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Letter
Circular
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MATHEMATICAL TABLES

(January 26, 1945)

Contents

General Information

The Mathematical Tables Project, which prepared the mathematical tables described herein (with the exception of MT15), had its inception in January 1938, and until March 1943, was operated by the Federal Works Agency, Work Projects Administration for the City of New York under the sponsorship of the National Bureau of Standards.

When the Work Projects Administration discontinued this project, the sponsoring agency took over its operation with the support of the Office of Scientific Research and Development, the determination of the program being vested in the Applied Mathematics Panel of the National Defense Research Committee. Under these new auspices, the major emphasis is being placed on the computation of tables directly related to the war effort; but it is also planned to devote any available time to the important task of finishing various tables which were under way when the Work Projects Administration was discontinued. Thus it has been possible to bring to final completion several volumes, now in the process of publication. A mailing list is maintained by the Bureau for those desiring to receive announcements of new tables as they become available.

It has been the policy of the Project to select for tabulation mathematical functions of fundamental importance in pure and applied mathematics. In the selection of the functions to be tabulated and in the choice of the most useful range and interval of the argument, the Project has had the advantage of a continual exchange of views with outstanding mathematicians, physicists and engineers, both here and abroad.

TABLES OBTAINABLE FROM THE NATIONAL BUREAU OF STANDARDS

The tables listed under this heading can be purchased from the Bureau at the prices indicated, unless otherwise stated. Payment is required in advance and remittance should accompany the order. When remitting, please make check or money order payable to the National Bureau of Standards.

The prices are for delivery in the United States and its possessions and in countries extending the franking privilege, that is, Canada and most of the Latin American countries. To other countries the price of MT2 to MT16, inclusiv is \$2.50 each; MT17, \$1.75; and MT18 to MT32, inclusive, 30 cents each; remittance to be payable in United States currency.

Copies of these publications (with the exception of MT11, MT15, MT18 and subsequent numbers, the editions of which are very limited) have been sent to various Government Depositories in the United States, such as public libraries in large cities, colleges, and universities, where they may be consulted.

Note.- The figures in square brackets denote the range and interval of the argument, and the number of decimal places or significant figures in the tabulated entries. The letters "D" and "S" stand for "decimal places" and "significant figures", respectively. In MT6, for instance, Si(x) is tabulated over the range from 0 to 9.999, at intervals of 0.001 to 10 decimal places, and Ei(x) is tabulated over the same range, for the same interval of the argument, to 10 significant figures.

MT1. Table of the First Ten Powers of the Integers from 1 to 1000*.

(1939) viii + 80 pages; heavy paper cover, $8\frac{1}{2}$ by $13\frac{3}{4}$ in. Out of prin

MT2. Tables of the Exponential Function e^x.

[-2.5(.0001)1; 18D]; [1(.0001)2.5(.001)5; 15D]; [5(.01)10; 12D];

[-0.0001(.000001)0.0001; 18D]; [-100(1)100; 19S].

[$-9 \times 10^{-n}(10^{-n})9 \times 10^{-n}$; for n = 10, 9, 8, 7; 18D].

(1939) xv + 535 pages; bound in buckram. Price \$2.00.

MT3. Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments.

[0(0.0001)1.9999; 9D]; [0(0.1)10; 9D]. Conversion table,

radians $\rightleftharpoons$ degrees. (1939) xvii + 405 pages; bound in buckram. Price \$2.

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*Superseded by the more extensive table, "B. A. Mathematical Tables, volume IX, Table of Powers Giving Integral Powers of Integers," (1940) British Association for the Advancement of Science.

MT4. Tables of Sines and Cosines for Radian Arguments.

[0(0.001)25; 8D]; [0(1)100; 8D]. [$1 \times 10^{-n}(10^{-n})9 \times 10^{-n}$; for n = 5, 4, 3, 2, 1; 15D]; [0(0.00001)0.01; 12D]. Conversion table, radians $\rightleftarrows$ degrees. Values of $p(1 - p)$.

(1940) xix + 275 pages; bound in buckram. Price $2.00.

MT5. Tables of Sine, Cosine, and Exponential Integrals, Volume I.

[0(0.0001)1.9999; 9D]; [0(0.1)10; 9D]. Auxiliary tables of $p(1 - p^2)/6$ and $p(1 - p)$.

(1940) xxvi + 444 pages; bound in buckram. Price $2.00.

MT6. Tables of Sine, Cosine, and Exponential Integrals, Volume II.

Si(x), Ci(x): [0(0.001)9.999; 10D]; [10(0.1)20(0.2)40; 10D].

Ei(x): [0(0.001)9.999; 10S]; [10(0.1)15; 10S and 11S].

-Ei(-x): [0(0.001)9.999; 9S]; [10(0.1)15; 14D].

Si($n\pi \pm h$), for n = 1, 2, 3; Ci($n\pi/2 \pm h$), for n = 1, 3, 5;

for h = [0(0.0001)0.01(0.001)0.05; 15D].

Auxiliary tables of $p(1 - p^2)/6$ and $p(1 - p)$. Multiples of $\pi/2$ and other constants.

(1940). xxxvii + 225 pages; bound in buckram. Price $2.00.

MT7. Table of Natural Logarithms, Volume I.

Logarithms of the integers from 1 to 50,000 to 16 decimal places.

(1941) xviii + 501 pages; bound in buckram. Price $2.00.

MT8. Tables of Probability Functions, Volume I.

$$H'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2} \quad \text{and} \quad H(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\alpha^2} d\alpha$$

[0(0.0001)1(0.001)5.6(various)5.946; 15D]. H'(x) and 1 - H(x), [4(0.01)10; 8S].
(1941) xxviii + 302 pages; bound in buckram. Price $2.00.

MT9. Table of Natural Logarithms, Volume II.

Logarithms of the integers from 50,000 to 100,000 to 16 decimal places.
(1941) xviii + 501 pages; bound in bcukram. Price $2.00.

MT10. Table of Natural Logarithms, Volume III.

Logarithms of the decimal numbers from 0.0001 to 5.0000 to 16 decimal places. (1941) xviii + 501 pages; bound in buckram. Price $2.00.

MT11. Tables of the Moments of Inertia and Section Moduli of Ordinary Angles, Channels, and Bulb Angles with Certain Plate Combinations.

(1941) xiii + 197 pages; bound in green cloth. Price $2.00.

MT12. Table of Natural Logarithms, Volume IV.

Logarithms of the decimal numbers from 5.0000 to 10.0000 to 16 decimal places.
$\log_e x$; [2(1)10; 40D].
$\log_e(1 + x)$, $-\log_e(1 - x)$ [$x = 10^{-n}(10^{-n})10^{-n+1}$; n = 1, 2, ... 13; 25D].
(1941) xxii + 506 pages; bound in buckram. Price $2.00.

MT13. Table of Sine and Cosine Integrals for Arguments from 10 to 100.

[10(0.01)100; 10D]. Multiples of $\pi/2$ [1(1)100; 15D].
Values of $p(1 - p)$ and $p(1 - p^2)/6$.
(1942) xxxii + 185 pages; bound in buckram. Price $2.00.

MT14. Tables of Probability Functions, Volume II.

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad \text{and} \quad \frac{1}{\sqrt{2\pi}} \int_{-x}^{x} e^{-\alpha^2/2} d\alpha$$

[0(0.0001)1(0.001)7.800(various)8.285; 15D]; and

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad \text{and} \quad \frac{2}{\sqrt{2\pi}} \int_x^{\infty} e^{-\alpha^2/2} d\alpha$$

[6(0.01)10; 7S].
(1942) xxi + 344 pages; bound in buckram. Price $2.00.

MT15. The Hypergeometric and Legendre Functions with Applications to Integral Equations of Potential Theory.....Chester Snow, National Bureau of Standards.

Compiled for workers in applied mathematics, its scope is intermedia between tables of numerical values of these functions and a treatise on their pure theory. The linear and quadratic transformations and analytic continuations of the ordinary hypergeometric function are derived and written out at length with special space devoted to the general associate Legendre functions, and to a smaller extent Heun's generalization of the hypergeometric function. Applications to potential theory (91 pages) where the potential is given on surfaces of revolution include most of th elementary separable coordinate systems. Use is made of the potential equation in a form invariant to inversions, and from the point of view of integral equations with Legendre's function, $Q_{m-\frac{1}{2}}$, of the second kind as nucleus. Some generalizations of Fourier's integral are obtained in whic the development function satisfies a second order differential equation o rather general form. These are utilized in obtaining the formal solution of various potential problems. A set of normal functions is constructed satisfying the Lamé-Wangerin equation in "annular" coordinates which include toroidal coordinates and oblate spheroidal as limiting cases.

Reproduced by photo-offset process from original handwritten manuscript. (1942) 319 pages; bound in heavy paper covers. Price $2.00.

MT16. Table of Arc Tan x.

The principal value of the inverse tangent may be defined by the integral

$$\text{Arc tan } x = \int_0^x \frac{du}{1+u^2}$$

The intervals between successive arguments have been so chosen, tha interpolation to 12 decimals may be made with the aid of the second cent differences which are tabulated alongside of the entries.

[0(0.001)7(0.01)50(.1)300(1)2,000(10)10,000; 12D]. Values of p(1 - and $p(1 - p^2)/6$. Conversion table, radians ⇄ degrees.
(1942) xxv + 169 pages; bound in buckram. Price $2.00

MT17. Miscellaneous Physical Tables: Planck's Radiation Functions, and Electronic Functions.

1. Planck's radiation functions
(Reprinted from J. Opt. Soc. Am , February 1940)

The functions evaluated by Tables I to IV are

$$R_\lambda = c_1\lambda^{-5}(e^{c_2/\lambda T} - 1)^{-1} \quad ; \quad R_{0-\lambda} = \int_0^\lambda R_\lambda d\lambda$$

$$N_\lambda = 2\pi c\,\lambda^{-4}(e^{c_2/\lambda T} - 1)^{-1} \quad ; \quad N_{0-\lambda} = \int_0^\lambda N_\lambda d\lambda$$

$R_\lambda d\lambda$ is the energy emitted by a black body at absolute temperature, T, on the Kelvin scale, per unit time, per unit area, in a wave-length interva from λ to $\lambda + d\lambda$, throughout the solid angle 2π steradians.

$N_\lambda d\lambda$ represents the number of photons emitted in the wave-length range from λ to $\lambda + d\lambda$, per unit time, per unit area, throughout the solid angle 2π steradians.

The tables evaluate these four functions of λT not only relative to their maximum values, but also for T = 1000 °K in absolute units (ergs or photons per cm² per second). The values are given to five significant figures, first and second differences being provided to assist in finding R_λ and N_λ by interpolation; and there is provided a convenient method of correcting for small changes in the value of c_2 from that (c_2 = 1.436 cm K°) used in computing the tables. The function N_λ is also evaluated for T = 1500, 2000, 2500, 3000, 3500, and 6000 °K.

2. Electronic functions

This table of 28 pages gives in five columns the values for the velocity of an electron relative to the velocity of light, β; $G = 1/(1-\beta^2)^{\frac{1}{2}}$; βG; the energy in electron kilovolts, and the curvature of the electron path in a magnetic field times the field $H\rho$. The total energy, effective mass, and momentum depend on the functions G and β G. The energies in electron volts cover the range from 6 to 10^{10} electron volts so that the tables are applicable in the full range of energies involved in nuclear and cosmic ray physics. In the range from 10,000 to 1.5 x 10^7 electron volts successive values differ by less than 1 percent in energy and curvature so that in nearly all practical problems the tables would be used without interpolation. As relativistic equations must be used throughout this range the tables will save a great deal of laborious computation.

(1941) vi + 58 pages; bound in buckram. Price $1.50.

MT18. <u>Table of the Zeros of the Legendre Polynomials of Order 1-16 and the Weight Coefficients for Gauss' Mechanical Quadrature Formula.</u>

A. N. Lowan, N. Davids, and A. Levenson.

Gauss' method of mechanical quadrature has the advantage over most methods of numerical integration in that it requires about half the number of ordinate computations. This is desirable when such computations are very laborious, or when the observations necessary to determine the average value of a continuously varying physical quantity are very costly. Gauss' classical result states that, for the range (-1, +1), the "best" accuracy with n ordinates is obtained by choosing the corresponding abscissae at the zeros $x_1, \ldots x_n$ of the Legendre polynomials $P_n(x)$. With each x_i is associated a constant a_i such that

$$\int_{-1}^{1} f(x)dx \sim a_1 f(x_1) + a_2 f(x_2) + \ldots + a_n f(x_n).$$

The accompanying table computed by the Mathematical Tables Project gives the roots x_i for each $P_n(x)$ up to $n = 16$, and the corresponding weight coefficients a_i, to 15 decimal places.

Reprinted from Bulletin of the American Mathematical Society, October 1942. 5 pages, with cover, 25 cents.

MT19. On the Function $H(m,a,x) = \exp(-ix)F(m + 1 - ia, 2m + 2; ix)$. With table of the confluent hypergeometric function and its first derivative..............................A. N. Lowan and W. Horenstein

Confluent hypergeometric functions occur in numerous physical problems. They appear in the solutions of Laplace's equation and the equations of wave motion and of diffusion when the physical problem involves knife edges, parabolic cylinders, paraboloids of revolution, general ellipsoids, etc. Of particular importance to the quantum physicist is their occurrence as the solutions of the radial part of Schrödinger's equation for a Coulomb field and continuous values of the energy. Thus they are needed for the calculation of the scattering of charged particles from atomic nuclei, or of their binding by nuclei. In a more general form they also occur in the problem of dissociation of molecules into atomic ions (and, of course, in the converse case of the formation of molecules).

This paper contains a table of $H(m,a,x)$ and its first derivative for values of x from 1 to 10 by steps of 1, for values of a from 1 to 10 by steps of 1, and for values of the integer m from 0 to 3; values in all cases are given to 7 significant figures.

Reprinted from Journal of Mathematics and Physics, December 1942. 20 pages, with cover, 25 cents.

MT20. Table of Integrals $\int_0^x J_0(t)dt$ and $\int_0^x Y_0(t)dt$......A. N. Lowan and M. Abramowitz.

Values of the two integrals are given for $x = 0(.01)10$ to 10 decimal places. Reprinted from Journal of Mathematics and Physics, May 1943. 12 pages, with cover, 25 cents.

MT21. Table of $Ji_0(x) = \int_x^{\infty} \frac{J_0(t)}{t} dt$ and related functions... A.N.Lowan, G. Blanch, and M. Abramowitz.

Table I: $Ji_0(x)$ to 10 decimal places and $F(x) = Ji_0(x) + \log_e \frac{1}{2}x$ to 12 decimal places for $x = 0(.1)3$ with even central differences of $F(x)$.

Table II: $Ji_0(x)$ to 10 decimal places, for $x = 3(.1)10(1)22$ with even central differences up to $x = 100$.

Table III: "Reduced" derivatives of $F(x)$ for $x = 10(1)21$ and $n = 0(1)13$, to 12 decimal places.

Reprinted from Journal of Mathematics and Physics, June 1943. 7 pages, with cover, 25 cents.

MT22. Table of Coefficients in Numerical Integration Formulae...A. N. Lowan and Herbert Salzer.

The values of $B_n^{(n)}(1)/n!$ and $B_n^{(n)}/n!$ where $B_n^{(n)}(1)$ denotes the n^{th} Bernoulli polynomial of the n^{th} order for $x = 1$ and $B_n^{(n)}$ denotes the n^{th} Bernoulli number of the n^{th} order, were computed for $n = 1, 2, \ldots 20$. The quantities $B_n^{(n)}(1)/n!$ are required in the Laplace formula of numerical integration employing forward differences, as well as in the Gregory formula. The quantities $B_n^{(n)}/n!$ are used in the Laplace formula employing backward differences.

Reprinted from Journal of Mathematics and Physics, June 1943.
2 pages, with cover, 25 cents.

MT23. Table of Fourier Coefficients.......... A. N. Lowan and J. Laderman.

Whenever $\varphi(x)$ is a known polynomial whose degree does not exceed 10, the present table of the functions

$$S(k, n) = \int_0^1 x^k \sin n\pi x \, dx \quad \text{and} \quad C(k, n) = \int_0^1 x^k \cos n\pi x \, dx$$

to 10D ($1 \leqslant k \leqslant 10$, $1 \leqslant n \leqslant 100$), will facilitate the evaluation of the first hundred Fourier Coefficients.

Reprinted from Journal of Mathematics and Physics, September 1943.
11 pages, with cover, 25 cents.

MT24. Coefficients for Numerical Differentiation with Central Differences... ...Herbert E. Salzer.

Coefficients are given for derivatives as far as the 52nd. For the first 30 derivatives, exact values are given for coefficients of the first 30 differences, and also exact values are given for some coefficients of differences beyond the 30th. For the other coefficients, values are given to 18 significant figures.

Reprinted from Journal of Mathematics and Physics, September 1943.
21 pages, with cover, 25 cents.

MT25. Seven-Point Lagrangian Integration Formulas,....G. Blanch and I. Rhodes.

Formulas (not requiring differences) are given for the integral f(x), when the latter has been approximated by polynomials of degree six, thus affording greater accuracy than by Simpson's rule; or for comparable accuracy, permitting the evaluation of integrand at fewer points in the region. Certain remainder terms are also considered.

Reprinted from Journal of Mathematics and Physics, December 1943.
4 pages, with cover, 25 cents.

MT26. A Short Table of the First Five Zeros of the Transcendental Equation $J_0(x)Y_0(kx) - J_0(kx)Y_0(x) = 0$.....A. N. Lowan and A. Hillman.

The first five zeros of the above equation were computed for $k = 1\ 1/2,\ 2,\ 2\ 1/2,\ 3,\ 3\ 1/2,\ 4$. Zeros are given in Table I. The products of these zeros by $(k - 1)$ are given in Table II.

Reprinted from Journal of Mathematics and Physics, December 1943.
2 pages, with cover, 25 cents.

MT27. Table of Coefficients for Inverse Interpolation with Central Differences ......Herbert E. Salzer.

Contains tables giving the five fourth order coefficients for $m = 0(.001)1.000$; to 10 decimals (good to about a unit in the last decimal) and also the ten sixth order coefficients for $m = 0(.1)1.0$; exact values. A condensed one-page table of fourth order coefficients at intervals of 0.1 is also given for convenience in cases where a larger table is unnecessary.

Reprinted from Journal of Mathematics and Physics, December 1943. 15 pages, with cover, 25 cents.

MT28. Table of $f_n(x) = \frac{n!}{(x/2)^n} J_n(x)$.....The Mathematical Tables Project.

Values are given for $x \leq 10$, and $n \leq 20$, in the following ranges: f_2 and f_3, $0(.01)10$; 9D ; f_4 and f_5, $0(.01)6.50$, $6.5(.1)10$; 9D f_6 to f_8, $0(.01)1.50$, $1.5(.1)10$; 9D ; f_9 to f_{20}, $0(.1)10$; 9D .

Modified second central differences are tabulated alongside the entries.

Reprinted from Journal of Mathematics and Physics, February 1944. 16 pages, with cover, 25 cents.

MT29. Table of Coefficients for Inverse Interpolation with Advancing Differences...... Herbert E. Salzer.

This table may be regarded as a companion table to MT27, "Table of Coefficients for Inverse Interpolation with Central Differences." It contains tables giving values to 10 decimal places for

4th and 5th order terms for $m = 0(.001)1.000$;
6th order terms for $m = 0(.01)1.00$; and
7th and 8th order terms for $m = 0(.1)1.0$.

All these values are correct to about half a unit in the tenth decimal place with the exception of the coefficient of $\frac{\Delta^2 \Delta^3}{\Delta \cdot \Delta}$ which is accurate to within a unit in the tenth decimal place. A condensed one-page table of coefficients of the fourth, fifth and sixth order term at intervals of 0.1, is also given for the sale of convenience.

Coefficients of the second and third order terms $\frac{m(1 - m)}{2}$ and $\frac{m(1 - m)(m - 2)}{6}$ respectively, were not tabulated since they are both tabulated at intervals of 0.0001 by the Mathematical Tables Project in the "Tables of Lagrangian Interpolation Coefficients" published by the Colum University Press, New York City. For m (there "p") ranging from 0 to 1, $\frac{m(1 - m)}{2}$ is given by $-A_{-1}$ in the three-point table and $\frac{m(1 - m)(m - 2)}{6}$ is given by A_{-1} in the four-point table.

Reprinted from Journal of Mathematics and Physics, May 1944. 28 pages, with cover, 25 cents.

MT30. A New Formula for Inverse Interpolation.......... ....H. E. Salzer.

This paper is devoted to the derivation of a formula for inverse interpolation in a table of equally spaced arguments. The resulting formula is more concise and convenient than those in existence. It involves neither differences nor polynomial coefficients other than small powers. In use it will be found much simpler and quicker than those given by Davis, Aitken, Steffensen and Milne-Thomson. In a sense, it is the analogue of the Lagrangian formula for direct interpolation without differences (that is, in terms of the tabular entries only) if the usual expression is rearranged in terms of powers of the argument p.

Reprinted from Bulletin of the American Mathematical Society, August 1944. 4 pages, with cover, 25 cents.

MT31. Coefficients for Interpolation within a Square Grid in the Complex Plane....A. N. Lowan and H. E. Salzer.

The accompanying table gives the exact values of the real and imaginary parts of the 3-point and 4-point coefficients for both p and q ranging from 0 to 1 at intervals of 0.1.

The present table is particularly useful when interpolation is to be carried out either for an isolated argument or for a large number of arguments which are irregularly scattered. Thus for a 4-point interpolation for a single argument, the process of interpolation based on the present table requires a total of 4 complex multiplications (equivalent of 16 ordinary multiplications) and 2 additions whereas the method of interpolation in the real and imaginary parts each considered as a function of x and y, using real 4-point Lagrangian interpolation coefficients requires 40 multiplications and 10 additions. The table is however not an efficient tool for carrying out an extensive and systematic process of subtabulation. For such task the latter method involving the use of real interpolation coefficients is almost twice as efficient from the standpoint of the total number of multiplications and additions.

Reprinted from Journal of Mathematics and Physics, August 1944. 11 pages, with cover, 25 cents.

MT32. Table of Coefficients for Differences in Terms of the Derivatives. ....Herbert E. Salzer.

The table lists the exact values of the coefficients $B_{m,s}$ for $m = 1, 2, \ldots 20$ and $s = m, m + 1, \ldots 20$, for calculating the m^{th} advancing difference by means of Markoff's formula. It will be useful to any one who wishes to construct difference tables for functions whose derivatives are known.

Reprinted from Journal of Mathematics and Physics, November 1944. 4 pages, with cover, 25 cents.

TABLES OBTAINABLE FROM THE SUPERINTENDENT OF DOCUMENTS

Orders for either of these two tables should be sent to the Superintendent of Documents, Government Printing Office, Washington, D. C., accompanied by remittance drawn to his order:

Coordinate Conversion Tables. Published as Technical Manual TM 4-238 of the War Department.

Table I:

A sin x and A cos x where x is in mils,

A, [0(1)100; 4D]; x, [0(1)800].

The mil is 1/1600 of a right angle. The unit in which A is expressed will determine the unit of the entry--if A is in yards, the entries in this table are to be taken as yards.

Since these values represent products of sin x and cos x by A, values of the functions for any value of A correct to 5 and sometimes 6 significant figures may be obtained from the table by simple addition.

Table II:

Sin x and cos x, [0(1)800; 8D].

This table may be used whenever higher accuracy than that afforded by the Table I is desired.

March 25, 1943. 338 pages, $5\frac{1}{2}$ by $8\frac{1}{2}$ in. 40 cents. (Supt. Doc.)

Hydraulic Tables (2d ed.) Published by the Corps of Engineers, War Department.

These tables were prepared and published primarily for the use of the various offices of the Corps of Engineers, U. S. Army, in connection with the design of flood protection works. The range of many existing tables had been found inadequate for the large rivers, great depths, and steep slopes encountered in flood control work. It was also found desirable to compute several new tables. The new tables will be especially useful in solving the Manning formula in its various forms

This handbook contains 46 hydraulic tables especially applicable to the solutions of problems of open channel flow. It is divided into three sections; Section I consists of an extension of Tables 106, 121, and 133 in "Handbook of Hydraulics" (1939) by Horace W. King; Section II is an extension of Tables 14-18 and 47 in "Hydraulic and Excavation Tables," 8th edition, 1940, by the Bureau of Reclamation; Section III, Miscellaneous New Tables, i. e., (a) fractional powers of numbers, (b) normal depth in channels of infinite width, and (c) Manning formula tables giving the velocity of water for various coefficients of roughness.

(1944) viii + 565 pages. Blue imitation leather flexible cover, $4\frac{1}{2}$ by $6\frac{3}{4}$ in. $1.00. (Supt. Doc.)

TABLES OBTAINABLE FROM THE COLUMBIA UNIVERSITY PRESS

The following four tables can be obtained from the Columbia University Press Morningside Heights, New York 27, N. Y.

Table of the Reciprocals of the Integers from 100,000 through 200,009.

Tables of reciprocals are in frequent use by spectroscopists for the conversion of wave lengths into wave numbers, and vice versa. They may be applied also to all manner of computations, especially with calculating machines, by using multiplication to perform an indicated division. To carry out these operations the computer may make use of the 7-figure reciprocals of Oakes or Cotsworth. However, in the interval from 100,000 to 200,000 the differences between the successive entries of these tables vary from 1000 to 250 and interpolation between the tabular entries is somewhat difficult. To facilitate the use of reciprocal numbers, it was therefore decided to expand by tenfold the scope of the existing tables in this interval.

Preparation of manuscript tables was begun in December, 1934, by Dr. C. C. Kiess of the National Bureau of Standards. As a result of numerous requests for the reproduction of the table, the manuscript was given to the Mathematical Tables Project in March 1940. This project recomputed the values, checked them against the National Bureau of Standards MS, and prepared the table for publication.

(1943) viii + 201 pages. Buckram cover. $4.00 (Col.Univ.Press).

Table of the Bessel Functions $J_0(z)$ and $J_1(z)$ for Complex Arguments.

Bessel functions of orders zero and one are encountered in the general solution of boundary value problems arising in the theory of potential, heat conduction, and wave motion, when the domain is bounded by a circle or a circular cylinder. In particular, they occur in the problem of propagation of electromagnetic waves with a straight wire as a guide (in an isotropic medium), the theory of the skin effect for poorly conducting wires, the problem of oscillatory motion of a sphere in a viscous medium, the vibration of a heavy chain in a resisting medium, and in many other boundary value problems.

In spite of their great importance in theory and application, Bessel functions have scarcely been tabulated for complex arguments. Aside from tables for purely imaginary arguments and for arguments in the form $r\sqrt{i}$, Hayashi's short tables of $J_0(z)$ and $J_1(z)$ are the only ones in existence. To meet the need for more adequate tables, these functions are tabulated in the present volume for argument-points defined in polar coordinates. Along each of the rays $\varphi = 0^o, 5^o, \ldots 90^o$, ten-place values of the real and imaginary parts of $J_0(z)$ and $J_1(z)$ are given for moduli ranging from 0 to 10 at intervals of 0.01.

(1943) xliv + 403 pages. Buckram cover. $5.00 (Col.Univ.Press).

<u>Table of Circular and Hyperbolic Tangents and Cotangents for Radian Arguments.</u>

The main table in this volume is devoted to the circular and hyperbolic tangents and cotangents, for radian arguments ranging from 0 to 2 at intervals of 0.0001. Supplementary tables for all four functions over the range from 0 to 10 at intervals of 0.1 are also included. These tables may be regarded as forming a companion volume to the "Tables of Circular and Hyperbolic Sines and Cosines " which appeared in 1939 (See MT3 on page 2 of this list). The entries in the latter volume are given to nine decimal places---equivalent to eight, nine, or ten significant figures alsmost everywhere. Since the tangent and cotangent vary from $-\infty$ to ∞ over a range of π, the tables in this volume are given to eight significant figures rather than to a fixed number of decimal places. An exception has been made in some entries near the origin and near $\frac{1}{2}\pi$, where a few additional significant figures are given for the sake of regularlity in format.

(1943) xxxviii + 410 pages. Buckram cover. $5.00 (Col.Univ.Press).

<u>Tables of Lagrangian Interpolation Coefficients.</u>

Several tables are available giving the coefficients required in interpolation with the aid of differences. However, there are no extensive tables for interpolating by means of the most fundamental method--the Lagrangian formula--which involves only the tabular entries themselves and requires no differences. The Lagrangian interpolation formula (without the remainder term) approximates a given function by a polynomial. The polynomial of degree <u>n-1</u>, determined by <u>n</u> tabular entries, is referred to as an <u>n-point</u> interpolation polynomial. The present volume extends the few existing tables of Lagrangian interpolation coefficients in these <u>n-point</u> polynomials for <u>n</u> = 3, 4, ..., 11 by giving the entries at smaller intervals of the argument and by making adequate provisions for interpolation near the beginning and near the end of a table.

Furthermore, all the coefficients of Everett's central difference formula involving differences up to the eighth order are identical with certain Lagrangian coefficients given in this table. In particular, Everett's second-order coefficients are here tabulated at the interval of 0.0001 - a smaller interval than in other existing tables. Coefficients of the Gregory-Newton and Newton-Gauss formulas may also be found in this volume.

(1944) xxxvi + 392 pages. Buckram cover. $5.00 (Col.Univ.Press).

TABLES AVAILABLE ELSEWHERE

The ten tables listed below can be consulted in libraries maintaining a file of mathematical and technical journals. No reprints of them are obtainable from the Bureau.

On the Computation of Second Differences of the Si(x), Ei(x), and Ci(x) Functions. Bulletin of the American Mathematical Society (531 W. 116th St., New York 27 N. Y.), 45, No. 8, 583-588 (August 1939). By Arnold N. Lowan.

On the Distribution of Errors in the nth Tabular Differences. Arnold N. Lowan and Jack Laderman. Annals of Mathematical Statistics (Institute of Mathematical Statistics, E. S. Olds, Secretary, Carnegie Inst. of Technology, Pittsburgh, Pa.), X, No. 4, 360-364 (December 1939).

Note on the Computation of the Differences of the Si(x), Ci(x), Ei(x), and -Ei(-x) Functions. Milton Abramowitz. Bulletin of the American Mathematical Society, 46, No. 4, 332-333 (April 1940).

Errors in Hayashi's Table of Bessel Functions for Complex Arguments. Arnold N. Lowan and Gertrude Blanch. Bulletin of the American Mathematical Society, 47, No. 4, 291-293 (April 1941).

Tables of Stellar Functions for "Point-Source" Models. G. Blanch, A. N. Lowan, R. E. Marshak, and H. A. Bethe. Published under the title: "The Internal Temperature-Density Distribution of the Sun" in the Astrophysical Journal, (Yerkes Observatory, Williams Bay, Wisconsin), 94, 37-45 (July 1941).

On the Inversion of the q-Series Associated with Jacobian Elliptic Functions. A. N. Lowan, G. Blanch, and W. Horenstein. Bulletin of the American Mathematical Society, 48, No. 10, 737-738 (October 1942).

A Table of Coefficients for Numerical Differentiation. Arnold N. Lowan, Herbert E. Salzer and Abraham Hillman. Bulletin of the American Mathematical Society, 48, No. 12, 920-924 (December 1942).

Roots of Sin z = z. A. P. Hillman and H. E. Salzer. (Gives the first ten non-zero roots of sin z = z in the first quadrant to six decimal places. Roots of sin z = z where z = x + iy.) Philosophical Magazine (Taylor & Francis, Ltd., Red Lion Court, Fleet Street, London, E.C.4, England), Ser. 7, xxxiv, 575 (August 1943).

Coefficients for Numerical Integration with Central Differences. Herbert E. Salzer. Philosophical Magazine, Ser. 7, xxxv, 262-264 (April 1944).

Formulas for Complex Interpolation. Arnold N. Lowan and H. E. Salzer. Quarterly of Applied Mathematics (Brown University, Providence, R.I.), II, No. 3, 272-274 (October 1944).