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MATHEMATICAL TABLES

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I. General Information

The tables described in this Letter Circular were prepared by the Computation Laboratory of the National Applied Mathematics Laboratories, which constitute a division of the National Bureau of Standards. Prior to July 1, 1947 the Computation Laboratory was called the Mathematical Tables Project. It has been operated by the Bureau continuously since March, 1943, and its work has been sponsored by the Bureau since the inception of the Project in January, 1938.

The Computation Laboratory provides a general mathematical computing service for federal agencies, educational institutions, and scientific organizations interested in applied mathematics. An important part of the work of the Computation Laboratory consists in the preparation of tables of basic mathematical functions with a view to facilitating the application of mathematical theory to practical problems.

Two major series of tables have been published to date. It has become customary to designate these two series respectively as the MT Series and the Columbia University Press Series. In addition, a number of tables have been prepared for the War Department, and a large number of miscellaneous shorter tables have been computed. A third major series of tables will be contained in the new Applied Mathematics Series, recently established by the Bureau.

The MT Series, the Columbia University Press Series, and two of the War Department tables are described in detail in this Letter Circular. In addition, a special list of the tables available on punched cards at the Computation Laboratory has also been included here. A systematic catalogue of the remaining tables is now being prepared, and will be published at a future date in a comprehensive bibliography of the Computation Laboratory.

Explanation of Symbolism: The figures in square brackets denote the range and interval of the argument, and the number of decimal places or significant figures in the tabulated entries. The letters "D" and "S" stand for "decimal places" and "significant figures", respectively. In MT6, for instance, Si(x) is tabulated over the range from 0 to 9.999, at intervals of 0.001 to 10 decimal places, and Ei(x) is tabulated over the same range, for the same interval of the argument, to 10 significant figures.

II. The MT Series

The volumes of this series are available from the Superintendent of Documents, Government Printing Office, Washington 25, D. C. Payment should be made to the Superintendent of Documents, using checks, money orders, or Superintendent of Documents' coupons. The prices are for delivery in the United States and its possessions and in countries extending the franking privilege; that is, Canada and most of the Latin-American countries. To other countries, one-third of the listed price of the publication should be included in the remittance to cover the cost of mailing.

Copies of these publications (with the exception of MT11, MT18, and subsequent numbers, the editions of which are very limited) have been sent to various government depositories in the United States, such as colleges and universities and public libraries in large cities, where they may be consulted.

MT2. Tables of the Exponential Function e^x.

[-2.5(0.0001)1; 18D] ; [1(0.0001)2.5(0.001)5; 15D] ;

[5(0.01)10; 12D]; [-0.0001(0.000001)0.0001; 18D] ;

[-100(1)100; 19S].

[-9×10^{-n} (10^{-n}) 9×10^{-n}; for n = 10, 9, 8, 7; 18D].

(1939) xv + 535 pages; bound in buckram. Price $3.00.

MT3. Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments.

[0(0.001)1.9999; 9D] ; [0(0.1)10; 9D]. Conversion table, radians $\rightleftarrows$ degrees.

(1939) xvii + 405 pages; bound in buckram. Price $2.50.

MT4. Tables of Sines and Cosines for Radian Arguments.

[0(0.001)25; 8D]; [0(1)100; 8D].

[1×10^{-n}(10^{-n})9×10^{-n}; for n = 5, 4, 3, 2, 1; 15D] ;

[0(0.00001)0.01; 12D]. Conversion table, radians $\rightleftarrows$ degrees. Values of p(1 -p).

(1940) xix + 275 pages; bound in buckram. Price $2.00.

MT5. Tables of Sine, Cosine, and Exponential Integrals, Volume I.

[0(0.0001)1.9999; 9D] ; [0(0.1)10; 9D] .

Auxiliary tables of $p(1-p^2)/6$ and $p(1-p)$.

(1940) xxvi + 444 pages; bound in buckram. Price \$2.75.

MT6. Tables of Sine, Cosine, and Exponential Integrals, Volume II.

Si(x), Ci(x): [0(0.001)9.999; 10D]; [10(0.1)20(0.2)40; 10D].

Ei(x): [0(0.001)9.999; 10S] ; [10(0.1)15; 10S and 11S].

-Ei(-x): [0(0.001)9.999; 9S]; [10(0.1)15; 14D].

$Si(n\pi \pm h)$, for n = 1,2,3; $Ci(n\pi/2 \pm h)$, for n = 1,3,5;

for h = [0(0.0001)0.01(0.001)0.05; 15D].

Auxiliary tables of $p(1-p^2)/6$ and $p(1-p)$.

Multiples of $\pi/2$ and other constants.

(1940) xxxvii + 225 pages; bound in buckram. Price \$2.00

MT7. Table of Natural Logarithms, Volume I.

Logarithms of the integers from 1 to 50,000

to 16 decimal places.

(1941) xviii + 501 pages; bound in buckram. Price \$3.00.

MT8. Tables of Probability Functions, Volume I.

$$H'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \quad \text{and} \quad H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha$$

[0(0.0001)1(0.001)5.6(various)5.946; 15D].

H'(x) and 1-H(x): [4(0.01)10; 8S].

(1941) xxviii + 302 pages; bound in buckram. Price \$2.00

MT9. Table of Natural Logarithms, Volume II.

Logarithms of the integers from 50,000 to 100,000

to 16 decimal places.

(1941) xviii + 501 pages; bound in buckram. Price \$3.00.

MT10. Table of Natural Logarithms, Volume III.

Logarithms of the decimal numbers from 0.0001 to 5.0000 to 16 decimal places.

(1941) xviii + 501 pages; bound in buckram. Price $3.00.

MT11. Tables of the Moments of Inertia and Section Moduli of Ordinary Angles, Channels, and Bulb Angles with Certain Plate Combinations.

(1941) xiii + 197 pages; bound in green cloth. Price $2.00.

MT12. Table of Natural Logarithms, Volume IV.

Logarithms of the decimal numbers from 5.0000 to 10.0000 to 16 decimal places.

$\mathrm{Log}_e x$, [2(1)10; 40D].

$\mathrm{Log}_e(1 + x)$, $-\mathrm{Log}_e(1 - x)$
[$x=10^{-n}(10^{-n})10^{-n+1}$; $n=1,2,\ldots,13$; 25D].

(1941) xxii + 506 pages; bound in buckram. Price $3.00.

MT13. Table of Sine and Cosine Integrals for Arguments from 10 to 100.

[10(0.01)100; 10D]. Multiples of $\pi/2$, [1(1)100; 15D].

Values of $p(1 - p)$ and $p(1 - p^2)/6$.

(1942) xxxii + 185 pages; bound in buckram. Price $2.00.

MT14. Tables of Probability Functions, Volume II.

$$\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \quad \text{and} \quad \frac{1}{\sqrt{2\pi}}\int_{-x}^{x} e^{-\alpha^2/2}\,d\alpha$$

[0(0.0001)1(0.001)7.800(various)8.285; 15D]; and

$$\frac{1}{\sqrt{2\pi}}e^{-x^2/2} \quad \text{and} \quad \frac{2}{\sqrt{2\pi}}\int_{x}^{\infty} e^{-\alpha^2/2}\,d\alpha$$

[6(0.01)10; 7S].

(1942) xxi + 344 pages; bound in buckram. Price $2.25.

MT16. Table of Arc Tan x.

The principal value of the inverse tangent may be defined by the integral

$$\text{Arc tan } x = \int_0^x \frac{du}{1 + u^2}$$

The intervals between successive arguments have been so chosen, that interpolation to 12 decimals may be made with the aid of the second central differences which are tabulated alongside of the entries.

[0(0.001)7(0.01)50(0.1)300(1)2,000(10)10,000; 12D].

Values of $p(1 - p)$ and $p(1 - p^2)/6$. Conversion table, radians $\rightleftarrows$ degrees.

(1942) xxv + 169 pages; bound in buckram. Price $2.00.

MT17. Miscellaneous Physical Tables: Planck's Radiation Functions, and Electronic Functions.

1. Planck's radiation functions
(Reprinted from J. Opt. Soc. Am., February 1940)
The functions evaluated by Tables I to IV are

$$R_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1} \ ; \quad R_{0-\lambda} = \int_0^\lambda R_\lambda \, d\lambda$$

$$N_\lambda = 2\pi c \lambda^{-4}(e^{c_2/\lambda T} - 1)^{-1} \ ; \quad N_{0-\lambda} = \int_0^\lambda N_\lambda \, d\lambda$$

$R_\lambda d\lambda$ is the energy emitted by a black body at absolute temperature, T, on the Kelvin scale, per unit time, per unit area, in a wave-length interval from λ to $\lambda + d\lambda$, throughout the solid angle 2π steradians.

$N_\lambda d\lambda$ represents the number of photons emitted in the wave-length range from λ to $\lambda + d\lambda$, per unit time, per unit area, throughout the solid angle 2π steradians.

The tables evaluate these four functions of λT not only relative to their maximum values, but also for T = 1000 °K in absolute units (ergs or photons per cm^2 per second). The values are given to five significant figures, first and second differences being provided to assist in finding R_λ and N_λ by interpolation; and there is provided a convenient method of correcting for small changes in the value of c_2 from that (c_2 = 1.436 cm K°) used in computing the tables. The function N_λ is also evaluated for T = 1500, 2000, 2500, 3000, 3500, and 6000°K.

MT17 contd. 2. Electronic functions

This table of 28 pages gives in five columns the values for the velocity of an electron relative to the velocity of light, β ; $G = 1/(1 - \beta^2)^{\frac{1}{2}}$; βG; the energy in electron kilovolts; and the curvature of the electron path in a magnetic field times the field, $H\rho$. The total energy, effective mass, and momentum depend on the functions G and βG. The energies in electron volts cover the range from 6 to 10^{10} electron volts so that the tables are applicable in the full range of energies involved in nuclear and cosmic ray physics. In the range from 10,000 to 1.5×10^7 electron volts successive values differ by less than 1 per cent in energy and curvature so that in nearly all practical problems the tables would be used without interpolation. As relativistic equations must be used throughout this range the tables will save a great deal of laborious computation.

(1941) vi + 58 pages; bound in buckram. Price $1.50.

MT18. Table of the Zeros of the Legendre Polynomials of Order 1-16 and the Weight Coefficients for Gauss' Mechanical Quadrature Formula.

A. N. Lowan, N. Davids, and A. Levenson.

Gauss' method of mechanical quadrature has the advantage over most methods of numerical integration in that it requires about half the number of ordinate computations. This is desirable when such computations are very laborious, or when the observations necessary to determine the average value of a continuously varying physical quantity are very costly. Gauss' classical result states that, for the range (-1, +1), the "best" accuracy with n ordinates is obtained by choosing the corresponding abscissae at the zeros $x_1, \ldots x_n$ of the Legendre polynomials $P_n(x)$. With each x_i is associated a constant a_i such that

$$\int_{-1}^{1} f(x)dx \cong a_1 f(x_1) + a_2 f(x_2) + \ldots + a_n f(x_n).$$

The accompanying table computed by the Computation Laboratory gives the roots x_i for each $P_n(x)$ up to n = 16, and the corresponding weight coefficents a_i, to 15 decimal places.

Reprinted from Bulletin of the American Mathematical Society, October 1942. 5 pages, with cover, 25 cents.

MT19. On the Function $H(m,a,x) = \exp(-ix)F(m + 1 - ia, 2m + 2; ix)$. With table of the confluent hypergeometric function and its first derivative............A. N. Lowan and W. Horenstein

Confluent hypergeometric functions occur in numerous physical problems. They appear in the solutions of Laplace's equation and the equations of wave motion and of diffusion when the physical problem involves knife edges, parabolic cylinders, paraboloids of revolution, general ellipsoids, etc. Of particular importance to the quantum physicist is their occurrence as the solutions of the radial part of Schrödinger's equation for a Coulomb field and continuous values of the energy. Thus they are needed for the calculation of the scattering of charged particles from atomic nuclei, or of their binding by nuclei. In a more general form they also occur in the problem of dissociation of molecules into atomic ions (and, of course, in the converse case of the formation of molecules).

This paper contains a table of $H(m,a,x)$ and its first derivative for values of x from 1 to 10 by steps of 1, for values of a from 1 to 10 by steps of 1, and for values of the integer m from 0 to 3; values in all cases are given to 7 significant figures.

Reprinted from Journal of Mathematics and Physics, December 1942. 20 pages, with cover, 25 cents.

MT20. Table of Integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$....A.N. Lowan and M. Abramowitz.

0(0.01)10; 10D]. Reprinted from Journal of Mathematics and Physics, May 1943. 12 pages, with cover, 25 cents.

MT21. Table of $Ji_0(x) = \int_x^\infty \frac{J_0(t)}{t}$ dt and related functions.... A. N. Lowan, G. Blanch and M. Abramowitz.

Table I: $Ji_0(x)$ to 10 decimal places and $F(x) = Ji_0(x) + \log_e \frac{1}{2}x$ to 12 decimal places for $x = 0(.1)3$ with even central differences of $F(x)$.
Table II: $Ji_0(x)$ to 10 decimal places, for $x = 3(.1)10(1)22$ with even central differences up to $x = 100$.
Table III: "Reduced" derivatives of $F(x)$ for $x = 10(1)21$ and $n = 0(1)13$, to 12 decimal places.

Reprinted from Journal of Mathematics and Physics, June 1943. 7 pages, with cover, 25 cents.

MT22. Table of Coefficients in Numerical Integration Formulae... A. N. Lowan and Herbert Salzer.

The values of $B_n^{(n)}(1)/n!$ and $B_n^{(n)}/n!$, where $B_n^{(n)}(1)$ denotes the n^{th} Bernoulli polynomial of the n^{th} order for $x = 1$, and $B_n^{(n)}$ denotes the n^{th} Bernoulli number of the n^{th} order, were computed for $n = 1,2, \ldots 20$. The quantities $B_n^{(n)}(1)/n!$ are required in the Laplace formula of numerical integration employing forward differences, as well as in the Gregory formula. The quantities $B_n^{(n)}/n!$ are used in the Laplace formula employing backward differences.

Reprinted from Journal of Mathematics and Physics, June 1943. 2 pages, with cover. (Out of print.)

MT23. Table of Fourier Coefficients............A. N. Lowan and J. Laderman.

Whenever $f(x)$ is a known polynomial whose degree does not exceed 10, the present table of the functions

$$S(k,n) = \int_0^1 x^k \sin n\pi x \, dx \quad \text{and}$$

$$C(k,n) = \int_0^1 x^k \cos n\pi x \, dx$$

to 10D ($1 \leq k \leq 10$, $1 \leq n \leq 100$), will facilitate the evaluation of the first hundred Fourier Coefficients.

Reprinted from Journal of Mathematics and Physics, September 1943. 11 pages, with cover. (Out of print.)

MT24. Coefficients for Numerical Differentiation with Central Differences.........................Herbert E. Salzer.

Coefficients are given for derivatives as far as the 52nd. For the first 30 derivatives, exact values are given for coefficients of the first 30 differences, and also exact values are given for some coefficients of differences beyond the 30th. For the other coefficients, values are given to 18 significant figures.

Reprinted from Journal of Mathematics and Physics, September 1943. 21 pages, with cover, 25 cents.

MT25. Seven Point Lagrangian Integration Formulas....G. Blanch and I. Rhodes.

Formulas (not requiring differences) are given for the integral f(x), when the latter has been approximated by polynomials of degree six, thus affording greater accuracy than by Simpson's rule; or for comparable accuracy, permitting the evaluation of integrand at fewer points in the region. Certain remainder terms are also considered.

Reprinted from Journal of Mathematics and Physics, December 1943. 4 pages, with cover, 25 cents.

MT26. A Short Table of the First Five Zeros of the Transcendental Equation $J_0(x)Y_0(kx) - J_0(kx)Y_0(x) = 0$.....A. N. Lowan and A. Hillman.

The first five zeros of the above equation were computed for $k = 1\ 1/2, 2, 2\ 1/2, 3, 3\ 1/2, 4$. Zeros are given in Table I. The products of these zeros by (k-1) are given in Table II.

Reprinted from Journal of Mathematics and Physics, December 1943. 2 pages, with cover, 25 cents.

MT27. Table of Coefficients for Inverse Interpolation with Central Differences.........................Herbert E. Salzer

Contains tables giving the five fourth order coefficients for $m = 0(.001)1.000$; to 10 decimals (good to about a unit in the last decimal), and also the ten sixth order coefficients for $m = 0(.1)1.0$; exact values. A condensed one-page table of fourth order coefficients at intervals of 0.1 is also given for convenience in cases where a larger table is unnecessary.

Reprinted from Journal of Mathematics and Physics, December 1943. 15 pages, with cover, 25 cents.

MT28. Table of $f_n(x) = \dfrac{n!}{(x/2)^n} J_n(x)$.

Values are given for $x \leq 10$, and $n \leq 20$, in the following ranges: f_2 and f_3, [0(.01)10; 9D]; f_4 and f_5, [0(.01)6.50,6.5(.1)10; 9D]; f_6 to f_8; [0(.01)1.50; 1.5 (.1)10; 9D]; f_9 to f_{20}, [0(.1)10; 9D].

Modified second central differences are tabulated alongside the entries.

Reprinted from Journal of Mathematics and Physics, February 1944. 16 pages, with cover, 25 cents.

MT29. Table of Coefficients for Inverse Interpolation with Advancing Differences................Herbert E. Salzer.

This table may be regarded as a companion table to MT27, "Table of Coefficients for Inverse Interpolation with Central Differences." It contains tables giving values to 10 decimal places for

4th and 5th order terms for $m = 0(.001)1.000$;
6th order terms for $m = 0(.01)1.00$; and
7th and 8th order terms for $m = 0(.1)1.0$.

A condensed one-page table of coefficients of the fourth, fifth and sixth order terms, at intervals of 0.1, is also given for the sake of convenience.

Note: Coefficients of the second and third order terms $\frac{m(1-m)}{2}$ and $\frac{m(1-m)(m-2)}{6}$ respectively, were not tabulated since they are both tabulated at intervals of 0.0001 in "Tables of Lagrangian Interpolation Coefficients", published by the Columbia University Press, New York City.(see listing in part IV, this Letter Circular). For m (there "p") ranging from 0 to 1, $\frac{m(1-m)}{2}$ is given by $-A_{-1}$ in the three-point table, and $\frac{m(1-m)(m-2)}{6}$ is given by A_{-1} in the four-point table.

Reprinted from Journal of Mathematics and Physics, May 1944. 28 pages, with cover, 25 cents.

MT30. A New Formula for Inverse Interpolation.......H. E. Salzer.

This paper is devoted to the derivation of a formula for inverse interpolation in a table of equally spaced arguments. The resulting formula is believed to be more concise and convenient than the earlier formulas. It involves neither differences nor polynomial coefficients other than small powers. In a sense, it is the analogue of the Lagrangian formula for direct interpolation without differences (that is, in terms of the tabular entries only) if the usual expression is rearranged in terms of powers of the argument p.

Reprinted from Bulletin of the American Mathematical Society, August 1944. 4 pages, with cover, 25 cents.

MT31. Coefficients for Interpolation Within a Square Grid in the Complex Plane......A. N. Lowan and H. E. Salzer.

The accompanying table gives the exact values of the real and imaginary parts of the 3-point and 4-point coefficients for both p and q ranging from 0 to 1 at intervals of 0.1.

The present table is particularly useful when interpolation is to be carried out either for an isolated argument or for a large number of arguments which are irregularly scattered. Thus for a 4-point interpolation for a single argument, the process of interpolation based on the present table requires a total of 4 complex multiplications (equivalent of 16 ordinary multiplications) and 2 additions, whereas the method of interpolation in the real and imaginary parts each considered as a function of x and y, using real 4-point Lagrangian interpolation coefficients, requires 40 multiplications and 10 additions. The table is however not an efficient tool for carrying out an extensive and systematic process of subtabulation. For such tasks the latter method involving the use of real interpolation coefficients is almost twice as efficient from the standpoint of the total number of multiplications and additions.

Reprinted from Journal of Mathematics and Physics, August 1944. 11 pages, with cover, 25 cents.

MT32. Table of Coefficients for Differences in Terms of the Derivatives................Herbert E. Salzer.

The table lists the exact values of the coefficients $B_{m,s}$ for $m = 1, 2, \ldots 20$ and $s = m, m + 1, \ldots 20$, for calculating the m^{th} advancing difference by means of Markoff's formula. It will be useful to anyone who wishes to construct difference tables for functions whose derivatives are known.

Reprinted from Journal of Mathematics and Physics, November 1944. 4 pages, with cover, 25 cents.

MT33. Table of Coefficients for Numerical Integration Without Differences..........A. N. Lowan and H. E. Salzer.

When a function f(x) is known for n equally spaced arguments at interval h, the value of the integral between the limits $x_0 + rh$ and $x_0 + sh$ may be obtained (by the integration of the well-known Lagrangian interpolation formula) in the form

MT33 contd.

$$\int_{x_0 + rh}^{x_0 + sh} f(x)\,dx \sim h \sum_{i = -[(n-1)/2]}^{[n/2]} \left\{ B_i^{(n)}(s) - B_i^{(n)}(r) \right\} f(x_0 + ih)$$

where [x] denotes the largest integer in x and $B_i^{(n)}(p)$ are polynomials of the n^{th} degree. The accompanying table gives the value of these polynomials $B_i^{(n)}(p)$ to ten decimals, for p ranging from $-[(n-1)/2]$ to $[n/2]$. For n = 3, 4 and 5 these polynomials are tabulated at intervals of 0.01; for n = 6 and 7, they are tabulated at intervals of 0.1.

Reprinted from Journal of Mathematics and Physics, February 1945. 21 pages, with cover, 25 cents.

MT34. Inverse Interpolation for Eight-, Nine-, Ten-, and Eleven-Point Direct Interpolation.....H. E. Salzer.

Formulas are given for inverse interpolation for functions that are tabulated at a uniform interval h, and which require direct interpolation polynomials ranging from the seventh to the tenth degree. Here f_i denotes $f(x_0 + ih)$ and the problem is to find p when we are given $f(x_0 + ph)$.

In a previous paper (see MT30), the author obtained all the terms involving the first six powers of $(f_p - f_0)$ in the expansion for p in the terms of f_p. and the tabular values, corresponding to the cases where three- to seven-point direct interpolation was required. In the present paper that same formula for p is extended to include all the terms involving the first ten powers of $(f_p - f_0)$ and, as before, quantities are defined in terms of f_p and the tabular entries to provide for inverse interpolation when eight-, nine-, ten-, or eleven-point formulas are required for direct interpolation. Although the greater part of the expression for p will hardly ever be needed in most practical problems, due to the rapidity of convergence and smallness of many of its terms, its full use can provide unusual accuracy in solving equations (both real and complex) up to the tenth degree when the values of the polynomials are tabulated near the root at equal intervals.

Reprinted from Journal of Mathematics and Physics, May 1945. 4 pages, with cover, 25 cents.

MT35. Table of Coefficients for Double Quadrature Without Differences, for Integrating Second Order Differential Equations........................ ......H. E. Salzer.

On the basis of a double quadrature of the Lagrange interpolation formula, a table of coefficients has been computed to determine a function at equally spaced points (to within an arbitrary $Ax + B$), when its second derivative is known at those points. The coefficients cover the cases where the second derivative may be approximated by a polynomial ranging from the second to tenth degrees (i.e., from three-point through eleven-point formulas), and are given exactly. Their chief value will occur in the numerical solution of ordinary linear differential equations of the second order, which can always be reduced to the form $y'' + g(x)\ y = h(x)$. They can also be employed to integrate the more general equation $y'' + \phi\ (x,y) = 0$. In every case it is necessary to begin with a few values of y'' which can always be found by the usual methods. There are indications that these coefficients can be used to extend the solution of a second order partial differential equation of the form $u_{xx} = u_t + \psi(u,x,t)$, provided that it is known at a rectangular array of points in the x-t plane and at two other points in the next row of values of t.

Reprinted from Journal of Mathematics and Physics, November 1945. 6 pages, with cover, 25 cents.

MT36. Formulas for Direct and Inverse Interpolation of a Complex Function Tabulated Along Equidistant Circular Arcs ....H. E. Salzer.

When an analytic function $f(z)$ may be approximated by a complex polynomial of degree n-1 passing through the values of the function at n points, according to the Lagrange-Hermite interpolation formula, it often happens that those n points are situated along the arc of a circle (equally spaced) and it is required to obtain $f(z)$ for z off the circle but near the arguments. An important case is when $f(z)$ is tabulated in polar form (including tabulation along the vertices of any regular polygon). The formulas that were obtained will facilitate direct interpolation when $f(z)$ is known at three, four, or five points. They furnish the real and imaginary parts of

$L_k^{(n)}\ (P)$ where $f(x) \sim \sum L_k^{(n)}\ (P) f(z_k)$ as functions of

MT36 contd.

$P^m = p_m + iq_m$ and θ. Here $P = (z - z_0)/h$, h being the distance between points z_k, and θ denotes the angle between successive chords joining the points z_k. For extensive use for a fixed θ, one can readily obtain $L_k^{(n)}(P)$ in the form

$$\sum C_k^{(m)}(p_m + iq_m).$$

A method for inverse interpolation is given, employing the coefficients of the polynomials $L_k^{(n)}(P)$ in an expansion derived in the author's "A New Formula for Inverse Interpolation", Bulletin Amer. Math. Soc., Aug. 1944, pp. 513-516 (see MT30).

Reprinted from Journal of Mathematics and Physics, November 1945. 3 pages, with cover, 25 cents.

MT37. On the Computation of Mathieu Functions....G. Blanch.

This paper gives a method of correcting the characteristic values of Mathieu's differential equation and the Fourier coefficients determining the solution, in the process of generating the latter. If the characteristic value is in error by g, the corrected characteristic value and Fourier coefficients have errors proportional to g^2. The method is applicable to other types of equations, when the coefficients of the power series solution or of the Fourier series are determined by a three-term recursion formula.

Reprinted from Journal of Mathematics and Physics, February 1946. 20 pages, with cover, 25 cents.

III. War Department Tables

These volumes are obtainable from the Superintendent of Documents; see part II of this Letter Circular for the terms of sale.

Coordinate Conversion Tables. Published as Technical Manual TM 4-238 of the War Department.

Table I:

A sin x and A cos x where x is in mils,

A = [0(1)100; 4D]; x = [0(1)800].

The mil is 1/1600 of a right angle. The unit in which A is expressed will determine the unit of the entry--if A is in yards, the entries in this table are to be taken as yards.

Since these values represent products of sin x and cos x by A, values of the functions for any value of A correct to 5 and sometimes 6 significant figures may be obtained from the table by simple addition.

Table II:

Sin x and cos x, [0(1)800; 8D].

This table may be used whenever higher accuracy than that afforded by the Table I is desired.

March 25, 1943. 338 pages, $5\frac{1}{2}$ by $8\frac{1}{2}$ in. 40 cents.

Hydraulic Tables (2d ed.) Published by the Corps of Engineers, War Department.

These tables were prepared and published primarily for the use of the various offices of the Corps of Engineers, U. S. Army, in connection with the design of flood protection works. The range of many existing tables had been found inadequate for the large rivers, great depths, and steep slopes encountered in flood control work. It was also found desirable to compute several new tables. The new tables will be especially useful in solving the Manning formula in its various forms.

This handbook contains 46 hydraulic tables especially aplicable to the solutions of problems of open channel flow. It is divided into three sections; Section I consists of an extension of Tables 106, 121, and 133 in "Handbook of Hydraulics" (1939) by Horace W. King; Section II is an extension of Tables 14-18 and 47 in "Hydraulic and Excavation Tables", 8th edition, 1940, by the Bureau of Reclamation; Section III, Miscellaneous New Tables, i.e., (a) fractional powers of numbers, (b) normal depth in channels of infinite width, and (c) Manning formula tables giving the velocity of water for various coefficients of roughness.

(1944) viii + 565 pages. Blue imitation leather flexible cover, $4\frac{1}{2}$ by 6 3/4 in. $1.50.

IV. The Columbia University Press Series

These tables can be obtained from the Columbia University Press, Morningside Heights, New York 27, New York. Further description of the tables is to be found in a pamphlet published by the Columbia University Press.

Table of Reciprocals of the Integers from 100,000 through 200,009.

Tabular entries given to 7S, with differences and proportional parts.

(1943) viii + 204 pp., bound in buckram. Price $4.00

Table of the Bessel Functions $J_0(z)$ and $J_1(z)$ for Complex Arguments.

The real and imaginary parts of $J_0(z)$ and $J_1(z)$ are tabulated to 10D for argument $z = \rho \exp i\varphi$, where $\rho = 0(.01)10$; $\varphi = 0^\circ(5^\circ)90^\circ$. The volume includes an extensive bibliography, contour lines and an auxiliary table of five-point Lagrangian interpolation coefficients, [0(.001)1; 10D].

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Table of Arc sin x.

[0(.0001).989(.00001)1; 12D], with second central differences. Auxiliary table of $f(v) = \{1/2\pi - \text{Arc}\ \sin(1-v)\} / \sqrt{2v}$, [0(.00001).0005; 13D], with differences. Volume includes: values of $\frac{1}{2}p(1-p)$; coefficients in Everett's interpolation formula; conversion tables, radians $\rightleftarrows$ degrees, minutes, seconds; multiples of $\pi/2$; and a bibliography.

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Table of Associated Legendre Functions.

$P_n^m(\cos\theta), \frac{d}{d\theta} P_n^m(\cos\theta)$; θ = [0°(1°)90°; 6S]. $P_n^m(z)$, $Q_n^m(z)$, $\frac{d}{dx}P_n^m(z)$, $\frac{d}{dx} Q_n^m(z)$; [z = x = 1(.1)10 and z = ix, x = 0(.1)10; 6S]. In all the above tables, n = 0 or 1(1)10; m = 0(1)4 but $\leq$ n. $P_{n+\frac{1}{2}}^m(x)$, $Q_{n+\frac{1}{2}}^m(x)$, $\frac{d}{dx} P_{n+\frac{1}{2}}^m(x)$, $\frac{d}{dx} Q_{n+\frac{1}{2}}^m(x)$; x = [1(.1)10; 4S, 5S, or 6S]; n = -1(1)4; m = 0(1)4. Various auxiliary tables and an extensive bibliography are also provided.

(1945) xlvi + 306 pp., bound in buckram. Price $5.00.

Tables of Fractional Powers.

Part I. Values of A^x

A = 2(1)9, x = [.001(.001).01(.01).99; 15D].
A = 10, x = [.001(.001)1; 15D].
A = π, x = [.001(.001)1; 15D, 15S], and $\pm$ x = 1/4, 1/3, 1/2, 2/3, 3/4, 1(1)12.
A = [.01(.01).99], x = [.001(.001).01(.01).99; 15D].
A = 10^{-3} P, P a prime between 100 and 1000, and x = [.001(.001).01(.01).99; 15D].

Part II. Values of x^a

a = $\pm$ 1/2, $\pm$ 1/3, $\pm$ 2/3, $\pm$ 1/4, $\pm$3/4; x = [0(.01)9.99 or 10;15D].
a = .01(.01).99, x = [0(.001) various; 0(.01).99; 7D].

This table is provided with an extensive bibliography.

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$P^m_{n+\frac{1}{2}}(x)$, $Q^m_{n+\frac{1}{2}}(x)$, $\frac{d}{dx} P^m_{n+\frac{1}{2}}(x)$, $\frac{d}{dx} Q^m_{n+\frac{1}{2}}(x)$; x = [1(.1)10; 4S, 5S, or 6S]; n = -1(1)4; m = 0(1)4. Various auxiliary tables and an extensive bibliography are also provided.

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A = [.01(.01).99], x = [.001(.001).01(.01).99; 15D].
A = 10^{-3} P, P a prime between 100 and 1000, and x = [.001(.001).01(.01).99; 15D].

Part II. Values of x^a

a = ± 1/2, ± 1/3, ± 2/3, ± 1/4, ±3/4; x = [0(.01)9.99 or 10; 15D].
a = .01(.01).99, x = [0(.001) various; 0(.01).99; 7D].

This table is provided with an extensive bibliography.

(1946) xxx + 490 pp., bound in buckram. Price $7.50.

Tables of Spherical Bessel Functions, Volume I.

$\sqrt{\pi/2x}\, J_\nu(x)$, $\pm\nu = \frac{1}{2}(1)\frac{25}{2}$: $x = 0(.01)10(.1)25$

$\pm\nu = \frac{27}{2}$: $x = 0(.01)10(.05)10.5(.1)25$

to 8S-10S for $x \leq 10$, mostly to 7S for $x > 10$

$x^{-\nu + 1/2} \sqrt{\pi/2x}\, J_\nu(x)$, $[x = 0(.01)x_\nu$; 8S-10S], $.5 \leq x_\nu \leq 2.5$.

With second and fourth central differences, and auxiliary tables of interpolation coefficients.

(1947) xxviii + 375 pp., bound in buckram. Price $7.50.

Tables of Spherical Bessel Functions, Volume II.

$\sqrt{\pi/2x}\, J_\nu(x)$, $\pm\nu = \frac{29}{2}(1)\frac{43}{2}$: $x = 0(.01)10(.1)25$

$\pm\nu = \frac{45}{2}(1)\frac{61}{2}$: $x = 10(.1)25$

to 8S-10S for $x \leq 10$, mostly to 7S for $x > 10$

$\Lambda_\nu(x) = \frac{2^\nu \Gamma(\nu + 1)}{x^\nu} J_\nu(x)$, $\nu = \frac{1}{2}\left(\frac{1}{2}\right)\frac{41}{2}$: [x = 0(.1)10; 9D]

$\nu = \frac{1}{2}(1)\frac{61}{2}$: [x = 10(.1)25; mostly 7S]

$-\nu = \frac{29}{2}(1)\frac{33}{2}$:

x = 0(.1)9.5(.05)10(.1)25; mostly 7S]

$-\nu = \frac{35}{2}(1)\frac{61}{2}$: [x = 0(.1)25; mostly 7S].

With second and fourth central differences, and auxiliary tables of interpolation coefficients.

Zeros of $J_\nu(x)$ and $J'_\nu(x)$
with corresponding values of $J'_\nu(x)$ at the zeros of $J_\nu(x)$ and the maxima and minima of $J_\nu(x)$.

(1947) xx + 328 pp., bound in buckram, Price $7.50.

V. Tables Available on Punched Cards

These tables are on file at the Computation Laboratory, 150 Nassau Street, New York, New York. Within the limits of the program of the Computation Laboratory, they will be duplicated upon request, provided the requester furnishes the blank cards. Correspondence should be sent directly to the Computation Laboratory.

1. $\sqrt{N}$, $\sqrt{10N}$, $\sqrt[3]{N}$: to 7D with first differences $1/N$: to 7S with first differences; [1,000(1)2,000(2)4,000(5)10,000]

2. $1/\sqrt{1-x^2}$, $x/\sqrt{1-x^2}$: x from 0 to 1 at varying intervals; first differences.

3. $\text{Log}_{10}N$; [0(1)23,000; 5D]

4. 10^x ; [0(.00001)1; 10D] with first differences

5. Sec θ, csc θ; [$0^\circ(1')90^\circ$; 10D]

6. Sin θ, tan θ; [$0^\circ(0^\circ.01)89^\circ.99$; 7D] with first differences

7. Cos θ ; [$0^\circ(1')90^\circ$; 8D] with $\Delta/60$

8. Sin θ, cos θ, where $\theta = \frac{b}{a}\tan\phi$; [$\phi = 0^\circ(1')90^\circ$; 8D] with $\Delta/60$

9. Sin θ, cos θ; [$0^\circ(0^\circ.01)90^\circ$; 15D] with δ^2

10. Areas and Circumferences of Circles
    Diameter = [0(.0001)10; 7D]

11. Real and imaginary parts of $Y_0(\rho e^{i\theta})$, $Y_1(\rho e^{i\theta})$,

    [$\rho = 0(.01)10$, $\theta = 0^\circ(5^\circ)90^\circ$; 10D]

12. $H(x) = (1/2\pi)\int_{-x}^{x} e^{-\frac{\alpha^2}{2}}\,d\alpha$,

    $H'(x)$; [0(.001)7.8; 15 D]

13. $E_1(z) = \int_z^\infty \frac{e^{-u}}{u}\,du$.

    A. $E_1(z) + \log z$; [$x = 0(.01)1$, $y = 0(.02)1$; 6D]

    B. $E_1(z)$; [$x = 0(.02)4$, $y = 0(.02)3(.05)10$; 6D]

    C. $e^x E_1(z)$; [$x = 3(.1)10$, $y = 0(.05)10$; 6D]

14. Solutions of Mathieu's Equation.
$Se_r(s, \text{Cos } \theta)$, $So_r(s, \text{Cos } \theta)$;

$s = 0$ to 100 at varying intervals.

$\theta = 0^\circ(1^\circ)90^\circ$

$r = 7(1)15$