 (1960), Z. Maric and P. Möbius, Nuclear Phys., 10, 135 (1959), and E. G. Fuller and E. V. Hayward, Phys. Rev. Letters, 1, 1507 (1958).

[2] G. Placzek, Marx Handb. der Radiologie, Vol. 6, part 2, p. 305 (1934).

works,[3] which will be referred to as FR59 and F60, respectively. An application of these techniques to various aspects of γ-ray nuclear scattering is presented in the following sections.

The treatment will be limited to the electric dipole interaction, which gives the main contribution to the scattering and is most prominent in the "giant resonance" range of photon energies (10-20 Mev). The treatment will include the effects of γ-ray polarization and can be easily extended to take into account nuclear polarization or alignment.

2. GENERAL FORMULAS

Let us define:

$\underset{\sim}{r}$ = radius vector from the center of mass of a nucleus to the center of its charge,

$\underset{\sim}{A}, \underset{\sim}{A}'$ = unit polarization vectors of the incident and scattered radiation,

$0_- = \left[H - E_i - \hbar\omega\right]^{-1}$ = reciprocal of resonance denominator pertaining to intermediate nuclear state following photon absorption, with H = Hamiltonian operator of nucleus, E_i = ground state energy of nucleus, $\hbar\omega$ = energy of incident radiation photon,

$0_+ = \left[H - E_i + \hbar\omega'\right]^{-1}$ = reciprocal of resonance denominator applicable when emission of the "scattered" photon energy $\hbar\omega'$ precedes the absorption of the incident photon; $\omega' = \omega$ for elastic scattering.

[3] U. Fano and G. Racah, Irreducible Tensorial Sets, Academic Press, (1959). U. Fano, J. Mathem. Physics, 1, 417 (1960).

The scattering operator may then be expressed, in the electric dipole approximation, as

$$S = \mathbf{r}\cdot\mathbf{A}'\; O_-\; \mathbf{r}\cdot\mathbf{A} \;+\; \mathbf{r}\cdot\mathbf{A}\; O_+\; \mathbf{r}\cdot\mathbf{A}' \tag{1}$$

According to the conventions of FR59, p. 22-23, the standard or contrastandard sets of vector or tensor components will be represented by letters with shill across in typing or in German type in print with an appropriate superior index. In this notation, (1) becomes

$$S = \mathfrak{r}^{[1]}\, \mathfrak{A}'^{(1)}\; O_-\; \mathfrak{r}^{[1]} \mathfrak{A}^{(1)} + \mathfrak{r}^{[1]}\, \mathfrak{A}^{(1)}\; O_+\; \mathfrak{r}^{[1]}\, \mathfrak{A}'^{(1)}. \tag{1'}$$

The parts of this operator that pertain to the radiation can be separated from those pertaining to the nucleus by the recoupling procedure given at the end of p. 47 of FR. This yields

$$S = \sum_{j=0}^{2} \left[\mathfrak{r}^{[1]} \times (O_- + (-1)^j\, O_+)\; \mathfrak{r}^{[1]}\right]^{[j]} \left[\mathfrak{A}'^{(1)} \times \mathfrak{A}^{(1)}\right]^{(j)} \tag{2}$$

The index j indicates the <u>angular</u> <u>momentum</u> <u>transferred</u> in the interaction (it was called j_t on p. 114 of FR59). The first factor on the right side of (2) may be called the 2^j-pole polarizability operator of the nucleus, and indicated by

$$\mathfrak{p}^{[j]} = \left[\mathfrak{r}^{[1]} \times (O_- + (-1)^j\, O_+)\; \mathfrak{r}^{[1]}\right]^{[j]}. \tag{2'}$$

Since $j = 0, 1, 2$, we distinguish a scalar, a vector, and a quadrupole polarizability; the vector polarizability vanishes in the static case, since $O_- - O_+ = 0$ for $\omega' = \omega = 0$, and it may often be disregarded as will be seen in Sec. 4 and 5.

Following Chap.18 of FR59 we consider here reduced sets of matrix elements of the operator S, between the initial and final nuclear states whose angular momentum quantum numbers I_i and I_f may be equal or different. Because the form (2) of S coincides with (18.6) of FR59, its reduced sets of matrix elements are given in accordance with (18.7) by[4]

$$\left[(I_f \mid \underset{\sim}{S} \mid I_i)\right]^{(j)} = (2j+1)^{-1/2}\,(I_f \| P^{[j]} \| I_i)\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)}, \quad (3)$$

where the second factor on the right is a reduced matrix element (Eq. 14.4 of FR59) of the 2^j-pole polarizability. This reduced matrix element vanishes whenever "triangular" selection rules, such as $|I_i - j| \leq I_f \leq I_i + j$ are violated; in particular its vanishes for $I_i = 0$ unless $I_f = j$.

If the scatterer nucleus is unpolarized, the desired scattering cross section σ is proportional to the squared modulus of the matrix element of S between an initial and a final state, summed (or averaged) over the magnetic quantum numbers of these states. Application of (18.10) and (18.19) of FR59 and of (3) above yields

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= C\ \Sigma_{m_i, m_f} \left|(I_f\, m_f \mid S \mid I_i\, m_i)\right|^2 = \\ &= C\ \Sigma_j \left[(I_f \mid \underset{\sim}{S} \mid I_i)\right]^{(j)*}\left[(I_f \mid \underset{\sim}{S} \mid I_i)\right]^{(j)} = \\ &= C\ \Sigma_j\ (2j+1)^{-1}\left|(I_f \| P^{[j]} \| I_i)\right|^2 \\ &\times \left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)*}\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)},\end{aligned}$$

where

$$C = \frac{e^4}{c^4}\,\frac{\omega\omega'^4}{2I+1} \quad (4)$$

As a result of the expansion of the interaction in Eq. (2), all the geometrical elements of the γ-ray angular distribution and polarization

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[4] The factor corresponding to $(2j + 1)^{-1/2}$ was omitted by mistake in (18.7) and (18.9) as printed in FR59.

are embodied in the last factor of (4), namely, in the hermitian products of $\left[\mathbf{A}'^{(1)} \times \mathbf{A}^{(1)}\right]^{(j)}$ by itself, for j = 0, 1, 2. The nuclear properties, represented by the 2^j-pole polarizabilities, enter as weight factors in the summation of the hermitian products of different degrees.

If the nuclear orientation were observed before and/or after the scattering, the calculation would proceed in close analogy to the derivation of (4). The invariant product of polarization vectors in (4) would be replaced by an invariant product of the polarization vectors and of the 2^K-pole tensors that describe the nuclear polarization. This new product contains a factor $\left[\left[\mathbf{A}'^{(1)} \times \mathbf{A}^{(1)}\right]^{(j')*} \times \left[\mathbf{A}'^{(1)} \times \mathbf{A}^{(1)}\right]^{(j)}\right]^{(K)}$, with $j' \neq j$. Accordingly the Σ_j in (4) would be replaced by a $\Sigma_{j',j}$ and would include "rectangular" polarizability factors $(I_f \| P^{[j']} \| I_i)^* \times (I_f \| P^{[j]} \| I_i)$, besides the quadratic factor $|(I_f \| P^{[j]} \| I_i)|^2$ which appears in (4). This result is noteworthy: Whereas the scalar, vector and tensor polarizabilities give separate contributions the cross section (4), so that their magnitudes can be determined by experiments with unpolarized nuclei as shown in the next section, the scattering by polarized nuclei depends on interference effects among the polarizabilities so that its measurement can provide the relative phases of the different polarizabilities.

The analysis of the scattering process carried out in this section differs from the analysis that underlies the familiar theories of angular correlation in nuclear physics. We have emphasized the angular momentum transferred from the radiation to the nucleus in the complete scattering process, and expressed the relevant nuclear properties in terms of 2^j-pole polarizabilities. This analysis is represented by the angular momentum diagram in figure 1a, where $\underset{\sim}{L}_i$ and $\underset{\sim}{L}_s$ represent the angular momenta of the incident and scattered radiation, respectively. (In the electric dipole approximation we have, of course, $L_i = L_s = 1$.) The triangle with sides $(\underset{\sim}{L}_i, \underset{\sim}{L}_s, \underset{\sim}{j})$ in figure 1a corresponds to the geometrical factor in eq. (4), the triangle $(\underset{\sim}{I}_i, \underset{\sim}{I}_f, \underset{\sim}{j})$ to the nuclear polarizability factor. Figure 1b represents, instead, the analysis of angular momentum relationships that is performed in the usual theory of angular correlations. This analysis emphasizes the angular momentum $\underset{\sim}{I}$ of the intermediate nuclear state and deals with the absorption and re-emission as separate stages of the scattering process; the separate stages correspond, respectively, to the triangles $(\underset{\sim}{I}_i, \underset{\sim}{L}_i, \underset{\sim}{I})$ and $(\underset{\sim}{I}, \underset{\sim}{L}_s, \underset{\sim}{I}_f)$. The treatment of FR59 develops primarily the analysis corresponding to Figure 1b which leads to the angular correlation formula (19.11), and deals only briefly with the analysis corresponding to Figure 1a which leads to equations (19.12, (19.13). The equivalence between the two types of formulas is established by a geometrical

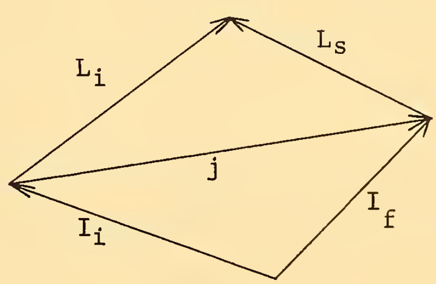

Figure 1a

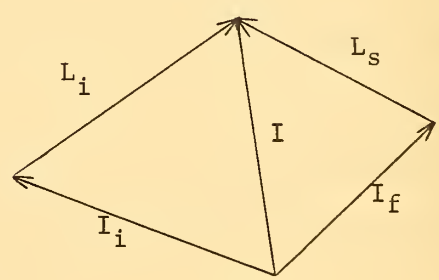

Figure 1b

identity due to Biedenharn (FR59, p. 114 and Appendix I), and is illustrated in the Appendix of this paper. Note that the analysis according to angular momentum transfer is particularly convenient in the treatment of scattering which may result from the alternative sequences absorption-emission and emission-absorption, represented by the two terms on the right of eq. (1). (Diagrams representing the emission-absorption sequence differ from those of Figure 1 by the interchange of $\underset{\sim}{L}_i$ and $\underset{\sim}{L}_s$.) The interference between the contributions of the two sequences is readily represented by the expression of the 2^j-pole polarizability in eq. (2).

3. THE GEOMETRICAL FACTOR

The product of polarization vectors in (4) is expressed in ordinary (non-standard) vector notation by

$$\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)*}\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)} =$$

$$= \begin{cases} (1/3)\ (\underset{\sim}{A}' \cdot \underset{\sim}{A})^2 & \text{for } j = 0, \\ (1/2)\left|\underset{\sim}{A}' \times \underset{\sim}{A}\right|^2 = (1/2)\left[1 - (\underset{\sim}{A}' \cdot \underset{\sim}{A})^2\right] & \text{for } j = 1, \\ (1/2)\left[1 + (1/3)\ (\underset{\sim}{A}' \cdot \underset{\sim}{A})^2\right] & \text{for } j = 2 \end{cases} \quad (5)$$

This formula is quite suitable for direct evaluation in two extreme cases, namely: (1) $\underset{\sim}{A}$ and $\underset{\sim}{A}'$ have definite orientations, i.e., the incident γ rays have a complete linear polarization and the scattered radiation is observed with an ideal analyzer for linear polarization (this case is unrealistic), or (2) the incident γ rays are unpolarized and no polarization analysis is made after scattering (this occurs most frequently). In case 2) one averages over two perpendicular orientations of $\underset{\sim}{A}$ and sums over two perpendicular orientations of $\underset{\sim}{A}'$, thereby obtaining the following values of (5)

$$(1/6)\ (1 + \cos^2\theta), \quad \text{for } j = 0,$$

$$(1/4)\ (2 + \sin^2\theta), \quad \text{for } j = 1,$$

$$(1/12)\ (13 + \cos^2\theta), \quad \text{for } j = 2, \tag{6}$$

where θ is the angle between the directions of incidence and of scattering.

A partially polarized incident beam may be represented by the incoherent superposition of two components with orthogonal polarization vectors $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ and relative intensities p_1 and $p_2 = 1 - p_1$. (The degree of polarization is, then, $p_1 - p_2$.) Polarization analysis of the scattered radiation may be similarly represented by a maximum response efficiency e_1 for detection of γ rays with a certain polarization $\underset{\sim}{A}_1'$ and by a minimum efficiency e_2 for γ rays with the orthogonal polarization $\underset{\sim}{A}_2'$. (The degree of selectivity of the analyzer is, then, $(e_1 - e_2)/(e_1 + e_2)$.) In this general case one must evaluate the expression

$$\sum_{i,s=1}^{2} p_i\, e_s \left[\underset{\sim}{A}_s^{\prime(1)} \times \underset{\sim}{A}_i^{(1)}\right]^{(j)*} \left[\underset{\sim}{A}_s^{\prime(1)} \times \underset{\sim}{A}_i^{(1)}\right]^{(j)}, \tag{6'}$$

which reduces to (6) when $p_1 = p_2 = 1/2$ and $e_1 = e_2 = 1$. Notice that $\underset{\sim}{A}' \cdot \underset{\sim}{A}$ in (5) must be replaced by $|\underset{\sim}{A}' \cdot \underset{\sim}{A}|$ when the basic polarizations are circular or elliptical, in which case the vectors $\underset{\sim}{A}_i$ and/or $\underset{\sim}{A}_s'$ are complex.

Alternatively, the distribution of scattered γ rays can be analyzed, with regard to both direction and polarization, by the general procedure of harmonic analysis given in Chapter 19 of FR59. This procedure represents the kind and degree of polarization of the incident or scattered γ rays by means of variables, such as the Stokes parameters, which are always observable whereas the polarization vectors $\underset{\sim}{A}$ and $\underset{\sim}{A}'$ are observable only for fully polarized radiation. These variables are, in essence, mean values of products of polarization vector components, $\Sigma_i\, p_i\left[\underset{\sim}{A}_i^{(1)} \times \underset{\sim}{A}_i^{(1)}\right]^{(k)}$ and $\Sigma_s\, e_s\left[\underset{\sim}{A}_s'^{(1)} \times \underset{\sim}{A}_s'^{(1)}\right]^{(k)}$, where the degree, k, of the product ranges from 0 to 2. The geometrical factor (5) is expressed in terms of these products by a recoupling transformation. With reference to FR59, p. 29, p. 37 and eq. (11.10) one finds that

$$\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)*}\left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(j)} =$$

$$= (2j+1)\, \Sigma_k (-1)^{j+k}\, \overline{W}\begin{pmatrix}1 & 1 & j\\ 1 & 1 & k\end{pmatrix}\left[\underset{\sim}{A}^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(k)} \cdot \left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}'^{(1)}\right]^{(k)} . \qquad (7)$$

The product $\left[\underset{\sim}{A}^{(1)} \times \underset{\sim}{A}^{(1)}\right]^{(k)} \cdot \left[\underset{\sim}{A}'^{(1)} \times \underset{\sim}{A}'^{(1)}\right]^{(k)}$ in (7) is a harmonic function of degree k of the Euler angles between frames of reference attached to the polarizer of the incident radiation and to the analyzer of the scattered radiation (FR59, Chap. 6). Accordingly the summation over k in (7) represents a harmonic expansion of the angular and polarization distribution of the scattered radiation. This harmonic expansion is obtained automatically in the theories that analyze the distribution according to the coupling scheme of Figure 1b, as emphasized in Chap. 19 of FR59. When the analysis develops according to the coupling scheme of Figure 1a, as was done in this paper, the harmonic expansion results from a recoupling transformation and takes the form of (19.13) of FR59, of which the right hand side of (7) is a special case. Note that the $\overline{W}$ coefficient in (7) is the same as one encounters when calculating the 2^k-pole interaction of two p-orbitals (with $\ell=1$) coupled with a resultant quantum number j.

We still have the task of obtaining an explicit expression for the scalar product on the right hand side of (7) properly averaged over the polarization intensities and detector efficiencies p_i and e_s. (This task has not been carried out in the examples of FR59, Chap. 19, where the final results were expressed in terms of density and detector matrices and of $\emptyset$ functions.) In this connection, it is not always convenient to characterize the polarizations by the vectors $\underset{\sim}{A}$ and $\underset{\sim}{A}'$ because these vectors are, in general, complex and their real and imaginary parts merely represent a pair of conjugated half-diameters of the polarization ellipse. Notice also that the polarization vector $\underset{\sim}{A}'$ which appears in cross section formulas is not properly a characteristic of the scattered radiation but that it characterizes the analyzer detector whose probability of response is being calculated; similarly $\underset{\sim}{A}$ may be regarded as a property of the polarizer that has prepared the incident beam. We shall express the cross section in terms of the following parameters:

a) the degrees of linear and circular polarization of the incident rays, $P_\ell = |\underset{\sim}{A}_1 \cdot \underset{\sim}{A}_1|(p_1-p_2)$ and $P_c = |\underset{\sim}{A}_1 \times \underset{\sim}{A}_1^*|\,(p_1-p_2)$;

b) the corresponding selectivities of the analyzer for linear and circular polarization of the scattered γ rays, $Q = |\underset{\sim}{A}_1' \cdot \underset{\sim}{A}_1'|(e_1-e_2)/(e_1+e_2)$ and $Q_c = |\underset{\sim}{A}_1' \times \underset{\sim}{A}_1'^*|(e_1-e_2)/(e_1+e_2)$,

c) the total response efficiency $\xi = e_1+e_2$, which constitutes only a normalization constant (simple summation over two alternative polarizations yields $\xi = 2$),

d) the angle Θ between the directions of incidence and scattering,

e) the angle ψ from the plane of incident linear polarization, identified by the polarizer orientation, to the plane of scattering,

f) the angle φ from the plane of scattering to the plane of linear polarization selected by the analyzer.

We write, then, in terms of these variables

$$\Sigma_{i,s=1}^{2} p_i e_s \left[\mathcal{A}_1^{(1)} \times \mathcal{A}_1^{(1)}\right]^{(k)} \cdot \left[\mathcal{A}_s'^{(1)} \times \mathcal{A}_s'^{(1)}\right]^{(k)} =$$

$$= \xi R_k(P_\ell, P_c, Q_\ell, Q_c, \psi, \Theta, \varphi). \qquad (8)$$

The functions R_k can be worked out in the real-standard representation of F60, and are in fact obtained by setting $I_o=E_o=1$ in (26) and (27) of F60. They are

$$R_o = 1/3$$

$$R_1 = (1/2) P_c Q_c \cos\Theta$$

$$R_2 = (1/12)(3\cos^2\Theta - 1) - (1/4)(P_\ell \cos 2\psi + Q_\ell \cos 2\varphi)\sin^2\Theta + \qquad (9)$$

$$+ (1/4) P_\ell Q_\ell \cos 2\psi \cos 2\varphi (1+\cos^2\Theta) - P_\ell Q_\ell \sin 2\psi \sin 2\varphi \cos\Theta.$$

Expressing the coefficient on the right side of (7) as a matrix

$$M_{jk} = (-1)^{j+k}\, \overline{W}\begin{pmatrix} 1 1 j \\ 1 1 k \end{pmatrix} = \begin{vmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/6 & -1/6 \\ 1/3 & -1/6 & 1/30 \end{vmatrix}, \qquad (10)$$

we can finally write the geometrical factor (6'), as a function of the observable variables of polarization and direction, in the form

$$\Sigma_{i,s=1}^{2} p_i e_s \left[\mathcal{A}_s'^{(1)} \times \mathcal{A}_i^{(1)}\right]^{(j)*} \left[\mathcal{A}_s'^{(1)} \times \mathcal{A}_i^{(1)}\right]^{(j)} =$$

$$= (2j+1)\ \Sigma_k M_{jk} R_k. \qquad (11)$$

This formula reduces to (6) when P_ℓ, P_c, Q_ℓ, and Q_c vanish and $\xi=2$.

Substitution into (4) of the geometrical factor thus evaluated yields the cross section for detection of the scattered radiation, by an analyzer of given characteristics, in the form

$$\frac{d\sigma}{d\Omega} = C\,\Sigma_{jk}\,|(I_f\|\,P^{[j]}\|I_i)|^2\; M_{jk\xi}\,R_k(P_\ell, P_c, Q_\ell, Q_c, \psi, \theta, \varphi)$$

$$= \Sigma_{k=0}^{2}\, c_k R_k \qquad (12)$$

Several points may be emphasized regarding this formula:

1) The zero-degree coefficient c_0 is 3 times the average value of the cross section over all the orientation variables. This was expected since the Σ_k represents an expansion into harmonic functions of the orientations.

2) When all polarization effects are absent or are averaged out, the experimental dependence of the cross section on $\cos^2\theta$ determines the value of the coefficient c_2.

3) Once c_2 is thus determined, the dependence of the cross section on all the linear polarization variables is completely determined through the expression (9) of R_2. Therefore: a) the observation of <u>linear</u> polarization effects contributes no <u>independent information</u> on the scattering process, and b) conversely, the properties of polarization considered in this section enable one to utilize the scattering process as a quantitative polarizer or analyzer tool once c_2 has been determined.

4) Experimental determination of c_1 <u>requires</u> the presence of <u>circular</u> polarization in the incident beam and the measurement of circular polarization of the scattered radiation.

5) Experimental determination of all three coefficients c_o, c_1, and c_2 (or equivalent information) is necessary and sufficient to determine the magnitudes of the three 2^j-pole polarizabilities, by inversion of (12), which yields

$$|(I_i \| P^{[j]} \| I_f)|^2 = \frac{1}{C\,\xi}\ \Sigma_k (M^{-1})_{jk}\, c_k \ , \tag{13}$$

where

$$(M^{-1})_{jk} = (-1)^{j+k}\,(2j+1)(2k+1)\ \overline{W}\begin{pmatrix} 1 & 1 & j \\ 1 & 1 & k \end{pmatrix} = \begin{vmatrix} 1/3 & 1 & 5/3 \\ 1 & 3/2 & -5/2 \\ 5/3 & -5/2 & 5/6 \end{vmatrix} .$$

6) All three coefficients c_k are equal when the vector and quadrupole polarizabilities vanish.

4. POLARIZABILITY ANALYSIS IN TERMS OF INTERMEDIATE STATES

We shall now consider the connection between the 2^j-pole polarizabilities $(I_f \| P^{[j]} \| I_i)$, which determine the γ-ray scattering, and other nuclear parameters. As is well known, dispersion relations tie the electric polarizability of a system to its absorption of radiation by electric dipole interaction. The usual dispersion relations pertain to scalar polarizability; but in our problem we deal also with a vector and a quadrupole polarizability. We should, therefore, derive a more general dispersion relation.

Since $\underset{\sim}{P}^{[j]} = \left[\underset{\sim}{r}^{[1]} \times (O_- + (-1)^j\, O_+)\ \underset{\sim}{r}^{[1]}\right]^{[j]}$, its reduced matrix elements $(I_f \| P^{[j]} \| I_i)$ can be expressed in terms of the reduced matrix elements of the dipole operator $\underset{\sim}{r}^{[j]}$ pertaining to transitions between the states I_i or I_f and an intermediate state. The intermediate state can be classified by its angular momentum quantum number $I = I_i + 1$, I_i, or $I_i - 1$, and by an additional quantum number n which may be a continuous variable.

The energy of this state will be indicated by E_{nI}. The intermediate state is an eigenstate of the operators O_- and O_+ which become now algebraic functions of E_{nI}. With these conventions, (15.15) of FR59 yields the following expression of the polarizabilities:

$$(I_f \| P^{[j]} \| I_i) = \tag{14}$$

$$= (-1)^{I_i+I_f+j} (2j+1)^{\frac{1}{2}} \Sigma_{nI} (I_f \| r^{[1]} \| nI) \left[\frac{1}{E_{nI}-E_i-\hbar\omega} + \frac{(-1)^j}{E_{nI}-E_i+\hbar\omega'} \right]$$

$$\times (nI \| r^{[1]} \| I_i) \; \overline{W}\begin{pmatrix} I_i I_f j \\ 1 \; 1 \; I \end{pmatrix}$$

This is a Kramers-Heisenberg dispersion formula, generalized through the $\overline{W}$ factor to represent vector and quadrupole polarizabilities besides the scalar one. (In the scalar case, j vanishes and $\overline{W}$ takes a simple form.) This formula provides little qualitative information unless it is discussed from the standpoint of a model that provides some assumption regarding the matrix elements and the energy levels E_{nI}. The coefficients $\overline{W}$ can be taken from tables[5].

Some simplification of (14) is attained for elastic scattering, when $I_f = I_i$, $\omega' = \omega$. The reduced matrix element can then be expressed in terms of the oscillator strength of the transition $I_i \rightarrow nI$ (averaged over magnetic quantum numbers)

[5a] A. Simon, J. H. Vandersluis, and L. C. Biedenharn, Tables of the Racah coefficients, Oak Ridge Nat. Lab. Report 1679 Special (1954).

[b)] M. Rotenberg, R. Bivins, N. Metropolis, and J. K. Wooten, The 3j and 6j Coefficients. Technology press, Cambridge, Mass. (1959). See particularly eqs. (2.22-25) for our application.

$$(I_f \| r^{[1]} \| nI)\ (nI \| r^{[1]} \| I_i) = (-1)^{I_i+1-I}\ |(nI \| r^{[1]} \| I_i)|^2 =$$

$$= (-1)^{I_i+1-I}\ 3(2I_i+1)\ \frac{\hbar^2}{2M\,(E_{nI}-E_i)} \left\langle f_{nI,I_i} \right\rangle \quad , \tag{15}$$

where M indicates the proton mass. When $I_f = I_i$, the $\overline{W}$ coefficient in (14) is the same as one finds in the expression of the 2j-pole interaction energy between systems with quantum numbers I_i and 1 coupled with resultant quantum number I. Ref. 5a, p. XIII-XIV, also provides a simplified form of this $\overline{W}$ coefficient which can be cast in the form

$$\overline{W}\begin{pmatrix} I_i & I_i & j \\ 1 & 1 & I \end{pmatrix} = \frac{(-1)^{I_i+I+1}}{\left[3(2I_i+1)\right]^{1/2}}\ F_j(I_i, I) \quad , \tag{16}$$

where

$$F_j(I_i,I) = \begin{cases} 1 \quad , & \text{for } j = 0 \\[2ex] \dfrac{I(I+1)-I_i(I_i+1)-2}{\left[8I_i(I_i+1)\right]^{1/2}} \quad , & \text{for } j = 1 \\[2ex] \dfrac{6\left[I(I+1)-I_i(I_i+1)-2\right]\left[I(I+1)-I_i(I_i+1)-1\right] - 16I_i(I_i+1)}{160\left[I_i(I_i+1)(2I_i-1)(2I_i+3)\right]^{1/2}} \quad , & \text{for } j = 2 . \end{cases} \tag{17}$$

Substitution of (15) and (16) into (14) yields

$$(I_i \| P^{[j]} \| I_i) = \tag{18}$$

$$= (-1)^j \left[3(2I_i+1)(2j+1)\right]^{1/2} \Sigma_I\ F_j(I_i,I) \left\{ \Sigma_n \frac{h^2}{2M(E_{nI}-E_i)} \left\langle f_{nI,I_i} \right\rangle \right.$$

$$\left. \times \left[\frac{1}{E_{nI}-E_i-\hbar\omega} + \frac{(-1)^j}{E_{nI}-E_i+\hbar\,\omega} \right] \right\}$$

Whereas the usual Kramers-Heisenberg dispersion formula provides a single equation, which relates the scalar polarizability to a function of the distribution of oscillator strengths, we have now two additional equations, which relate the vector and quadrupole polarizabilities to two different functions of the oscillator strength distribution. Experimental determination of the three polarizabilities, as functions of ω, yields then additional information on the distribution of oscillator strength. More specifically, it is known that the Kramers-Heisenberg formula can be inverted, at least in principle, to determine the spectral distribution of oscillator strength, $\Sigma_I \langle f_{nI,I_i} \rangle$. Since the contributions of three values of I, namely I_i and $I_i \pm 1$, are pooled in this spectral distribution, one may inquire whether the availability of data on the three polarizabilities, with j=0,1,2, would enable us to sort out the contributions of the transitions to states with different I. This would be possible if the expression in the braces of (18) were a function of I but not of j, since (18) would then constitute a system of three linear equations in the three unknown values of the braces for $I=I_i, I_i \pm 1$. However, the braces include a factor $(-1)^j$ so that (18) includes in fact six separate functions of I, namely $\Sigma_n \langle f_{nI,I_i} \rangle \hbar^2/2m(E_{nI}-E_i \pm \hbar\omega)$ for three values of I and two alternative signs in the last factor.

5. POLARIZABILITY ANALYSIS IN THE BOHR-MOTTELSON MODEL

It is a characteristic property of the states of a rotational band that the reduced matrix elements of a tensorial operator between different pairs of states of the band are proportional to a single matrix element, evaluated in the "intrinsic" coordinate system.[6] The proportionality coefficients are determined by the following geometric considerations.

In the intrinsic coordinate system consider the ground state of the nucleus, call K_i its (non-negative) angular momentum quantum number about the z axis of this system, and indicate this state by $|K_i)$. To this state corresponds in the laboratory system a band of rotational states indicated by $|K_i I_i m_i)$, which are represented in terms of $|K_i)$ and of wave functions of the symmetrical top $D^{(I_i)}{}_{K_i m_i}(\psi, \theta, \varphi)$. Similarly a tensorial set of operators $\mathcal{T}^{[k]}_q$, such as the set of components of the 2^k-pole polarizability, is represented in the laboratory system by $\Sigma_{q'}\ \mathcal{T}^{[k]}_{q'}\ D^{(k)}_{q'q}\ (\psi, \theta, \varphi)$, where $\mathcal{T}^{[k]}$ indicates an operator of the set in the intrinsic system. It follows that a matrix element $(K_f I_f m_f |\ \mathcal{T}^{[k]}_q\ | K_i I_i m_i)$ pertaining to the laboratory system is a linear combination of matrix elements $(K_f | \mathcal{T}^{[k]}_{q'} | K_i)$ of the intrinsic system, whose coefficients are integrals over D functions expressed in terms of Wigner coefficients. (Notice, however, that this procedure is meaningful only insofar as the operator $T^{[k]}_q$ is independent of the energy of the final state $K_f I_f m_f$. In our problem this condition holds only in the approximation where ω' in O_+ may be taken equal to ω, i.e. when $\hbar(\omega - \omega') = E_f - E_i \sim 0$. This assumption will be made in the following.)

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[6] A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd., 27, No. 16 (1953), p. 109.

We shall be interested here in matrix elements between states of the same rotational band, in which case $K_f = K_i$ and $(K_f|\mathcal{T}^{[k]}_{q'}|K_i)$ vanishes unless $q' = 0$ or $q' = 2K_i$. The matrix elements with $q' = 2K_i$ arise insofar as the tensorial operator can reverse the sign of the spin in the intrinsic system. Considering that $|q'| \leqslant k$, that $k \leqslant 2$ in our problem, that $K_i = 1$ does not occur in the lowest band of deformed nuclei, and that $K_i = 1/2$ occurs only in a few instances, we shall disregard henceforth the contributions from $q' = 2K_i \neq 0$.

Under this restrictive condition, all matrix elements $(K_i I_f m_f|\mathcal{T}^{[k]}_q|K_i I_i m_i)$ are proportional to the single matrix element $(K_i\,|T^{[k]}_0|\,K_i)$ of the intrinsic system. Here we need only consider the reduced matrix elements $(K_i I_f\,||\,T^{[k]}||K_i I_i)$, since the dependence of the matrix elements on the magnetic quantum numbers (m_i, m_f) is given by the Wigner-Eckart theorem (FR59, Eq. (14.4))which has been taken into account implicitly in Sect. 2. These matrix elements are given by

$$(K_i I_f\,||\,T^{[k]}||K_i I_i) = \sqrt{2I_i+1}\;(I_i K_i k0\,|\,I_i k I_f K_i)\;(K_i|T^{[k]}_0\,|\,K_i). \qquad (19)$$

In the case of the polarizability matrix of (3) and (4), this equation reads

$$(K_i I_f\,||\,P^{[j]}||K_i I_i) = \qquad (20)$$

$$= \sqrt{2\,I_i+1}\;\;(I_i K_i j0\,|\,I_i j I_f K_i)\;(K_i|\,\mathcal{P}^{[j]}_0|K_i).$$

A sum rule follows from this result, namely that the total cross section for elastic plus Raman scattering within a rotational band depends only on the polarizability in the intrinsic coordinate system:

$$\Sigma_{I_f} \, |(K_i I_f \| P^{[j]} \| K_i I_i)|^2 =$$

$$= (2I_i+1)|(K_i | \not{P}^{[j]}_0 | K_i)|^2 \quad . \tag{21}$$

Analysis in the intrinsic system. The analysis of polarizability in terms of dipole transitions to intermediate states, carried out in general form in Sect. 4, can also be carried out for the Bohr-Mottelson model in the intrinsic system. Here there is no further reason to utilize standard notation and we can express the polarizability operators in ordinary coordinates. However we keep the complex combinations of coordinates $x\pm iy$ because they lead to transitions from the ground state K_i only to intermediate states (n,K) with $K = |K_i \pm 1|$, respectively.

Translation from standard notation to ordinary coordinates yields

$$\not{P}^{[j]}_0 = \left[\underset{\sim}{\not{r}}^{[1]} \times (0_- + (-1)^j 0_+)\, \underset{\sim}{\not{r}}^{[1]}\right]^{[j]}_0 = \tag{22}$$

$$\frac{1}{2\sqrt{3}}\left[(x-iy)(0_-+0_+)(x+iy)+(x+iy)(0_-+0_+)(x-iy)+2z(0_-+0_+)z\right], \text{ for } j = 0,$$

$$\frac{1}{2\sqrt{2}}\left[-(x-iy)(0_--0_+)(x+iy)+(x+iy)(0_--0_+)(x-iy)\right], \qquad \text{for } j = 1,$$

$$\frac{1}{2\sqrt{6}}\left[(x-iy)(0_-+0_+)(x+iy)+(x+iy)(0_-+0_+)(x-iy)-4z(0_-+0_+)z\right], \text{for } j = 2.$$

Indicating the intermediate states by nK we have, then,

$$(K_i|\not{P}_0^{[0]}|K_i) = \frac{1}{\sqrt{3}} \Sigma_n \Bigg[\frac{E_{n,K_i+1} - E_o}{(E_{n,K_i+1} - E_o)^2 - \hbar^2\omega^2} |(n,K_i+1 \,|\, x+iy \,|\, K_i)|^2 +$$

$$+ \frac{E_{n,|K_i-1|} - E_o}{(E_{n,|K_i-1|} - E_o)^2 - \hbar^2\omega^2} |(n,\ K_i-1|\ x-iy|K_i)|^2 +$$

$$+ \frac{E_{nK_o} - E_{K_o}}{(E_{nK_o} - E_o)^2 - \hbar^2\omega^2}\, 2\, |(nK_i|z|\,K_i)|^2 \Bigg],$$

$$(K_i|\not{P}_0^{[1]}|K_i) = -\frac{1}{\sqrt{2}} \Sigma_n \Bigg[\frac{\hbar^2\omega^2}{(E_{n,K_i+1} - E_o)^2 - \hbar^2\omega^2} |(n,K_i+1|\ x+iy\,|\,K_i)|^2 -$$

$$- \frac{\hbar^2\omega^2}{(E_{n,|K_i-1|} - E_o)^2 - \hbar^2\omega^2} |(n,|K_i-1||\ x-iy|K_i)|^2 \Bigg],$$

$$(K_i|\not{P}_0^{[2]}|K_i) = \frac{1}{\sqrt{6}} \Sigma_n \Bigg[\frac{E_{n,K_i+1} - E_o}{(E_{n,K_i+1} - E_o)^2 - \hbar^2\omega^2} |(n,K_i+1|x+iy|K_i)|^2 +$$

$$+ \frac{E_{n,|K_i-1|} - E_o}{(E_{n,|K_i-1|} - E_o)^2 - \hbar^2\omega^2} |(n,|K_i-1||\ x-iy|K_i)|^2 -$$

$$-4 \frac{E_{nK_i} - E_o}{(E_{nK_i} - E_o)^2 - \hbar^2\omega^2} |(nK_i|\ z\,|\,K_i)|^2 \Bigg]. \quad (23)$$

Notice that the matrix element of $\not{P}^{[1]}_0$ (vector polarizability) vanishes unless the excitation spectra to states with $K = |K_i \pm 1|$ differ. No indication of such a difference has yet been detected for giant resonance transitions, and none could occur for the even-even nuclei in which $K_i = 0$ and $|K_i - 1| = K_i + 1$. Similarly the matrix element of $\not{P}^{[2]}_0$ (quadrupole polarizability) vanishes unless there is a difference between transverse and longitudinal excitations. A probable cause of similarity between the spectra with $K = |K_i \pm 1|$ is indicated in the next Section.

6. CORE POLARIZABILITY IN THE PRESENCE OF A "SPECTATOR" PARTICLE

Situations occur in which a nucleus may be regarded as consisting of a single particle and of a more symmetrical core which are weakly coupled. In this event the nuclear polarizability should be primarily determined by the large core and should exhibit the core's symmetry. Examples of this kind may be the Bi^{209}_{83} nucleus with a single proton outside the spherical "double-magic" Pb^{208}_{82} core, and any odd-A deformed nucleus in which the elongated spinless residue of mass A-1 is presumably unaffected by any reversal of the direction of rotation of the odd particle. It will be shown here how, when such conditions prevail, the polarizability of the whole nucleus reduces to the polarizability of the core and exhibits the core's symmetry.

In a Bi^{209}_{83}-type nucleus, the ground state may be represented by the coupling scheme symbol $(0, I_i; I_i)$, where 0 is the angular momentum of the core, and I_i indicates both the angular momentum of the odd proton and the total angular momentum of the whole nucleus. Similarly a state obtained by electric dipole excitation of the core, with spin $I = I_i - 1$, I_i, or $I_i + 1$, may be represented by $(nl, I_i; I)$. The reduced matrix element of the dipole operator $\underset{\sim}{r}$ between these two nuclear states can be expressed in terms of the corresponding matrix element for the core alone by the projection formula (15.7) of FR59.

$$(nl, I_i; I \| r^{[1]} \| 0, I_i; I_i) = (-1)^{2I_i} (nl \| r^{[1]} \| 0) \left[(2I_i+1)(2I+1)\right]^{1/2}$$

$$\times \ \overline{W}\begin{pmatrix} I & I_i & 1 \\ 0 & 1 & I_i \end{pmatrix} =$$

$$= (-1)^{I-I_i+1} (2I+1)^{1/2} \sqrt{\frac{1}{3}} (nl \| r^{[1]} \| 0), \tag{24}$$

where the dependence on I is reduced to the weight and phase factor $(-1)^{I-I_i+1} (2I+1)^{1/2}$ If this result is entered in the polarizability formula (14), with $I_f = I_i$, and if the <u>weak coupling approximation</u> is made, $E_{nI} = E_{nl}$ independently of I, the sum over I can be carried out analytically and the polarizability is seen to vanish for $j \neq 0$. That is, the spherically symmetric core contributes no vector or tensor polarizability, as was to be expected. (To verify this result, one must take into account the conjugation property of the reduced matrix element (24), namely Eq. (14.8) of FR59, and a special case of the orthonormality Eq. (11.15) of FR59, namely $\left[3(2I_i+1)\right]^{1/2} \Sigma_i (2I+1)(-1)^{I_i+1+I} \overline{W}(I_i I_i j/11 I) = \delta_{jo}$.)

In an odd-A deformed nucleus, the ground state with spin K_i may be represented, in the intrinsic coordinate system, by the coupling scheme symbol $(0,K_i;K_i)$, in which 0 indicates the spin of the elongated residue of mass A-1 and the first index K_i represents the angular momentum of the odd particle. A state obtained by transverse electric dipole excitation of the spinless residue may be represented by $(nl,K_i;K_i\pm 1)$. Insofar as the odd particle is weakly coupled to the residue, the excitation energy and the dipole matrix element of the transition will be independent of whether the transition induces an angular momentum parallel or opposite to that of the odd particle, i.e. whether their resultant is K_i+1 or K_i-1. In this event the vector (j=1) polarizability will vanish, according to (23), as it would do for an even-A spinless nucleus.

APPENDIX

VECTOR DIAGRAM OF THE BIEDENHARN RECOUPLING IDENTITY

The connection between the ordinary angular correlation formula and the formulas obtained by emphasizing the angular momentum transferred in the scattering process is illustrated by Figure 2. In this figure is reproduced the same pair of vector triangles, $(\underset{\sim}{I}_i,\underset{\sim}{L}_i,\underset{\sim}{I})$ and $(\underset{\sim}{I},\underset{\sim}{L}_s,\underset{\sim}{I}_f)$, as was shown in Figure 1b. Consider now that the directions of $\underset{\sim}{I},\underset{\sim}{L}_i$, and $\underset{\sim}{L}_s$, are not identified in the scattering process; indeed even the magnitude of $\underset{\sim}{I}$ is not identified, and those of $\underset{\sim}{L}_i$ and $\underset{\sim}{L}_s$

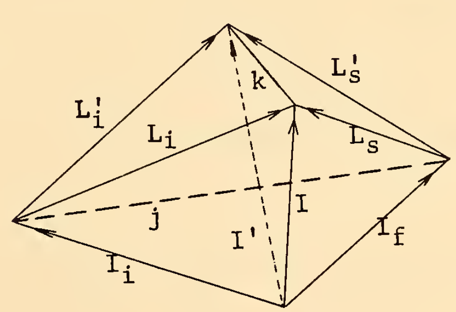

Figure 2

would vary if the limitation to dipole transitions were relaxed. The interference effects arising from alternative vectors $\underset{\sim}{I}, \underset{\sim}{L}_i$, and $\underset{\sim}{L}_s$ are represented by adding to the diagram a second pair of triangles, $(\underset{\sim}{I}_i, \underset{\sim}{L}_i', \underset{\sim}{I}')$ and $(\underset{\sim}{I}', \underset{\sim}{L}_s', \underset{\sim}{I}_f)$. The possible magnitudes of the vector $\underset{\sim}{k} = \underset{\sim}{L}_i - \underset{\sim}{L}_i' = \underset{\sim}{L}_s' - \underset{\sim}{L}_s'$, namely 0,1, or 2, correspond to the degrees of the various terms in the harmonic analysis of the distribution of scattered radiation (Chap. 19 of FR59). The 9 vectors listed thus far form a triangular bipyramid consisting of two tetrahedra with the common base $(\underset{\sim}{I}, \underset{\sim}{I}', \underset{\sim}{k})$. The edges of these two tetrahedra correspond, respectively, to the indices of the two $\overline{W}$ coefficients that appear in a scattering formula constructed on the pattern of Eq. (19.11) of FR59.

The angular momentum transfer $\underset{\sim}{j}$ is represented by the diagonal that joins the opposite vertices of the bipyramid. The vector triangles $(\underset{\sim}{I}_i, \underset{\sim}{j}, \underset{\sim}{I}_f)$ and $(\underset{\sim}{L}_i, \underset{\sim}{j}, \underset{\sim}{L}_s)$ are the same as are shown in Figure 1a. Notice that these two triangles, together with $(\underset{\sim}{L}_i', \underset{\sim}{j}, \underset{\sim}{L}_s')$, subdivide the bipyramid into three tetrahedra with the common edge $\underset{\sim}{j}$. The edges of the tetrahedron $(\underset{\sim}{L}_i, \underset{\sim}{L}_i', \underset{\sim}{L}_s, \underset{\sim}{L}_s', \underset{\sim}{j}, \underset{\sim}{k})$ correspond to the indices of the $\overline{W}$ coefficient in (7), those of $(\underset{\sim}{I}_i, \underset{\sim}{L}_i, \underset{\sim}{I}_f, \underset{\sim}{L}_s, \underset{\sim}{I}, \underset{\sim}{j})$ to the indices of the $\overline{W}$ in (14), those of the third tetrahedron to the indices of the additional $\overline{W}$ that appears when the square of (14) is substituted into (4).

Altogether, the edges of the five tetrahedra that can be seen in Figure 2 correspond to the indices of the five $\overline{W}$ coefficients of the Biedernharn identity in (19.12) of FR59. The identity of a product of two $\overline{W}$ and of a sum of products of three $\overline{W}$ relates to the fact that the bipyramid can be subdivided alternatively into two or into three tetrahedra.

U.S. DEPARTMENT OF COMMERCE
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