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A Users' Guide to the OMNITAB Command "STATISTICAL ANALYSIS"

U.S. DEPARTMENT OF COMMERCE
National Bureau of Standards

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A USERS' GUIDE TO THE OMNITAB COMMAND "STATISTICAL ANALYSIS"

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Abstract

This Technical Note is the first of a series of interpretive notes for a number of commands in the OMNITAB system that have the automatic printout feature. Others planned in the series include commands FIT and POLYFIT, TWOWAY analysis, ONEWAY analysis, and CORRELATION.

These notes aim to be self-contained so that users may have sufficient information on hand for the understanding of the statistics computed and to use them for their immediate applications. , Computation formulas are given in the text, and a number of statistical tables are reproduced in the Appendix for the convenience of users.

Key Words: Computing, statistical; documentation for users; OMNITAB; statistics.

Introduction

The purpose of this Technical Note is to give some supplemental statistical information for the OMNITAB command STATISTICAL ANALYSIS described in NBS Handbook 101 and NBS Technical Note 552.[1/] This command (and several others) gives automatic printout of a variety of statistics computed from a set of observations. The interpretations of these statistics and their uses are described in some detail in this Note.

This Technical Note aims to be self-contained in the sense that users may gain sufficient understanding from the descriptions of these statistics for their immediate applications. References to popular statistical texts are given whenever available and to papers in statistical literature only when discussions are not readily found in text books. Computation formulas are provided. A number of statistical tables referenced are reproduced in the Appendix for the convenience of the user.

The printout for STATISTICAL ANALYSIS used in this Note is based on OMNITAB II, version 5.01, dated May 29, 1971. Future changes in printout format and algorithms, however, would not vitiate the usefulness of the descriptions and interpretations given for these statistics in this Note.

We assume that readers already know how to use OMNITAB and are familiar with the OMNITAB User's Reference Manual. We only note a few related items of interest.

A. The command STATISTICAL ANALYSIS works on a column of numbers. These numbers may be read into the desired column by the commands READ or SET, or may be stored in a column as a result of another command, e.g., residuals stored as a result of the command FIT.

B. The normal size of the worksheet is 201 rows by 62 columns. If the length of the data exceeds 201, then the arrangement of the worksheet has to be changed by the command DIMENSION.

1/ OMNITAB: A Computer Program for Statistical and Numerical Analysis, Nat'l. Bur. of Standards Handbook 101 (1966, reissued 1968).

OMNITAB II User's Reference Manual, NBS Technical Note 552 (1971).

C. If STATISTICAL ANALYSIS is to be used repeatedly for several columns of different lengths, the command RESET must be used to set the length of each column.

The first part of the following lists the command STATISTICAL ANALYSIS in its various forms, and the second part describes and interprets statistics given in the printout.

Frequently users may wish to use weights in conjunction with their observations. The command "STATISTICAL ANALYSIS" can be used with a specified column of weights. The use of such weights, however, raises problems in the strict interpretation of <u>some</u> of the statistics that are printed automatically. We shall not attempt to interpret all such items, except to give the rule of computation used in the weighted cases:

a. If the weights are either 0's or 1's, the observations with zero weights are excluded from computation. The number of observations, N, is taken to be those with weights one.

b. If arbitrary weights, other than 0's and 1's, are given, then all the deviations of the observation are computed from the weighted mean, i.e., at all places where the term "Dev." is used,

DEV = OBSERVATION - WEIGHTED MEAN.

These "deviations" are used in the formula given in the text, with or without weights, regardless as to whether the resulting number is meaningful.

I. The Command [2/]

The simplest form of the command for statistical analysis is:

STATIStical analysis of values in column (C)

or, in an abbreviated form

STATIS (C)

At least two pages of automatic printout will result from this command, as shown in the example in Appendix B. The first page lists values of 53 statistics computed from the observations in Column C, and the second page lists, among others, ranks of these observations and ordered observations. A detailed discussion on the use of these statistics is given in the next section under GENERAL COMMENTS.

If storage of these statistics in specified columns is desired for later use, e.g., plotting of deviations from the mean, the command can be modified by adding either 1 or 4 arguments as follows:

STATIStical analysis of column 2, put statistics in column 10

(and next three columns)

STATIStical analysis of column 2, put statistics in columns 10, 11, 12, 13

The results of the above two commands are in fact equivalent. The second form, however, can be used to specify the storage in specified columns that are not in a sequence.

In the above example, values of the 53 statistics will be stored in the rows of column 10 as shown in Appendix B, the ranks of the observations in column 11, the ordered observations in column 12, and the residuals, or deviations from the mean, in column 13.

In the three forms of the command given above, all weights are set equal to 1. If a column of weights is used, then the number of arguments must be increased to include

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2/ For a detailed description of this command, see pp. 115-122, OMNITAB II USer's Reference Manual, NBS Technical Note 552, issued October 1971.

the column of weights as follows:

STATIStical analysis of column 2, weights in column 3, put statistics in no columns -4*

STATIStical analysis of column 2, weights in column 3, put statistics in column 10 (and next three columns)

STATIStical analysis of column 2, weights in column 3, put statistics in columns 10, 11, 12, 13

A summary of the number of arguments in various forms of this command is given below.

NUMBER OF ARGUMENTS IN COMMAND STATIS

	No Storage	Consecutive Storage	Specified Storage
No weights (weights=1)	1	2	5
Weights specified	-3*	3	6

* Last column number is preceded by a minus sign.

In each form of STATIS described above, an "S" in front of the command (SSTATIS) suppresses the automatic printout.

The example in Appendix B used the following commands:

```
OMNITAB   DAVIS-HARRISON RELATIVE HUMIDITY DATA PIKES PEAK POSITIVE DEW POINT
READ      1, 2
      84 data points
STATIS    2, 10, 11, 12, 13
PRINT     10, 11, 12, 13
STOP
```

(Identification numbers of data points are read into column 1.)

II. General Comments

Given a column of N numbers, what can we say about them? Evidently the answer depends on the information we are looking for, and different people would ask different questions. The command "Statistical analysis" is designed to answer almost all the questions that might be raised and can be answered - a 99.44% extractor.

Hence at least two pages of statistics are printed out automatically for each statistical analysis of a column of N numbers (unless printout is suppressed). Some of these statistics, such as the average, standard deviation, are nearly always required by the user; others may provide information which he did not ask for. We therefore feel that it is to the advantage of the user to make a practice to glance over the sheets and look for surprises that may contradict his general feeling or impression about the data. In this sense additional information may be gained through the use of this command.

For convenience, the statistics computed are classified into eight groups:

1. Frequency distribution
2. Measures of location
3. Measures of dispersion
4. Linear trend statistics
5. Tests for non-randomness
6. Other statistics
7. Ranked observations
8. Ordered observations

We will give a brief general discussion of the statistics given within each group, and the uses they may be put to. Obviously these discussions cannot be exhaustive and references are given whenever appropriate, in addition to those given in the printout in parenthesis. We shall use the attached printout as an example for our discussion.

OMNITAB DAVIS-HARRISON R.H. DATA, PIKES PEAK POS. D.P. LOW

The first line of the printout, as is always the case for OMNITAB for every page, is used for name, date, title of the experiment, or any other labels of identification.

STATISTICAL ANALYSIS OF COL 2 N=84

The second line gives the name of the command and the number of data points, N=NRMAX. This is useful to check for duplication or omission of cards.

FREQUENCY DISTRIBUTION (1-6) 5 25 35 8 1 0 0 4 4 2

The next line gives the frequency distribution which shows the number of data points that fall in each of 10 equal intervals of the range of all the data, listed in ascending sequence of intervals. Items in parentheses refer to page number in NBS Handbook 91.

The frequency distribution is helpful in:

(1) looking at the shape of distribution, e.g., skewness, peakedness, etc., and in

(2) detecting outliers at one or both ends.

Usually if an end point is separated by one or more zeroes from the main part of the data in the frequency distribution, it is suspected as an outlier. If it can be verified that the outlier is present because of a mistake in recording or typing, or because of violation of experimental conditions, obviously the point should be deleted. Other than that the course of action is not clear. We suggest as a first reference the three-page, "Some Remarks on Wild Observations" by William H. Kruskal [1].

The data shown are a partial set of calibration data for a whirling psychrometer made by William Ferrel in 1886 on Pikes Peak, Colorado. There appear to be 10 high values separated by two zero class intervals from the remaining 74 points. A close look at these points is in order before we proceed to further analysis.

MEASURES OF LOCATION (2-2)

UNWEIGHTED MEAN	=	6.3734391-01
WEIGHTED MEAN	=	6.3734391-01
MEDIAN	=	6.2915000-01
MID-RANGE	=	6.6844999-01
25 PCT UNWTD TRIMMED MEAN	=	6.2885945-01
25 PCT WTD TRIMMED MEAN	=	6.2885945-01

For a description of these six statistics all of which are measures of location, we first introduce a column of weights, and let W_i stand for the weight for the ith row corresponding to the number X_i. For later use, we also denote the number of non-zero weights as "NZW".

The weighted mean is therefore:

$$\text{WEIGHTED MEAN} = \frac{\sum_{i=1}^{N} (W_i X_i)}{\sum_{i=1}^{N} W_i}$$

If all $W_i = 1$ (or $W_i = k$) then we obtain the unweighted mean

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

For the weighted mean, the user must have reason to select the weights he wishes to attach to the X values. Some of the common ones could be:

1. To assign a weight of zero to a value to be omitted in the average,
2. To assign weights proportional to the number of observations which the X_i are averages of, or
3. To assign weights inversely proportional to variances of individual X_i values, provided these variances are fairly well established.

From the expression for weighted mean above, it is evident that the computation is carried out in a self-normalizing manner, and the sum of weights need not be 1.0 in the input.

We may continue our discussion on the median, the midrange, and the 25% trimmed mean as special cases of weighted mean, although the weights used are incorporated in the computing routine and do not need to be specified.

For N odd, set $W_i = 1$ for $i = \frac{N+1}{2}$, $W_i = 0$ for all other i; we obtain the median. For N even, the value of the median is computed with $W_i = 1$ for $i = \frac{N}{2}$, $i = \frac{N}{2} + 1$, and zero otherwise.

The midrange is defined as the average of the two extreme values, hence the weights are 1's for the smallest and largest values in the column of values of X, and 0's for the remaining values.

The selection of these three measures of location is not without justification. To begin with, each measure represents the solution of a particular minimization criterion. If one wishes to choose the measure of location, m, such that

$$\sum_{i=1}^{N} (X_i - m)^2 = \min, \text{ UNWEIGHTED MEAN is the resulting estimate of m,}$$

$$\sum_{i=1}^{N} |X_i - m| = \min, \text{ MEDIAN is the result,}$$

$$\max_i |X_i - m| = \min, \text{ MIDRANGE is the result.}$$

Furthermore, each measure is the "best" estimator of the location parameter for certain distribution from which the observed values were obtained (as a random sample). Heuristically the "best" estimator may be interpreted as the most "stable" estimator for the parameter in repeated samples from the same population. Thus the unweighted mean is the "best" if these values are normally distributed, the median is the "best" if the distribution is double exponential, and the midrange the best if rectangular.

Hence if we know approximately how our data are distributed, we can select the proper measure of location. Most of the time, however, we do not enjoy this privilege, and wish that we could use an estimate that is "robust" for a reasonable number of likely distributions. A "robust" estimator is particularly important if the data may be contaminated by outliers, or by a cluster of points originating from another distribution.

The 25 percent trimmed mean is such a robust estimator. Both the upper and lower 25% of the ordered values are "trimmed" off, and the mean is computed from the middle 50% of the values. In our example, 21 values from each end are assigned weight of 0's, and the mean is computed from the central 42 values all of which fall within the second and third intervals of the frequency distribution. We note that the ten high values do not contribute at all to the 25% trimmed mean. We note also that the value of the median, itself an extreme trimmed mean, is very close to the value of the 25% trimmed mean.

What it amounts to is that we are paying an insurance premium, half of the observed values, to minimize the chance that one or more aberrant values may unduly disturb the mean. Also, parallel to insurance premium payments, the protection is for the long run, since for just one series of observations we do not know if the "trimming" is worth the cost.

Measures of location is a large subject in itself. The measures of location computed and printed out in STATISTICAL ANALYSIS offer one a chance to inspect and compare these measures, as well as a choice of one or another measure for the class of unimodal continuous distributions considered (distributions with a single peak). For a recent discussion we recommend Crow and Siddiqui [2].

MEASURES OF DISPERSION (2-6)

STANDARD DEVIATION	=	3.2401807-02
S. D. OF MEAN	=	3.5353269-03
RANGE	=	1.4670000-01
MEAN DEVIATION	=	2.1074478-03
VARIANCE	=	1.0498771-03
COEFFICIENT OF VARIATION	=	5.0838811-00

As in the case of measures of location, we have a column of N values of X_i, and a column of weights W_i. If the weights are not specified, $W_i = 1$ for the computed results. Zero weights may be specified to delete one or more numbers. Here we are interested in some measure of the scatter of values of X_i.

RANGE

The RANGE of N values is defined as the difference between the highest and the lowest of these values, i.e. $X_{max} - X_{min}$. If we have only two values of X, the difference between the two numbers is all the information we have about the dispersion. The same is reasonably true for small sets of numbers. However, as N gets larger, the range is no longer an efficient measure of dispersion. Evidently, the range is not affected by weights.

VARIANCE

The most important measure of dispersion, the VARIANCE computed from the set of N values, is

$$S^2 = \sum_{i=1}^{N} (X_i - \bar{X})^2/(N-1) = \sum_{i=1}^{N} (DEV_i)^2/(N-1)$$

or, if weights are used, is

$$S^2 = \sum_{i=1}^{N} (X_i - \text{WEIGHTED MEAN})^2 W_i / (\text{NZW} - 1)$$

$$= \sum_{i=1}^{N} (\text{DEV}_i)^2 W_i / (\text{NZW} - 1),$$

where the weighted mean is computed as

$$\sum_{i=1}^{N} W_i X_i \Big/ \sum_{i=1}^{N} W_i \ .$$

We shall return to the weighted case later

There are a number of reasons for S^2 to be the most popular measure of dispersion, viz.,

1. S^2 is an efficient and unbiased estimator of σ^2, the variance.
2. S^2 is additive for independent variables or observations, hence its use in propagation of errors, and pooling of estimates of variances.
3. S^2 is mathematically more tractable in comparison with other measures.

It is impossible to discuss even a fraction of the properties of uses of S^2 in the limited space here. We suggest as a start, in addition to the section of NBS Handbook 91 given in parentheses, that one look over the 15 items listed under variance and those under standard deviation in the index of NBS Special Publication 300, Vol 1., reference [3].

STANDARD DEVIATION

The positive square root of the variance computed from N numbers is the computed STANDARD DEVIATION, S in symbol. S is of the same unit as the observed values and is usually used as the measure of precision of these values. Two comments here are appropriate:

1. The standard deviation computed is the standard deviation of X_i about the mean $\bar{X}$, and is referred to as the "standard deviation of a single observation."

   With increasing N, S approaches a constant value which is the standard deviation of the population, or of the measurement process.

2. The standard deviation S is only an estimate computed from N numbers, and its "firmness" depends entirely on N. It is incomplete to report the value of S without also indicating in some manner the value of N from which S is computed.

STANDARD DEVIATION OF MEAN (Also called Standard Error of the Mean)

The S. D. OF MEAN is computed as

$$\text{S. D. OF MEAN} = S \,/\, \sqrt{N}$$

or, in the case of the weighted mean

$$\text{S. D. OF WEIGHTED MEAN} = S \,/\, \sqrt{\sum_i W_i}$$

We note that, contrary to S which approaches a constant value with increasing N, the S. D. of mean approaches zero with increasing N. The number of measurements, N, is therefore an important item to report together with the mean and the standard deviation of the mean.

WEIGHTED CASES FOR VARIANCE AND STANDARD DEVIATION

Whenever a column of weights is specified, then the STATIS command works only on those numbers with non-zero weights, and this fact is stated in the third line of the printout. Hence, if the column of weights consists of 1's and 0's only, the weighted and unweighted statistics appearing in the printout are the same.

The main reason for using weights is to combine observations having different precisions. A convenient way to think about this problem may be described as follows:

Suppose each X_i is actually an average of a number of observations, say N_i. If the numbers N_i for the groups are different from group to group, then a weighted average with the weights proportional to N_i is a proper mean to compute, such that the weighted average is equal to the simple average of the original data set. Assuming that there is no between group variation and that the variance for the individual observations are the same and equal to σ^2, or assuming

$$\mathrm{VAR}(X_i) = \sigma_i^{\,2} = \sigma^2/N_i \;,$$

we select W_i to be inversely proportional to σ_i^2, or proportional to N_i. Hence we can think of the column of weights as the number of observations that enter into each of the averages X_i.

One can show that the variance computed by

$$S^2 = \sum_{i=1}^{N} (DEV_i)^2 N_i / (NZW - 1)$$

appearing in the printout is an unbiased estimate of σ^2, the underlying common variance for an individual observation. The standard deviation appearing in the printout corresponds also to this interpretation.

To obtain the estimated variance of a particular X_i, we will have to divide S^2 by N_i, i.e.,

$$VAR(X_i) = S^2/N_i \ , \text{ or } \ S^2/W_i$$

and

$$S.D.(X_i) = S / \sqrt{N_i} \ , \text{ or } \ S / \sqrt{W_i} \ .$$

The S. D. of mean for the weighted case is the standard deviation of the weighted mean computed by

$$S.\ D.\ OF\ MEAN = S / \sqrt{\sum_i W_i}$$

COEFFICIENT OF VARIATION

The coefficient of variation appearing in the print-out is expressed in percent and is defined as

$$COEFFICIENT\ OF\ VARIATION = 100(S/\bar{X}).$$

Sometimes the term "relative standard deviation" is used for the same expression. For the weighted case, the weighted mean is used in the denominator.

The coefficient of variation is a constant when magnitudes of random errors are proportional to averages of the observations, and hence is a useful measure of dispersion to summarize data when the same coefficient of variation applies to a set of different means. Its use should be restricted to this type of data only.

Since S is positive by definition, the coefficient of variation computed will be negative if $\bar{X}$ is negative. The use of coefficient of variation is therefore meaningful only when all the observations take positive values, or if $\bar{X}$ is bounded away from zero.

MEAN DEVIATION

$$\text{MEAN DEVIATION} = \sum_{i=1}^{N} |X_i - \bar{X}|/N ,$$

where $\bar{X}$ is the weighted mean if weights are specified.

The mean deviation has been popular with physical scientists for a long time: it is easy to compute for small N and it is less affected by outliers than the standard deviation. If the observations are exactly normally distributed, then the computed standard deviation S is a more efficient measure of dispersion than the mean deviation. But if the observations are not normally distributed, or are contaminated by outliers, then the advantage of the estimator S over that of the mean deviation is reduced rapidly.

For a general discussion of efficiency of the mean deviation as compared to other measures of dispersions in contaminated distributions see Tukey (Ref. 4).

A convenient way to check the normality of one's data is to remember that the ratio of mean deviation to standard deviation is about 0.8 for large samples from normally distributed values. Percentage points of the distribution of this ratio is given in Table V which can be used to test for departure from normality.

For data that are normally distributed, an approximate relationship between the mean deviation and the standard deviation computed is as follows:

$$\text{STD. DEVIATION} = \text{MEAN DEVIATION} \times \frac{\pi}{2} \times \frac{N}{N-1}$$

Hence a comparison of the standard deviation computed from data and that computed from the expression above provides another check on the normality assumption of the data.

```
A TWO-SIDED 95 PCT CONFIDENCE INTERVAL FOR MEAN IS
                  6.3031-01 TO 6.4438-01   (2-2)
A TWO-SIDED 95 PCT CONFIDENCE INTERVAL FOR S.D. IS
                  2.8026-02 TO 3.8586-02   (2-7)
```

The next two lines of printout give a two-sided 95% confidence interval for the mean μ, and a two-sided 95% confidence interval for the standard deviation σ, assuming the observations are normally distributed. The upper and lower limits are computed by the following formulas:

	upper limits	lower limits
μ	$X_u = \bar{X} + tS / \sqrt{N}$	$X_L = \bar{X} - tS / \sqrt{N}$
σ	$S_u = B_u S$	$S_L = B_L S$

where t (α = 0.05, n-1) is the Student's t-factor given in Table II, and B_u, B_L(α = 0.05, n-1) are factors for two-sided confidence limits for σ given in Table III in the appendix.

Since interval estimation is a large subject in itself, we refer to Chapters 1 and 2 of reference [6] for the concepts and interpretations of these intervals. Suffioe it to note here that the confidence level used, 95% in these cases, is associated with the procedure used in the computation such that 95% of the intervals so computed would bracket the mean μ (or standard deviation σ) in repeated sets of observations that are normally distributed. For a particular set of data, the intervals so computed may or may not bracket the mean (or standard deviation).

LINEAR TREND STATISTICS (5-1)

SLOPE	=	-2.4760868-04
S.D. OF SLOPE	=	1.4412009-04
SLOPE/S.D. OF SLOPE = T	=	-1.7180820+00
PROB EXCEEDING ABS VALUE OF OBS T	=	.090

A set of data is always taken in some sequence, usually ordered in time, or by entry of data from the design. The linear trend statistics check on the drift, if any, of the data against the order of the sequence. Essentially the data are plotted against the integers 1, ..., N, and a line is fitted by the method of least squares. The computation formulas for the slope and the standard deviation of the slope are the following:

$$\text{Slope} = 12 \sum_{i=1}^{N} i(\text{DEV}_i)/N(N^2 - 1)$$

$$\text{S.D. of slope} = \frac{1}{\sqrt{N-2}} \left| \frac{12 \sum_{i=1}^{N} (\text{DEV}_i)^2}{N(N^2-1)} - (\text{SLOPE})^2 \right|^{1/2} .$$

If the assumption is no trend in the sequence, then

$$t = \frac{\text{Slope} - 0.0}{\text{S.D. of slope}}$$

is distributed as Stûdent's t with (N-2) degrees of freedom. The probability of observing a slope of the magnitude calculated or greater while in fact there is no trend is given in the next line. In this example the probability is only .090, giving some evidence against the assumption of no trend (including but not necessarily limited to linear trend). A plot of the data or residuals becomes almost mandatory at this juncture of analysis: to reveal possible causes of the trend. (In searching for trend when the data are entered in some meaningful order, we should bear in mind that the trend in the data may also be caused by chance with the probability indicated).

TESTS FOR NON-RANDOMNESS

NO. OF RUNS UP AND DOWN	=	47
EXPECTED NO OF RUNS	=	55.7
S.D. OF NO OF RUNS	=	3.82
MEAN SQ SUCCESSIVE DIFF	=	3.6380990-04
MEAN SQ SUCC DIFF/VAR	=	.347

DEVIATIONS FROM WTD MEAN

NO OF + SIGNS	=	22
NO OF - SIGNS	=	62
NO OF RUNS	=	14
EXPECTED NO OF RUNS	=	33.5
S.D. OF RUNS	=	3.51
DIFF./S.D. OF RUNS	=	-5.550

Since "randomness" is an elusive and difficult concept to define and test, our tests here are restricted to the detection of certain important patterns or non-randomness that may be inherent in a set of data which is ordered in some meaningful way. In the linear trend statistics described above, the test is for long-term trend from the beginning to the end, and is based on normal theory; in this section the tests are for the detection of short-term patterns, such as grouping of data into sets, cyclic variations, and the like. Moreover, the tests used do not depend on any assumption on the form of distribution.

Let us take our sequence of observations: $X_1, X_2, \ldots, X_n$ and look at the algebraic signs of the differences between adjacent numbers, i.e., the signs of $(X_{i+1} - X_i)$. There are (N-1) such differences, giving rise to a sequence of (N-1) "+" or "-" signs. A subsequence of successive "+" signs is called a run up and a subsequence of "-" signs is called a run down. Assuming that all N! possible arrangements of the N numbers are equally likely, then it can be shown that:

$$\text{EXPECTED NO. OF RUNS} = \frac{1}{3}(2N-1), \text{ and}$$

$$\text{S.D. OF NO. OF RUNS} = [(16N-29)/90]^{1/2} .$$

Furthermore, for $N > 20$, the number of runs may be regarded as normally distributed and Table I in the Appendix can be used to check if the departure from expected is excessive. (See for instance, p. 354, Reference [7].) Hence the ratio:

$$\frac{\text{OBSERVED NO. OF RUNS - EXPECTED NO OF RUNS}}{\text{S.D. OF NO. OF RUNS}}$$

gives a measure of the non-randomness of the sequence. Short-term drifts would tend to yield a smaller number of runs than expected. Cyclic data could yield either larger or smaller number of runs, depending on the period of the cycle.

The mean square successive difference is another test for the presence of correlation between adjacent observed values, and is defined as:

$$\text{MEAN SQ SUCC DIFF} = \sum_{i=1}^{N} (X_{i+1} - X_i)^2/(N-1) .$$

The ratio:

MEAN SQ SUCC DIFF/VAR

appearing on the next line has an expected value of 2. To test if there are correlations between adjacent values, it is simpler to consider $r = \frac{1}{2}$ (MEAN SQ SUCC. DIFF./VAR) which has an expected value of 1 and standard deviation of $\sqrt{(N-2)/(N^2-1)}$. For $N > 20$ this ratio is approximately normally distributed. Hald [7], gives a table of percentage points for $N < 20$ for this ratio which is reproduced as Table IV in the appendix. A ratio significantly less than one is an indication of positive correlation between adjacent values, and a ratio greater than one that of negative correlation.

Hald (page 358, reference [7]) illustrates the use of this statistic for checking gradual changing of the mean of the population while the variance remains constant. The estimate S^2 of σ^2 will tend to be too large because S^2 includes also variations of the population mean. On the other hand, since the mean square successive difference includes only the differences between successive values, the effect of gradual changes in the population mean will be relatively small.

If we compute the values of $(X_i - \bar{X})$ and count the number of "+" and "-" signs, we obtain the first two lines under DEVIATIONS FROM WTD MEAN:

$$\text{NO OF + SIGNS} = M_1$$

$$\text{NO OF - SIGNS} = M_2$$

(If a value of X_i is an integer and is equal to $\bar{X}$, a "+" sign is counted, hence $M_1 + M_2 = N$ always). In the weighted case, the weighted mean is subtracted from the observations.

Each change in the sequence of signs constitute a run. The total number of runs in a sequence is equal to the number of change of signs plus one, and is given in the next line. Again assuming all arrangement of "+" and "-" signs are equally likely, then it can be shown that (p. 170, reference [8]):

$$\text{EXPECTED NO OF RUNS} = 1 + \frac{2M_1M_2}{N}$$

$$\text{S.D. OF RUNS} = \left[\frac{2M_1M_2(2M_1M_2 - N)}{N^2(N-1)}\right]^{1/2}$$

The distribution of NO OF RUNS becomes approximately normal for large samples, and is usually good enough for practical purposes for M_1, M_2 > 10. The last line of this section gives

$$\frac{\text{NO OF RUNS - EXPECTED NO OF RUNS}}{\text{S.D. OF RUNS}}$$

which can be used to check for abnormally large departure of the number of runs from the expected. Table I in the appendix can be used for this purpose.

OTHER STATISTICS

MINIMUM	=	5.9509999-01
MAXIMUM	=	7.4180000-01
BETA ONE	=	3.7310967+00
BETA TWO	=	5.9296834+00
WTD SUM OF VALUES	=	5.3536888+01
WTD SUM OF SQUARES	=	3.4208556+01
WTD SUM OF DEVS SQUARED	=	8.7139798-02
STUDENT'S T	=	1.8027863+02
WTD SUM ABSOLUTE VALUES	=	5.3536888+01
WTD AVE ABSOLUTE VALUES	=	6.3734391-01

The smallest and largest values in the set of data are printed out as the minimum and maximum, respectively, as the first two lines in OTHER STATISTICS.

The two statistics BETA ONE and BETA TWO are related to the third and fourth moments of the distribution. They are included here to give an indication of the normality of the observed data, and are calculated by the following formulas:

$$\text{BETA ONE} = b_1 = \left[\frac{1}{N}\sum_{i=1}^{N}(\text{DEV}_i)^3\right]^2 \Big/ \left[\frac{1}{N}\sum_{i=1}^{N}(\text{DEV}_i)^2\right]^3,$$ estimating the value of β_1 for the population;

$$\text{BETA TWO} = b_2 = \frac{1}{N}\sum_{i=1}^{N}(\text{DEV}_i)^4 \Big/ \left[\frac{1}{N}\sum_{i=1}^{N}(\text{DEV}_i)^2\right]^2,$$ estimating the value of β_2 for the population.

Weights, if used, are ignored in these computations except for the calculation of (DEV_i).

The value b_1 is a measure of asymmetry (skewness) of the distribution of observed values, and the value b_2 is a measure of its "peakedness" (kurtosis). In a normal population, $\beta_1 = 0$, and $\beta_2 = 3$. For $\beta_1 > 0$, the distribution has a longer tail than the normal distribution. For $\beta_2 > 3$, the distribution has a sharper peak than normal; for $\beta_2 < 3$, a flatter peak than normal. For uniform distribution, β_2 is equal to 1.8. However, since these two statistics fluctuate considerably from sample to sample, they are useful only for moderately large groups of data. Percentage points of the distributions of $\sqrt{b_1}$ and b_2 are given in Table Vb and Table Vc respectively. Tables and discussions of tests of skewness and kurtosis are given in reference [9].

Since a test for normality can be considered simultaneously as a test for the presence of outliers, these statistics have also been proposed for such tests, especially when more than one spurious observation may be involved. For a discussion see reference [10].

If the observed values are believed to center about zero, the STUDENT'S t statistic can be used to check this assumption. It is computed as:

$$\text{STUDENT'S T} = \frac{\text{WTD MEAN} - 0.0}{\text{S.D. OF MEAN}} = \frac{.63734391-0}{.0035353269}$$

Percentiles of the "t" distribution are given in Table II in the appendix.

The remaining five items in OTHER STATISTICS are self-explanatory. These are computed as follows:

$$\text{WTD SUM OF VALUES} = \sum_{i=1}^{N} W_i X_i$$

$$\text{WTD SUM OF SQUARES} = \sum_{i=1}^{N} W_i X_i^2$$

$$\text{WTD SUM OF DEVS SQUARED} = \sum_{i=1}^{N} (DEV_i)^2 W_i$$

$$\text{WTD SUM ABSOLUTE VALUES} = \sum_{i=1}^{N} |W_i X_i|$$

$$\text{WTD AVE ABSOLUTE VALUES} = \frac{1}{N} \sum_{i=1}^{N} |W_i X_i| \; .$$

The above completes the descriptions of the various statistics appearing on the first page of the printout.

RANKED OBSERVATIONS				ORDERED OBSERVATIONS		
I	X(I)	RANK	X(I) - MEAN	NO.	X(J)	X(J+1) - X(J)

Two blocks of information are presented on the next page of printout, with titles "OBSERVATIONS" and "ORDERED OBSERVATIONS" appearing on top of the left and right groups of columns respectively. There are five, or four, columns under "OBSERVATIONS", depending on whether weights are used or not used. The first column "I" indexes the observations in the sequence they are read into the program. The second column X(I) lists the values of observations in floating point format. The third column lists the ranks of the values when ranked from smallest to largest. If two or more values are the same, these are given in the same rank number, calculated as the average of the ranks if these numbers are slightly different. In our example, X(67) and X(80) share the rank 56.5, and X(23), X(41) and X(78) share the rank 33.0.

The fourth column lists the differences X(I) - MEAN in floating point format. The weighted mean is used if weights are specified. These weights are printed as W(I) in the fifth column, only when weights are specified.

Since there are ten high values spearated from the main body of data in our example (as shown in FREQUENCY DISTRIBUTION), we certainly would like to know what these values are. From the column of ranks we find that ten consecutive values X(11) thru X(20) have consecutive ranks 75 thru 84, with positive X(I) - MEAN values approximately an order of magnitude larger than the others. Mr. Harrison checked into the original data and found a note to the effect that these values were obtained in a heated cabin, whereas the other values were obtained in open air.

An easier way to spot these ten values is to look at the second column X(J) in the next block under the title ORDERED OBSERVATIONS. The column X(J) orders the observations in an ascending sequence, hence the ten largest values appear at the end of the column. Under the heading "NO." the index number "I" associated with the observations are listed so that specific ones can be identified. The last column X(J+1) - X(J) gives the successive differences between adjacent ordered observations.

From the apparent grouping of adjacent observations (e.g. X(1) through X(10) are all at the low end, etc.) of this set of data, it seems that there is considerable heterogeneity (and perhaps a between day component) which should be taken into consideration in further analysis.

COLUMN 10	COLUMN 11	COLUMN 12	COLUMN 13
84.000000	3.0000000	.595099999	-.030343913
.	.	.	.
.	.	.	.
.	.	.	.

The command we used for this example

STATISTICAL 2, 10, 11, 12, 13

stored the results of computations in columns 10, 11, 12, and 13. These columns are printed on the next page for ready reference. The 43 statistics plus the 10 values of the histogram appear in column 10 in the order listed on page 119 of the OMNITAB II User's Reference Manual. The ranks of these values are printed as column 11; the ordered values in column 12; and the deviations (residuals) in column 13. These values can be extracted from the respective columns for use in subsequent computations. For example, the residuals can be plotted against the order of observations; the mean and standard deviations may be "DEFINED" to rows of other columns as elements of a table where several sets of data are summarized; etc. since it is impossible to list even a small fraction of what one can do with these stored values, we refer the user to the "self-teaching" feature of OMNITAB to experiment to his satisfaction that OMNITAB is indeed an easy and effective system to use in statistical or data analysis. Self-teaching is discussed on pages 39-40 of the OMNITAB II User's Reference Manual.

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APPENDIX A

TABLE I (A-1 Ref. 6). Cumulative Normal Distribution

TABLE II (A-4 Ref. 6). Percentiles of the t Distribution

TABLE III (A-20 Ref. 6). Factors for Computing Two-Sided Confidence Limits for σ

TABLE IV (13.14 Ref. 7). Fractiles in the Distribution of $r = q^2/s^2$

(r = MEAN SQ SUCC DIFF/2 VAR)

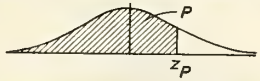
TABLE V (34 Ref. 5). Tests for Departure from Normality

A. Percentage points of the distribution of
a = (mean deviation)/(standard deviation)

B. Percentage points of the distribution of $\sqrt{b_1} = m_3/m_2^{3/2}$

C. Percentage points of the distribution of $b_2 = m_4/m_2^2$

TABLE I. CUMULATIVE NORMAL DISTRIBUTION — VALUES OF *P*



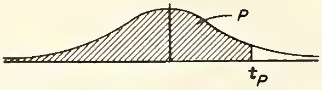
Values of *P* corresponding to z_p for the normal curve.

z is the standard normal variable. The value of *P* for $-z_p$ equals one minus the value of *P* for $+z_p$, e.g., the *P* for -1.62 equals $1 - .9474 = .0526$.

z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Adapted from Experimental Statistics by Mary G. Natrella, 1963, NBS Handbook 91, U. S. Government Printing Office, Washington, D. C., 20015.

TABLE II. PERCENTILES OF THE t DISTRIBUTION



df	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	.325	.727	1.376	3.078	6.314	12.706	31.821	63.657
2	.289	.617	1.061	1.886	2.920	4.303	6.965	9.925
3	.277	.584	.978	1.638	2.353	3.182	4.541	5.841
4	.271	.569	.941	1.533	2.132	2.776	3.747	4.604
5	.267	.559	.920	1.476	2.015	2.571	3.365	4.032
6	.265	.553	.906	1.440	1.943	2.447	3.143	3.707
7	.263	.549	.896	1.415	1.895	2.365	2.998	3.499
8	.262	.546	.889	1.397	1.860	2.306	2.896	3.355
9	.261	.543	.883	1.383	1.833	2.262	2.821	3.250
10	.260	.542	.879	1.372	1.812	2.228	2.764	3.169
11	.260	.540	.876	1.363	1.796	2.201	2.718	3.106
12	.259	.539	.873	1.356	1.782	2.179	2.681	3.055
13	.259	.538	.870	1.350	1.771	2.160	2.650	3.012
14	.258	.537	.868	1.345	1.761	2.145	2.624	2.977
15	.258	.536	.866	1.341	1.753	2.131	2.602	2.947
16	.258	.535	.865	1.337	1.746	2.120	2.583	2.921
17	.257	.534	.863	1.333	1.740	2.110	2.567	2.898
18	.257	.534	.862	1.330	1.734	2.101	2.552	2.878
19	.257	.533	.861	1.328	1.729	2.093	2.539	2.861
20	.257	.533	.860	1.325	1.725	2.086	2.528	2.845
21	.257	.532	.859	1.323	1.721	2.080	2.518	2.831
22	.256	.532	.858	1.321	1.717	2.074	2.508	2.819
23	.256	.532	.858	1.319	1.714	2.069	2.500	2.807
24	.256	.531	.857	1.318	1.711	2.064	2.492	2.797
25	.256	.531	.856	1.316	1.708	2.060	2.485	2.787
26	.256	.531	.856	1.315	1.706	2.056	2.479	2.779
27	.256	.531	.855	1.314	1.703	2.052	2.473	2.771
28	.256	.530	.855	1.313	1.701	2.048	2.467	2.763
29	.256	.530	.854	1.311	1.699	2.045	2.462	2.756
30	.256	.530	.854	1.310	1.697	2.042	2.457	2.750
40	.255	.529	.851	1.303	1.684	2.021	2.423	2.704
60	.254	.527	.848	1.296	1.671	2.000	2.390	2.660
120	.254	.526	.845	1.289	1.658	1.980	2.358	2.617
∞	.253	.524	.842	1.282	1.645	1.960	2.326	2.576

Adapted by permission from *Introduction to Statistical Analysis* (2d ed.) by W. J. Dixon and F. J. Massey, Jr., Copyright, 1957, McGraw-Hill Book Company, Inc. Entries originally from Table III of *Statistical Tables* by R. A. Fisher and F. Yates, 1938, Oliver and Boyd, Ltd., London.

TABLE III. FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom ν	$\alpha = .05$		$\alpha = .01$		$\alpha = .001$	
	B_U	B_L	B_U	B_L	B_U	B_L
1	17.79	.3576	86.31	.2969	844.4	.2480
2	4.859	.4581	10.70	.3879	33.29	.3291
3	3.183	.5178	5.449	.4453	11.65	.3824
4	2.567	.5590	3.892	.4865	6.938	.4218
5	2.248	.5899	3.175	.5182	5.085	.4529
6	2.052	.6143	2.764	.5437	4.128	.4784
7	1.918	.6344	2.498	.5650	3.551	.5000
8	1.820	.6513	2.311	.5830	3.167	.5186
9	1.746	.6657	2.173	.5987	2.894	.5348
10	1.686	.6784	2.065	.6125	2.689	.5492
11	1.638	.6896	1.980	.6248	2.530	.5621
12	1.598	.6995	1.909	.6358	2.402	.5738
13	1.564	.7084	1.851	.6458	2.298	.5845
14	1.534	.7166	1.801	.6549	2.210	.5942
15	1.509	.7240	1.758	.6632	2.136	.6032
16	1.486	.7308	1.721	.6710	2.073	.6116
17	1.466	.7372	1.688	.6781	2.017	.6193
18	1.448	.7430	1.658	.6848	1.968	.6266
19	1.432	.7484	1.632	.6909	1.925	.6333
20	1.417	.7535	1.609	.6968	1.886	.6397
21	1.404	.7582	1.587	.7022	1.851	.6457
22	1.391	.7627	1.568	.7074	1.820	.6514
23	1.380	.7669	1.550	.7122	1.791	.6568
24	1.370	.7709	1.533	.7169	1.765	.6619
25	1.360	.7747	1.518	.7212	1.741	.6668
26	1.351	.7783	1.504	.7253	1.719	.6713
27	1.343	.7817	1.491	.7293	1.698	.6758
28	1.335	.7849	1.479	.7331	1.679	.6800
29	1.327	.7880	1.467	.7367	1.661	.6841
30	1.321	.7909	1.457	.7401	1.645	.6880
31	1.314	.7937	1.447	.7434	1.629	.6917
32	1.308	.7964	1.437	.7467	1.615	.6953
33	1.302	.7990	1.428	.7497	1.601	.6987
34	1.296	.8015	1.420	.7526	1.588	.7020
35	1.291	.8039	1.412	.7554	1.576	.7052
36	1.286	.8062	1.404	.7582	1.564	.7083
37	1.281	.8085	1.397	.7608	1.553	.7113
38	1.277	.8106	1.390	.7633	1.543	.7141
39	1.272	.8126	1.383	.7658	1.533	.7169
40	1.268	.8146	1.377	.7681	1.523	.7197
41	1.264	.8166	1.371	.7705	1.515	.7223
42	1.260	.8184	1.365	.7727	1.506	.7248
43	1.257	.8202	1.360	.7748	1.498	.7273
44	1.253	.8220	1.355	.7769	1.490	.7297
45	1.249	.8237	1.349	.7789	1.482	.7320
46	1.246	.8253	1.345	.7809	1.475	.7342
47	1.243	.8269	1.340	.7828	1.468	.7364
48	1.240	.8285	1.335	.7847	1.462	.7386
49	1.237	.8300	1.331	.7864	1.455	.7407
50	1.234	.8314	1.327	.7882	1.449	.7427

Adapted with permission from *Biometrika*, Vol. 47, (1960), from article entitled "Tables for Making Inferences About the Variance of a Normal Distribution" by D. V. Lindley, D. A. East, and P. A. Hamilton.

TABLE III(Continued). FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom ν	$\alpha = .05$		$\alpha = .01$		$\alpha = .001$	
	B_U	B_L	B_U	B_L	B_U	B_L
51	1.232	.8329	1.323	.7899	1.443	.7446
52	1.229	.8343	1.319	.7916	1.437	.7466
53	1.226	.8356	1.315	.7932	1.432	.7485
54	1.224	.8370	1.311	.7949	1.426	.7503
55	1.221	.8383	1.308	.7964	1.421	.7521
56	1.219	.8395	1.304	.7979	1.416	.7539
57	1.217	.8408	1.301	.7994	1.411	.7556
58	1.214	.8420	1.298	.8008	1.406	.7573
59	1.212	.8431	1.295	.8022	1.402	.7589
60	1.210	.8443	1.292	.8036	1.397	.7605
61	1.208	.8454	1.289	.8050	1.393	.7621
62	1.206	.8465	1.286	.8063	1.389	.7636
63	1.204	.8475	1.283	.8076	1.385	.7651
64	1.202	.8486	1.280	.8088	1.381	.7666
65	1.200	.8496	1.277	.8101	1.377	.7680
66	1.199	.8506	1.275	.8113	1.374	.7694
67	1.197	.8516	1.272	.8125	1.370	.7708
68	1.195	.8525	1.270	.8137	1.366	.7722
69	1.194	.8535	1.268	.8148	1.363	.7735
70	1.192	.8544	1.265	.8159	1.360	.7749
71	1.190	.8553	1.263	.8170	1.356	.7761
72	1.189	.8562	1.261	.8181	1.353	.7774
73	1.187	.8571	1.259	.8191	1.350	.7787
74	1.186	.8580	1.257	.8202	1.347	.7799
75	1.184	.8588	1.255	.8212	1.344	.7811
76	1.183	.8596	1.253	.8222	1.341	.7822
77	1.182	.8604	1.251	.8232	1.338	.7834
78	1.181	.8612	1.249	.8242	1.336	.7845
79	1.179	.8620	1.247	.8252	1.333	.7856
80	1.178	.8627	1.245	.8261	1.330	.7868
81	1.176	.8635	1.243	.8270	1.328	.7878
82	1.176	.8642	1.241	.8279	1.325	.7889
83	1.174	.8650	1.239	.8288	1.323	.7899
84	1.173	.8657	1.238	.8297	1.320	.7909
85	1.172	.8664	1.236	.8305	1.318	.7920
86	1.171	.8671	1.235	.8314	1.316	.7930
87	1.170	.8678	1.233	.8322	1.313	.7939
88	1.168	.8684	1.231	.8331	1.311	.7949
89	1.167	.8691	1.230	.8338	1.309	.7959
90	1.166	.8697	1.228	.8346	1.307	.7968
91	1.165	.8704	1.227	.8354	1.305	.7977
92	1.164	.8710	1.225	.8362	1.303	.7987
93	1.163	.8716	1.224	.8370	1.301	.7996
94	1.162	.8722	1.222	.8377	1.298	.8004
95	1.161	.8729	1.221	.8385	1.297	.8013
96	1.160	.8734	1.219	.8392	1.295	.8022
97	1.159	.8741	1.218	.8399	1.293	.8031
98	1.158	.8746	1.217	.8406	1.291	.8039
99	1.158	.8752	1.216	.8413	1.289	.8047
100	1.157	.8757	1.214	.8420	1.288	.8055

TABLE IV. Fractiles in the distribution of $r = q^2/s^2$.

n	Probability in per cent		
	0.1	1.0	5.0
4	0.295	0.313	0.390
5	0.208	0.269	0.410
6	0.182	0.281	0.445
7	0.185	0.307	0.468
8	0.202	0.331	0.491
9	0.221	0.354	0.512
10	0.241	0.376	0.531
11	0.260	0.396	0.548
12	0.278	0.414	0.564
13	0.295	0.431	0.578
14	0.311	0.447	0.591
15	0.327	0.461	0.603
16	0.341	0.475	0.614
17	0.355	0.487	0.624
18	0.368	0.499	0.633
19	0.381	0.510	0.642
20	0.393	0.520	0.650

Adapted with permission from Statistical Theory with Engineering Applications by A. Hald, 1952, John Wiley and Sons, Inc., New York.

Table V. *Tests for departure from normality*

A. *Percentage points of the distribution of $a=$ (mean deviation)/(standard deviation)*

Size of sample n	$n-1$	Percentage points: Upper 1%	Upper 5%	Upper 10%	Lower 10%	Lower 5%	Lower 1%	Mean	Standard deviation
11	10	·9359	·9073	·8899	·7409	·7153	·6675	·81805	·05784
16	15	·9137	·8884	·8733	·7452	·7236	·6829	·81128	·04976
21	20	·9001	·8768	·8631	·7495	·7304	·6950	·80792	·04419
26	25	·8901	·8686	·8570	·7530	·7360	·7040	·80590	·04011
31	30	·8827	·8625	·8511	·7559	·7404	·7110	·80456	·03697
36	35	·8769	·8578	·8468	·7583	·7440	·7167	·80360	·03447
41	40	·8722	·8540	·8436	·7604	·7470	·7216	·80289	·03241
46	45	·8682	·8508	·8409	·7621	·7496	·7256	·80233	·03068
51	50	·8648	·8481	·8385	·7636	·7518	·7291	·80188	·02919
61	60	·8592	·8434	·8349	·7662	·7554	·7347	·80122	·02678
71	70	·8549	·8403	·8321	·7683	·7583	·7393	·80074	·02487
81	80	·8515	·8376	·8298	·7700	·7607	·7430	·80038	·02332
91	90	·8484	·8353	·8279	·7714	·7626	·7460	·80010	·02203
101	100	·8460	·8344	·8264	·7726	·7644	·7487	·79988	·02094
201	200	·8322	·8229	·8178	·7796	·7738	·7629	·79888	·01491
301	300	·8260	·8183	·8140	·7828	·7781	·7693	·79855	·01220
401	400	·8223	·8155	·8118	·7847	·7807	·7731	·79838	·01058
501	500	·8198	·8136	·8103	·7861	·7825	·7757	·79828	·00947
601	600	·8179	·8123	·8092	·7873	·7838	·7776	·79822	·00865
701	700	·8164	·8112	·8084	·7878	·7848	·7791	·79817	·00801
801	800	·8152	·8103	·8077	·7885	·7857	·7803	·79813	·00749
901	900	·8142	·8096	·8071	·7890	·7864	·7814	·79811	·00707
1001	1000	·8134	·8090	·8066	·7894	·7869	·7822	·79808	·00670

B. *Percentage points of the distribution of $\sqrt{b_1}=m_3/m_2^{\frac{3}{2}}$*

Size of sample n	Percentage points 5%	1%	Standard deviation	Size of sample n	Percentage points 5%	1%	Standard deviation	Size of sample n	Percentage points 5%	1%	Standard deviation
25	·711	1·061	·4354	200	·280	·403	·1706	1000	·127	·180	·0772
30	·661	·982	·4052	250	·251	·360	·1531	1200	·116	·165	·0705
35	·621	·921	·3804	300	·230	·329	·1400	1400	·107	·152	·0653
40	·587	·869	·3596	350	·213	·305	·1298	1600	·100	·142	·0611
45	·558	·825	·3418	400	·200	·285	·1216	1800	·095	·134	·0576
50	·533	·787	·3264	450	·188	·269	·1147	2000	·090	·127	·0547
				500	·179	·255	·1089				
60	·492	·723	·3009	550	·171	·243	·1039	2500	·080	·114	·0489
70	·459	·673	·2806	600	·163	·233	·0995	3000	·073	·104	·0447
80	·432	·631	·2638	650	·157	·224	·0956	3500	·068	·096	·0414
90	·409	·596	·2498	700	·151	·215	·0922	4000	·064	·090	·0387
100	·389	·567	·2377	750	·146	·208	·0891	4500	·060	·085	·0365
				800	·142	·202	·0863	5000	·057	·081	·0346
125	·350	·508	·2139	850	·138	·196	·0837				
150	·321	·464	·1961	900	·134	·190	·0814				
175	·298	·430	·1820	950	·130	·185	·0792				
200	·280	·403	·1706	1000	·127	·180	·0772				

N.B. As the sampling distribution of $\sqrt{b_1}$ is symmetrical about zero, the same values, with negative sign, correspond to the lower limits.

Table V. *Tests for departure from normality* (*continued*)

C. *Percentage points of the distribution of* $b_2 = m_4/m_2^2$

Size of sample n	Percentage points				Size of sample n	Percentage points			
	Upper 1%	Upper 5%	Lower 5%	Lower 1%		Upper 1%	Upper 5%	Lower 5%	Lower 1%
200	3·98	3·57	2·51	2·37	1000	3·41	3·26	2·76	2·68
250	3·87	3·52	2·55	2·42	1200	3·37	3·24	2·78	2·71
300	3·79	3·47	2·59	2·46	1400	3·34	3·22	2·80	2·72
350	3·72	3·44	2·62	2·50	1600	3·32	3·21	2·81	2·74
400	3·67	3·41	2·64	2·52	1800	3·30	3·20	2·82	2·76
450	3·63	3·39	2·66	2·55	2000	3·28	3·18	2·83	2·77
500	3·60	3·37	2·67	2·57					
550	3·57	3·35	2·69	2·58	2500	3·25	3·16	2·85	2·79
600	3·54	3·34	2·70	2·60	3000	3·22	3·15	2·86	2·81
650	3·52	3·33	2·71	2·61	3500	3·21	3·14	2·87	2·82
700	3·50	3·31	2·72	2·62	4000	3·19	3·13	2·88	2·83
750	3·48	3·30	2·73	2·64	4500	3·18	3·12	2·88	2·84
800	3·46	3·29	2·74	2·65	5000	3·17	3·12	2·89	2·85
850	3·45	3·28	2·74	2·66					
900	3·43	3·28	2·75	2·66					
950	3·42	3·27	2·76	2·67					
1000	3·41	3·26	2·76	2·68					

OMNITAB DAVIS-HARRISON R. H. DATA, PIKES PEAK, POS. D. P. LOW

STATISTICAL ANALYSIS OF COL 2 N = 84

FREQUENCY DISTRIBUTION (1-6) 5 25 35 8 1 0 0 4 4 2

MEASURES OF LOCATION (2-2)			MEASURES OF DISPERSION (2-6)		
UNWEIGHTED MEAN	=	6.3734391-01	STANDARD DEVIATION	=	3.2401807-02
WEIGHTED MEAN	=	6.3734391-01	S.D. OF MEAN	=	3.5353269-03
MEDIAN	=	6.2915000-01	RANGE	=	1.4670000-01
MID-RANGE	=	6.6844999-01	MEAN DEVIATION	=	2.1074478-02
25 PCT UNWTD TRIMMED MEAN	=	6.2885945-01	VARIANCE	=	1.0498771-03
25 PCT WTD TRIMMED MEAN	=	6.2885945-01	COEFFICIENT OF VARIATION	=	5.0838811+00

A TWO-SIDED 95 PCT CONFIDENCE INTERVAL FOR MEAN IS 6.3031-01 TO 6.4438-01 (2-2)
A TWO-SIDED 95 PCT CONFIDENCE INTERVAL FOR S.D. IS 2.8026-02 TO 3.8586-02 (2-7)

LINEAR TREND STATISTICS (5-1)			OTHER STATISTICS		
SLOPE	=	-2.4760868-04	MINIMUM	=	5.9509999-01
S.D. OF SLOPE	=	1.4412009-04	MAXIMUM	=	7.4180000-01
SLOPE/S.D. OF SLOPE = T	=	-1.7180720+00	BETA ONE	=	3.7310967+00
PROB EXCEEDING ABS VALUE OF OBS T	=	.090	BETA TWO	=	5.9296834+00
			WTD SUM OF VALUES	=	5.3536889+01
TESTS FOR NON-RANDOMNESS			WTD SUM OF SQUARES	=	3.4208556+01
			WTD SUM OF DEVS SQUARED	=	8.7139798-02
NO OF RUNS UP AND DOWN	=	47	STUDENT'S T	=	1.8027863+02
EXPECTED NO OF RUNS	=	55.7	WTD SUM ABSOLUTE VALUES	=	5.3536889+01
S.D. OF NO OF RUNS	=	3.82	WTD AVE ABSOLUTE VALUES	=	6.3734391-01
MEAN SQ SUCCESSIVE DIFF	=	3.6380990-04			
MEAN SQ SUCC DIFF/VAR	=	.347			

DEVIATIONS FROM WTD MEAN

NO OF + SIGNS	=	22
NO OF - SIGNS	=	62
NO OF RUNS	=	14
EXPECTED NO OF RUNS	=	33.5
S.D. OF RUNS	=	3.51
DIFF./S.D. OF RUNS	=	-5.550

NOTE - ITEMS IN PARENTHESES REFER TO PAGE NUMBER IN NBS HANDBOOK 91

RANKED OBSERVATIONS

I	X(I)	RANK	X(I)-MEAN
1	6.0699999-01	3.0	-3.0343913-02
2	6.0870000-01	5.0	-2.8643906-02
3	6.0860000-01	4.0	-2.8743908-02
4	6.1340000-01	11.0	-2.3943909-02
5	6.1080000-01	7.0	-2.6543908-02
6	6.1380000-01	12.0	-2.3543909-02
7	6.1250000-01	10.0	-2.4843909-02
8	6.1220000-01	9.0	-2.5143906-02
9	6.1099999-01	8.0	-2.6343912-02
10	6.1040000-01	6.0	-2.6943907-02
11	7.2130000-01	80.0	8.3956093-02
12	7.0779999-01	78.0	7.0456088-02
13	7.0209999-01	77.0	6.4756088-02
14	7.0039999-01	76.0	6.3056089-02
15	6.9809999-01	75.0	6.0756087-02
16	7.2420000-01	81.0	8.6856090-02
17	7.2679999-01	82.0	8.9456089-02
18	7.4180000-01	84.0	1.0445609-01
19	7.4070000-01	83.0	1.0335609-01
20	7.1990000-01	79.0	8.2556091-02
21	6.2249999-01	27.5	-1.4843911-02
22	6.2540000-01	35.0	-1.1943907-02
23	6.2520000-01	33.0	-1.2143910-02
24	6.2670000-01	38.0	-1.0643907-02
25	6.2180000-01	25.0	-1.5543908-02
26	6.1780000-01	17.0	-1.9543909-02
27	6.2159999-01	24.0	-1.5743911-02
28	6.1920000-01	20.0	-1.8143907-02
29	6.1910000-01	19.0	-1.8243909-02
30	6.2500000-01	31.0	-1.2343906-02
31	6.1880000-01	18.0	-1.8543907-02
32	6.2329999-01	30.0	-1.4043912-02
33	6.2249999-01	27.5	-1.4843911-02
34	6.2040000-01	22.0	-1.6943909-02
35	6.2069999-01	23.0	-1.6643912-02
36	6.1680000-01	16.0	-2.0543911-02
37	6.1409999-01	13.0	-2.3243912-02
38	6.2909999-01	42.0	-8.2439110-03
39	6.2310000-01	29.0	-1.4243908-02
40	6.2220000-01	26.0	-1.5143909-02

ORDERED OBSERVATIONS

NO.	X(J)	X(J+1)-X(J)
56	5.9509999-01	1.0000169-04
55	5.9519999-01	1.1799999-02
1	6.0699999-01	1.6000047-03
3	6.0860000-01	1.0000169-04
2	6.0870000-01	1.6999990-03
10	6.1040000-01	3.9999932-04
5	6.1080000-01	1.9999593-04
9	6.1099999-01	1.2000054-03
8	6.1220000-01	2.9999763-04
7	6.1250000-01	9.0000033-04
4	6.1340000-01	3.9999932-04
6	6.1380000-01	2.9999763-04
37	6.1409999-01	1.2000054-03
73	6.1530000-01	8.9999288-04
77	6.1619999-01	6.0000271-04
36	6.1680000-01	1.0000020-03
26	6.1780000-01	1.0000020-03
31	6.1880000-01	2.9999763-04
29	6.1910000-01	1.0000169-04
28	6.1920000-01	5.0000101-04
72	6.1970000-01	6.9999695-04
34	6.2040000-01	2.9999763-04
35	6.2069999-01	9.0000033-04
27	6.2159999-01	2.0000339-04
25	6.2180000-01	3.9999932-04
40	6.2220000-01	2.9999763-04
21	6.2249999-01	0.0000000
33	6.2249999-01	6.0000271-04
39	6.2310000-01	1.9999593-04
32	6.2329999-01	1.7000064-03
30	6.2500000-01	1.9999593-04
23	6.2520000-01	0.0000000
41	6.2520000-01	0.0000000
78	6.2520000-01	2.0000339-04
22	6.2540000-01	5.9999526-04
50	6.2599999-01	2.0000339-04
54	6.2620000-01	5.0000101-04
24	6.2670000-01	9.0000033-04
69	6.2760000-01	7.9999864-04
76	6.2840000-01	5.9999526-04

OBSERVATIONS

I	X(I)	RANK	X(I)-MEAN
41	6.2520000-01	33.0	-1.2143910-02
42	6.3079999-01	49.0	-6.5439120-03
43	6.3760000-01	63.0	2.5609136-04
44	6.3299999-01	52.0	-4.3439120-03
45	6.3029999-01	47.0	-7.0439130-03
46	6.3010000-01	46.0	-7.2439089-03
47	6.3900000-01	65.0	1.6560927-03
48	6.4229999-01	70.0	4.9560890-03
49	6.3000000-01	45.0	-7.3439106-03
50	6.2599999-01	36.0	-1.1343911-02
51	6.2920000-01	43.0	-8.1439093-03
52	6.2980000-01	44.0	-7.5439066-03
53	6.2899999-01	41.0	-8.3439127-03
54	6.2620000-01	37.0	-1.1143908-02
55	5.9519999-01	2.0	-4.2143911-02
56	5.9509999-01	1.0	-4.2243913-02
57	6.3140000-01	50.0	-5.9439093-03
58	6.4399999-01	72.0	6.6560879-03
59	6.4390000-01	71.0	6.5560937-03
60	6.3259999-01	51.0	-4.7439113-03
61	6.3919999-01	66.0	1.8560886-03
62	6.4170000-01	69.0	4.3560937-03
63	6.4120000-01	68.0	3.8560927-03
64	6.5300000-01	73.0	1.5656091-02
65	6.4110000-01	67.0	3.7560910-03
66	6.3550000-01	60.0	-1.8439069-03
67	6.3440000-01	56.5	-2.9439107-03
68	6.6230000-01	74.0	2.4956092-02
69	6.2760000-01	39.0	-9.7439066-03
70	6.3070000-01	48.0	-6.6439062-03
71	6.3540000-01	59.0	-1.9439086-03
72	6.1970000-01	21.0	-1.7643906-02
73	6.1530000-01	14.0	-2.2043906-02
74	6.3400000-01	55.0	-3.3439100-03
75	6.3380000-01	54.0	-3.5439059-03
76	6.2840000-01	40.0	-8.9439079-03
77	6.1619999-01	15.0	-2.1143913-02
78	6.2520000-01	33.0	-1.2143910-02
79	6.3490000-01	58.0	-2.4439096-03
80	6.3440000-01	56.5	-2.9439107-03

ORDERED OBSERVATIONS

NO.	X(J)	X(J+1)-X(J)
53	6.2899999-01	1.0000169-04
38	6.2909999-01	1.0000169-04
51	6.2920000-01	6.0000271-04
52	6.2980000-01	1.9999593-04
49	6.3000000-01	1.0000169-04
46	6.3010000-01	1.9999593-04
45	6.3029999-01	4.0000677-04
70	6.3070000-01	9.9994242-05
42	6.3079999-01	6.0000271-04
57	6.3140000-01	1.1999980-03
60	6.3259999-01	3.9999932-04
44	6.3299999-01	7.0000440-04
83	6.3370000-01	1.0000169-04
75	6.3380000-01	1.9999593-04
74	6.3400000-01	3.9999932-04
67	6.3440000-01	0.0000000
80	6.3440000-01	5.0000101-04
79	6.3490000-01	5.0000101-04
71	6.3540000-01	1.0000169-04
66	6.3550000-01	5.9999526-04
81	6.3609999-01	1.2000054-03
82	6.3730000-01	2.9999763-04
43	6.3760000-01	6.9999695-04
84	6.3829999-01	7.0000440-04
47	6.3900000-01	1.9999593-04
61	6.3919999-01	1.9000024-03
65	6.4110000-01	1.0000169-04
63	6.4120000-01	5.0000101-04
62	6.4170000-01	5.9999526-04
48	6.4229999-01	1.6000047-03
59	6.4390000-01	9.9994242-05
58	6.4399999-01	9.0000033-03
64	6.5300000-01	9.3000010-03
68	6.6230000-01	3.5799995-02
15	6.9809999-01	2.3000017-03
14	7.0039999-01	1.6999990-03
13	7.0209999-01	5.6999996-03
12	7.0779999-01	1.2100004-02
20	7.1990000-01	1.4000013-03
11	7.2130000-01	2.8999969-03

OBSERVATIONS

I	X(I)	RANK	X(I)-MEAN
81	6.3609999-01	61.0	-1.2439117-03
82	6.3730000-01	62.0	-4.3906271-05
83	6.3370000-01	53.0	-3.6439076-03
84	6.3829999-01	64.0	9.5608830-04

ORDERED OBSERVATIONS

NO.	X(J)	X(J+1)-X(J)
16	7.2420000-01	2.5999993-03
17	7.2679999-01	1.3900004-02
19	7.4070000-01	1.0999963-03
18	7.4180000-01	

COLUMN 10	COLUMN 11	COLUMN 12	COLUMN 13
84.000000	3.0000000	.59509999	-.030343913
84.000000	5.0000000	.59519999	-.028643906
.63734391	4.0000000	.60699999	-.028743908
.63734391	11.000000	.60860000	-.023943909
.62915000	7.0000000	.60870000	-.026543908
.66844999	12.000000	.61040000	-.023543909
.62885945	10.000000	.61080000	-.024843909
.62885945	9.0000000	.61099999	-.025143906
.032401807	8.0000000	.61220000	-.026343912
.0035353269	6.0000000	.61250000	-.026943907
.14670000	80.000000	.61340000	.083956093
.021074478	78.000000	.61380000	.070456088
.0010498771	77.000000	.61409999	.064756088
5.0838811	76.000000	.61530000	.063056089
.63031229	75.000000	.61619999	.060756087
.64437553	81.000000	.61680000	.086856090
.028026464	82.000000	.61780000	.089456089
.038586470	84.000000	.61880000	.10445609
*-2.4760868-04	83.000000	.61910000	.10335609
* 1.4412009-04	79.000000	.61920000	.082556091
-1.7180720	27.500000	.61970000	-.014843911
.089557514	35.000000	.62040000	-.011943907
47.000000	33.000000	.62069999	-.012143910
55.666667	38.000000	.62159999	-.010643907
3.8224483	25.000000	.62180000	-.015543908
* 3.6380990-04	17.000000	.62220000	-.019543909
.34652618	24.000000	.62249999	-.015743911
22.000000	20.000000	.62249999	-.018143907
62.000000	19.000000	.62310000	-.018243909
14.000000	31.000000	.62329999	-.012343906
33.476190	18.000000	.62500000	-.018543907
3.5094137	30.000000	.62520000	-.014043912
-5.5496990	27.500000	.62520000	-.014843911
.59509999	22.000000	.62520000	-.016943909
.74180000	23.000000	.62540000	-.016643912
3.7310967	16.000000	.62599999	-.020543911
5.9296834	13.000000	.62620000	-.023243912
53.536889	42.000000	.62670000	-.0082439110
34.208556	29.000000	.62760000	-.014243908
.087139798	26.000000	.62840000	-.015143909
* 1.8027863+02	33.000000	.62899999	-.012143910
53.536889	49.000000	.62909999	-.0065439120
.63734391	63.000000	.62920000	* 2.5609136-04
0.	52.000000	.62980000	-.0043439120
0.	47.000000	.63000000	-.0070439130
0.	46.000000	.63010000	-.0072439089
0.	65.000000	.63029999	.0016560927
0.	70.000000	.63070000	.0049560890
0.	45.000000	.63079999	-.0073439106
0.	36.000000	.63140000	-.011343911

COLUMN 10	COLUMN 11	COLUMN 12	COLUMN 13
5.0000000	43.000000	.63259999	-.0081439093
25.000000	44.000000	.63299999	-.0075439066
35.000000	41.000000	.63370000	-.0083439127
8.0000000	37.000000	.63380000	-.011143908
1.0000000	2.0000000	.63400000	-.042143911
0.	1.0000000	.63440000	-.042243913
0.	50.000000	.63440000	-.0059439093
4.0000000	72.000000	.63490000	.0066560879
4.0000000	71.000000	.63540000	.0065560937
2.0000000	51.000000	.63550000	-.0047439113
	66.000000	.63609999	.0018560886
	69.000000	.63730000	.0043560937
	68.000000	.63760000	.0038560927
	73.000000	.63829999	.015656091
	67.000000	.63900000	.0037560910
	60.000000	.63919999	-.0018439069
	56.500000	.64110000	-.0029439107
	74.000000	.64120000	.024956092
	39.000000	.64170000	-.0097439066
	48.000000	.64229999	-.0066439062
	59.000000	.64390000	-.0019439086
	21.000000	.64399999	-.017643906
	14.000000	.65300000	-.022043906
	55.000000	.66230000	-.0033439100
	54.000000	.69809999	-.0035439059
	40.000000	.70039999	-.0089439079
	15.000000	.70209999	-.021143913
	33.000000	.70779999	-.012143910
	58.000000	.71990000	-.0024439096
	56.500000	.72130000	-.0029439107
	61.000000	.72420000	-.0012439117
	62.000000	.72679999	*-4.3906271-05
	53.000000	.74070000	-.0036439076
	64.000000	.74180000	* 9.5608830-04

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DIMENSION      250  20
READ           1,2
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3836    .6087     .08583
3846    .6086     .1078
3856    .6134     .01329
3866    .6108    -.04528
3876    .6138     .1262
3886    .6125    -.04922
3896    .6122    -.07450
3906    .6110     .02926
3916    .6104    -.01599
3926    .7213     .02208
3936    .7078     .07162
3946    .7021     .04266
3956    .7004     .04604
3966    .6981     .03404
3976    .7242     .01416
3986    .7268     .03680
3996    .7418     .02983
4006    .7407     .01029
4016    .7199     .01746
4066    .6225     .03826
4076    .6254     .04812
4086    .6252     .01361
4096    .6267    -.02906
4106    .6218     .003828
4116    .6178     .04067
4126    .6216     .02104
4136    .6192     .003814
4146    .6191    -.1778
4246    .6250    -.04999
4256    .6188     .08734
4266    .6233     .01487
4276    .6225     .02140
4286    .6204    -.02172
4296    .6207    -.06341
4306    .6168     .01848
4316    .6141    -.07344
4586    .6291    -.02217
4596    .6231     .05215
4606    .6222     .1344
4616    .6252     .02088
4626    .6308    -.003182
4636    .6376     .04432
4646    .6330    -.03469
4656    .6303    -.06815
4666    .6301    -.02484
4676    .6390     .001270
4686    .6423     .007393
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4696      .6300      -.002256
4706      .6260       .01151
4716      .6292       .05266
4726      .6298       .6804
4736      .6290      -.02382
4746      .6262      -.07195
4756      .5952       .08198
4766      .5951       .08620
4846      .6314      -.04368
4856      .6440       .03386
4866      .6439       .02495
4936      .6326      -.03362
4946      .6392      -.03325
4956      .6417       .01094
4966      .6412      -.04209
5036      .6530      -.004807
5046      .6411       .02106
5056      .6355      -.005957
5066      .6344      -.01485
5076      .6623       .09334
5236      .6276       .03809
5246      .6307       .01394
5256      .6354       .03019
5266      .6197       .03389
5276      .6153       .05951
5326      .6340       .01945
5316      .6338       .05811
5336      .6284       .07580
5366      .6162      -.05455
5376      .6252      -.04439
5396      .6349       .07879
5406      .6344       .07899
5416      .6361       .05468
5446      .6373       .05507
5456      .6337       .04438
5466      .6383       .01897
STATISTICAL             2,10,11,12,13
PRINT                   10,11,12,13
STOP
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16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

This Technical Note is the first of a series of interpretive notes for a number of commands in the OMNITAB system that have the automatic printout feature. Others planned in the series include commands FIT and POLYFIT, TWOWAY analysis, ONEWAY analysis, and CORRELATION.

These notes aim to be self-contained so that users may have sufficient information on hand for the understanding of the statistics computed and to use them for their immediate applications. Computation formulas are given in the text, and a number of statistical tables are reproduced in the Appendix for the convenience of users.

17. KEY WORDS (Alphabetical order, separated by semicolons) Computing, statistical; documentation for users; OMNITAB; statistics.

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