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METHOD OF DESIGNING AN OBJECTIVE FOR AN ASTRONOMICAL TELESCOPE

Given:

N_D , the index of refraction for the D spectrum line of the crown glass

$\nu = \frac{N_D - 1}{N_F - N_C}$, the dispersion constant for the crown glass

N'_D , the index of refraction for the D spectrum line of the flint glass

$\nu' = \frac{N'_D - 1}{N'_F - N'_C}$, the dispersion constant for the flint glass.

d, the diameter of the objective.

In accordance with the usual practice, F, the focal length of the complete objective should be approximately 15 times its diameter. This is the ratio commonly adopted for an astronomical objective although if one has much skill in figuring the objective a much smaller ratio may be employed. For example, Mr. J. Clacey at the Bureau of Standards has constructed an objective 5 inches in diameter with a focal length of only 20.5 inch which gave good definition at the center of the field. However unless there are reasons which make imperative the selection of a smaller ratio it is advisable to not depart from the normal ratio of 15.

Determine f and f', the focal lengths of the crown and flint components respectively by the equations:

$$f = F\left(\frac{\nu - \nu'}{\nu}\right) \qquad (1)$$

$$f' = F\left(\frac{\nu' - \nu}{\nu'}\right) \qquad (2)$$

Form the equation:

$$\left(\frac{\nu}{\nu'}\right)^3 \frac{1}{N_D(N_D-1)}\left\{\left(\frac{N_D+2}{N_D-1}\right)X^2 - 4(N_D+1)X + (3N_D+2)(N_D-1) + \frac{N_D^3}{N_D-1}\right\} - \frac{1}{N'_D(N'_D-1)}\left\{\left(\frac{N'_D+2}{N'_D-1}\right)X'^2 + 4(N'_D+1)\alpha X' + (3N'_D+2)(N'_D-1)\alpha^2 + \frac{N'^3_D}{N'_D-1}\right\} = 0 \qquad (3)$$

and eliminate α and X' by the equations:

$$\alpha = \frac{-2f'}{f} - 1 \qquad (4)$$

$$X' = \left(\frac{N'_D-1}{N_D-1}\right)\frac{f'}{f}X - \left(\frac{N'_D-1}{N_D-1}\right)\frac{f'}{f} - 1 \qquad (5)$$

This yields a quadratic in X which may be solved and which will have two real solutions if the glasses selected are suitable for a telescope objective. Either solution will determine values of the radii for an objective free from spherical aberration. The root smaller in absolute value should always be selected since it gives the objective with the flatter curves which are better adapted for a large aperture. Having determined X, the value of X' may be obtained by means of equation (5).

Then the radii are given by equations 6, 7 and 8:

$$r_1 = \frac{2(N_D-1)f}{1 + X} \tag{6}$$

$$r_2 = \frac{2(N_D-1)f}{X - 1} \tag{7}$$

$$r_3 = \frac{2(N'_D-1)f'}{X' - 1} \tag{8}$$

The objective which has now been designed is the type in which the crown component is the one nearer the object and the two surfaces in contact have the same radius of curvature. An ordinary crown ($N_D = 1.524$, $\nu = 59$), or a barium crown ($N_D = 1.575$, $\nu = 56$), may be used with a flint ($N_D = 1.620$, $\nu = 36$). (The values of the indices given in the above sentence are only approximate and will be deviated from considerably in the glasses made by different manufacturers.)

If the computation is correctly carried through, r_1 will be positive, which indicates that the first surface is convex, r_2 will be negative which signifies a convex surface for the crown and concave surface for the flint component. The third radius may be either positive or negative and very large in comparison with r_1 and r_2. If r_3 is positive it indicates a concave surface; if negative, a convex surface. With many pairs of glasses which may be selected, r_3 will be so large that it will be a sufficiently good approximation to make the last surface plane.

Equations 9 and 10 enable one to determine the thicknesses of blanks necessary for the crown and flint components respectively:

$$\text{Thickness of crown} = \frac{d^2}{8r_1} - \frac{d^2}{8r_2} + k \qquad (9)$$

If r_3 is positive,

$$\text{thickness of flint} = \frac{-d^2}{8r_2} + \frac{+d^2}{8r_3} + k \qquad (10a)$$

If r_3 is negative,

$$\text{thickness of flint} = \frac{-d^2}{8r^2} + k \qquad (10b)$$

The constant k is the thickness of the blank at its thinnest part, i.e., the edge of the crown or the center of the flint. It must be sufficiently large to give the necessary mechanical strength to the finished lens. It is suggested that .04 times the diameter is a suitable value for k although this value may be deviated from considerably.

The lens as designed is to be assembled with the two equal radii of crown and flint nested and with the crown component turned toward the star or distant object. The values of the radii given by these formulae are approximate and the final objective should be figured or parabolized if complete elimination of spherical aberration is to be assured. This is commonly done on the outside surface of the flint lens. The reader is referred to Smithsonian Contributions to Knowledge, Vol. XXXIV, for detailed directions by which an amateur may learn to grind, polish and figure glass surfaces.

Illustrative Example

To design a 10 inch objective using barium crown, $N_D = 1.5734$, $\nu = 57.6$ and flint, $N'_D = 1.6202$, $\nu = 37.5$ glasses.

$N_D = 1.5734$

$\nu = 57.6$

$N'_D = 1.6202$

$\nu' = 37.5$

$d = 10$

Then $F = 15 \times 10 = 150$ inches.

$$f = 150 \left(\frac{57.6 - 37.5}{57.6}\right) = +52.3$$

$$f' = 150 \left(\frac{37.5 - 57.6}{37.5}\right) = -80.4$$

When values of N_D, N'_D, ν and ν' are substituted in equation (3) one obtains the equation

$$4.016(6.232 X^2 - 10.294 X + 3.353 + 6.793) \tag{11}$$

$$-0.995(5.837 X'^2 + 10.481 \alpha X' + 4.255 \alpha^2 + 6.858) = 0$$

From equations (4) and (5)

$$\alpha = 2.074 \tag{12}$$

$$X' = -1.668 X + 0.663$$

Introducing these values in equation (11) and collecting terms

$25.025\ X^2$	$-\ 41.336\ X$	$+\ 15.474$
		$+\ 27.279$
-16.060	$+\ 12.802$	$-\ 2.551$
	$+\ 35.969$	$-\ 14.336$
		$-\ 18.302$
		$-\ 6.825$
Adding		
$8.965\ X^2$	$+\ 7.435\ X$	$+\ 0.739 = 0$

This quadratic has the two roots

$$X = -0.114 \quad \text{or} \quad -0.713$$

The first root is the one which gives the desired solution.

From equation (12) $\quad X' = +0.853$

From equations (6), (7) and (8)

$$r_1 = \frac{2(0.5734)(52.3)}{0.886} = +67.7 \text{ inches}$$

$$r_2 = \frac{2(0.5734)(52.3)}{-1.114} = -53.8 \text{ inches}$$

$$r_3 = \frac{2(0.6202)(-80.4)}{-0.147} = +678.0 \text{ inches}$$

From equations (9) and (10a)

Thickness of crown component

$$\frac{100}{8(67.7)} - \frac{100}{8(-53.8)} + 0.4 = 0.82 \text{ inches}$$

Thickness of flint component

$$-\frac{100}{8(-53.8)} + \frac{100}{8(678)} + 0.4 = 0.65 \text{ inches.}$$