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Letter Circular LC 22
(Communication B522)
Gage Section
Bureau of Standards
Washington, D. C.
August 15, 1918
15 Sheets

THE MEASUREMENT OF TAPER GAGES.

The measurement of conical plug and ring gages, and others having combinations of two or more tapers or of cones with cylinders and planes, is not as simple and straightforward a procedure as is the case with plain plug, ring and snap gages. In most cases the latter may be measured by means of a micrometer, precision gage blocks, or a measuring machine; whereas, accurate inspection of the former necessitates, in addition, the application of special measuring apparatus and methods. Herein are presented and illustrated the various methods commonly applied by the Gage Section, Bureau of Standards, to the measurement of taper gages.

For convenience in describing the various methods which are used in their inspection, taper gages may be divided into the following five classes: plugs having a single taper, double taper, and triple taper; taper rings; and profiles of certain types.

MEASUREMENT OF A SIMPLE TAPERED PLUG.

A method commonly used in the shop for measuring the simple type of conical plug shown in Fig. 1 consists in measuring over the sharp corners n, n, with a micrometer and the angles at the ends with a bevel protractor. For the inspection of gages having tolerances of 0.001 inch or less this method is inadequate since the corners have a tendency to become rounded and thus a reading which is less than the correct dimension is obtained. Further, the angle measurement given by the bevel protractor is not reliable to within two minutes, which amounts to 0.0006 inch in a length of one inch. Consequently, the computation of the diameter at another point on the gage is not reliable to within 0.0012 multiplied by the distance between this point and the end at which the diameter measurement was taken.

First Method. A standard method which gives reliable results is the following: The plug is firmly clamped on a good surface plate with the small end against the plate as shown in Fig. 1. A pair of small, accurately ground cylinders of equal diameter are then placed on gage blocks of equal height and in contact with the gage. A measurement M_1 over the cylinders is taken by means of a micrometer caliper or gage block combination; a similar measurement M_2 is taken at another height and, for the purpose of checking, a third measurement M_3 should be taken. It is essential that the gage be held firmly against the plate in order to prevent its lifting because of the wedging action of the cylinders when the measurement is made, and it is set on the small end so that the cylinders will adjust themselves into the angles under the pressure of the micrometer.

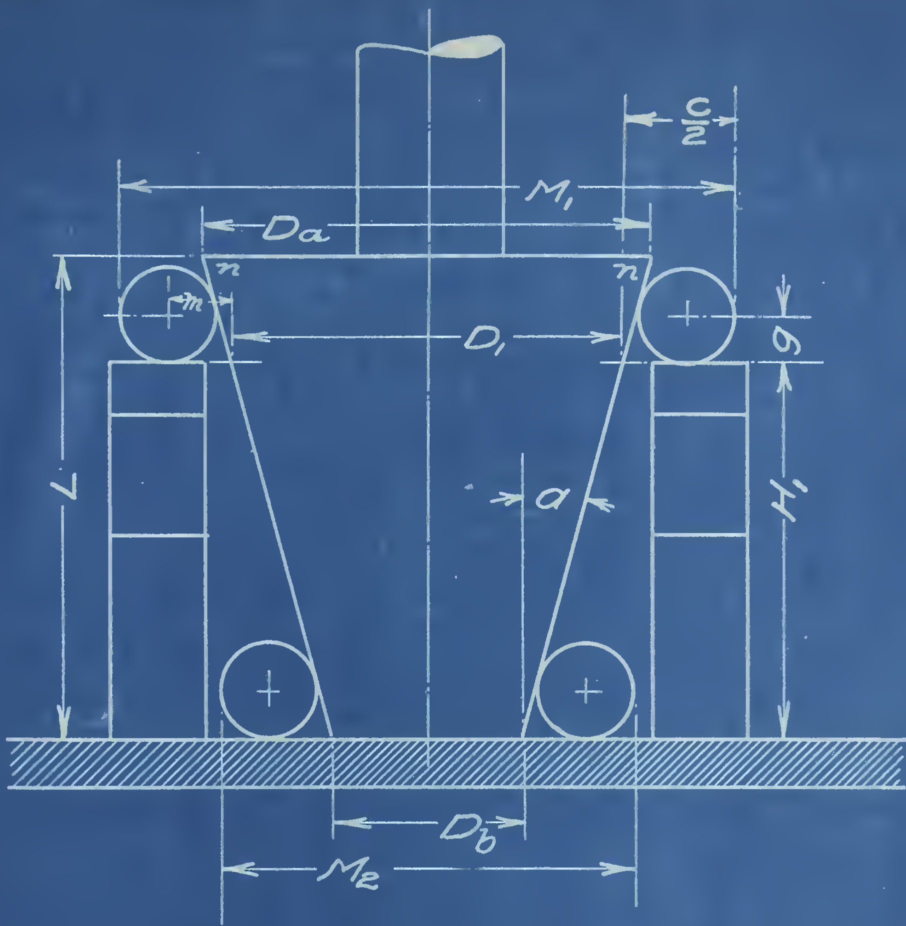

Fig. 1.

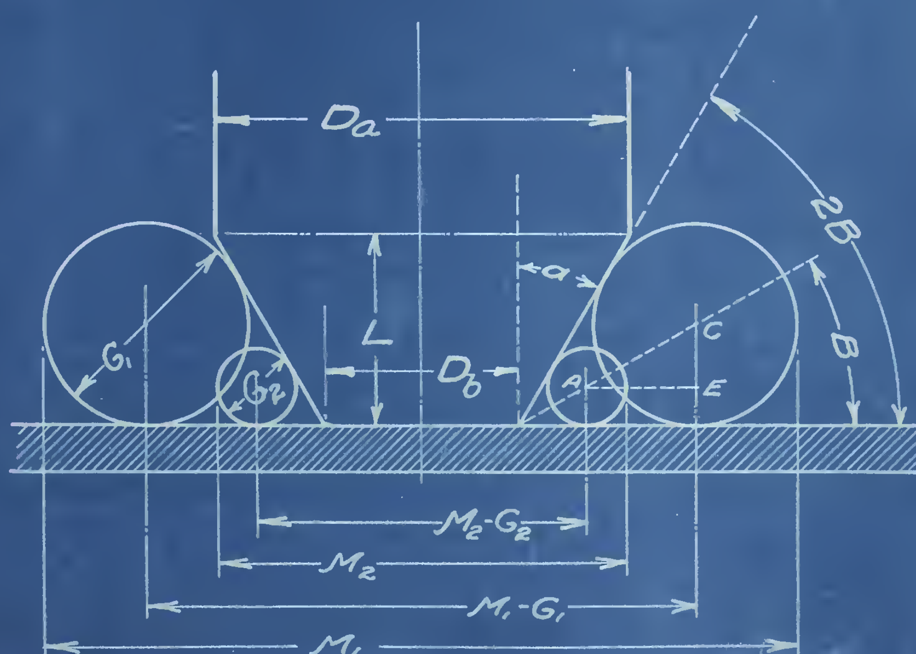

Fig. 2.

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From the measurements thus obtained the taper and the required diameters are computed as follows:

Let G = diameter of cylinders
g = corresponding radius
T = taper per inch
a = angle of taper = 1/2 total angle of taper on plug
H_1 = distance between two positions of measurement
M_1, M_2 = micrometer measurements
D_1, D_b = corresponding diameters on gage

(1) Then $$T = \frac{M_1 - M_2}{H_1}, \text{ and } T = 2 \tan a$$

(2) and $$D_1 = M_1 - G\,(1 + \cot 1/2\,(90° - a))$$

The latter formula is derived with reference to Fig. 1 as follows: Let C represent the amount to be subtracted from the micrometer measurement to give the diameter of the plug at the same height as the gage blocks on which the cylinders were supported.

Then
$$C = 2g + 2m = G + G \cot 1/2\,(90° - a)$$
$$C = G\,(1 + \cot 1/2\,(90° - a))$$
$$D_1 = M_1 - C$$
$$= M_1 - G\,(1 + \cot 1/2\,(90° - a))$$

(3) $$D_b = M_2 - G\,(1 + \cot 1/2\,(90° - a)), \text{ etc.}$$

To determine the diameter of the plug at any point other than that at which it was measured, the taper per inch is multiplied by the distance between the positions of the known and required diameters and the product is either added to or subtracted from the known diameter according to the relative positions of the two diameters. For example:

(4) $$D_a = T_1\,(L - H_1) + D_1$$

<u>Second Method.</u> It is sometimes more convenient, especially when the tapered surface is short, to use two pairs of cylinders of known and different diameters supported at the same height. The two large cylinders are placed in the angles formed by the gage and the surface plate and a measurement M_1 is taken over them. The two smaller cylinders are then substituted for the larger ones and a second measurement M_2 is taken. A third reading over a pair of another size is desirable for checking. The diameter D_a over the cylindrical portion is determined by direct measurement. From these measurements the angle of taper and the diameter at any point on the taper may be computed.

The half-angle of taper (a) is given by the formula:

(5) $$a = 90° - 2B, \text{ and } T = 2 \tan a$$

$$\tan B = \frac{G_1 - G_2}{(M_1 - M_2) - (G_1 - G_2)}$$

in which G_1 = diameter of large cylinders
G_2 = diameter of small cylinders
M_1 = measurement over large cylinders
M_2 = measurement over small cylinders

Referring to Fig. 2 this formula is derived as follows:

$M_1 - G_1$ = distance between centers of large wires
$M_2 = G_2$ = distance between centers of small wires

In the triangle ACE,

$$AE = \frac{(M_1 - G_1) - (M_2 - G_2)}{2}$$

$$CE = \frac{G_1 - G_2}{2}$$

$$\tan B = \frac{CE}{AE} = \frac{\frac{G_1 - G_2}{2}}{\frac{(M_1 - G_1) - (M_2 - G_2)}{2}} = \frac{G_1 - G_2}{(M_1 - M_2) - (G_1 - G_2)}$$

The diameter at the small end of the plug is given by the formula derived under the first method:

$$D_b = M_1 - G_1 \left(1 + \cot 1/2 \left(90° - a\right)\right)$$

(6) or $$D_b = M_1 - G_1 (1 + \cot B)$$

(7) The height L is given by the formula $L = \frac{D_a - D_b}{2} \cot a$

Summary.
Type: Plug having single taper - first method
Reference: Fig. 1
Required Dimensions: D_a, D_b, T, L
Measure: L, H_1, G, M_1, M_2
Formulae: (1), (2), (3), (4)

Type: Plug having single taper - second method
Reference: Fig. 2
Required Dimensions: D_a, D_b, T, L
Measure: G_1, G_2, M_1, M_2, D_a
Formulae: (5), (6), (7)

MEASUREMENT OF PLUG HAVING DOUBLE TAPER.

In Fig. 3 is shown the method commonly used for the measurement of a gage consisting of two tapers which make different angles with the axis of the gage. The complete inspection of such a gage requires the determination of the following dimensions:

D_a, diameter at large end of gage
D_b, diameter at intersection of tapers
D_c, diameter at small end of gage
T_1, taper per inch of first tapered surface
T_2, taper per inch of second tapered surface
a_1, angle of first taper
a_2, angle of second taper
L_1, length of first taper along axis of gage
L_2, length of second taper along axis of gage
L_3, length of cylindrical portion

In the inspection of a gage of this type the individual tapers are measured according to the methods described above under the inspection of a single taper. The diameters D_1, D_2, D_3, and D_4; the heights H_1 and H_4; and the dimensions T_1, T_2, a_1, and a_2 are thus determined. The diameter D_c and the overall length L are measured directly by means of micrometers.

The remaining dimensions are then determined with reference to Figs. 3 and 3a by the following relations:

(8) $$L_3 = H_4 - \frac{D_4 - D_c}{T_2}$$

(9) $$L_2 = E - F + \frac{D_4 - D_c}{T_2}$$

in which $$H_1 = E + H_4$$

and $$F = \frac{ET_2 - (D_1 - D_4)}{T_2 - T_1}$$

(10) $$L_1 = L - (L_2 + L_3)$$

(11) $$D_b = D_4 + T_2 (E - F) \text{ or } D_b = D_1 - FT_1$$

(12) $$D_a = D_1 + (L - H_1) T_1$$

The expression giving the value of F is derived with reference to Fig. 3a as follows:

$E = H_1 - H_4$, distance between the two known diameters D_1 and D_4

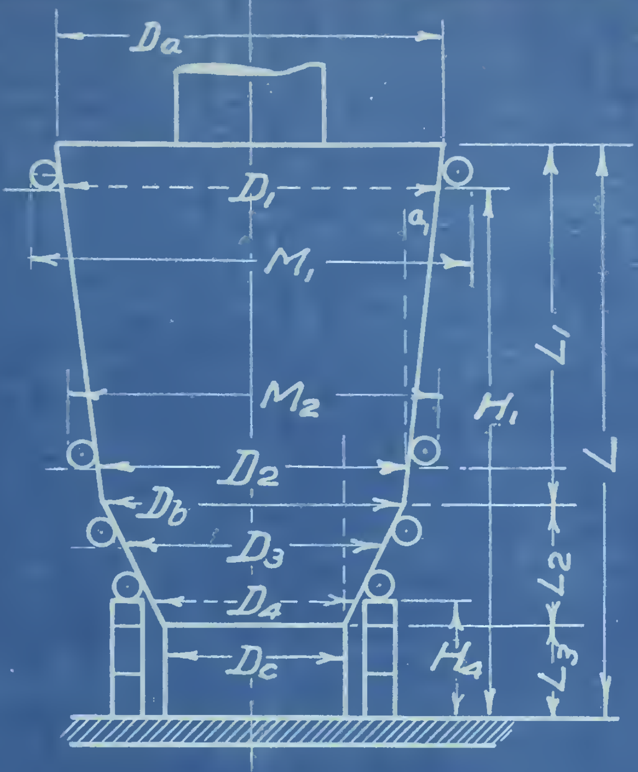

Da
D1
a1
M1
M2
H1
L1
L
D2
Db
D3
L2
D4
Dc
H4
L3

Fig 3.

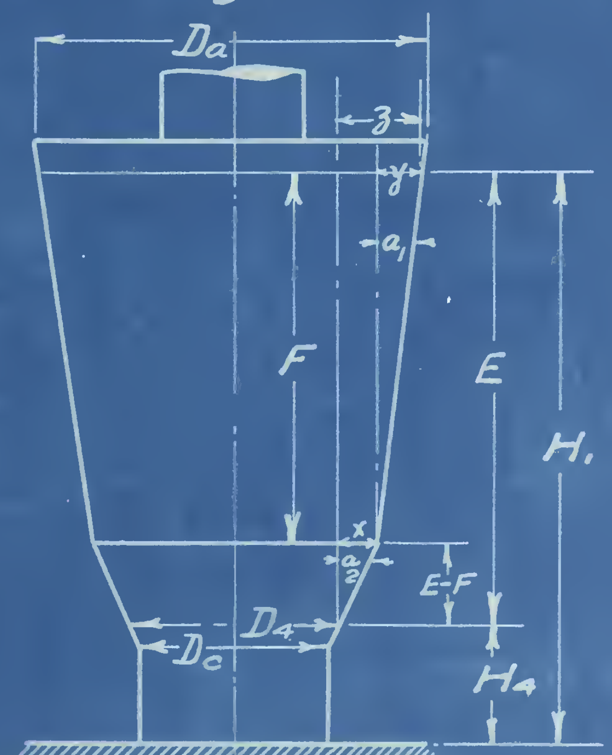

Da
z
y
a1
F
E
H1
x
a2
E-F
D4
Dc
H4

Fig. 3a.

$$z = \frac{D_1 - D_4}{2}$$

$$x = (E - F) \tan a_2$$

$$y = F \tan a_1$$

$$z = x + y = \frac{D_1 - D_4}{2}$$

Therefore $$\frac{D_1 - D_4}{2} = (E - F) \tan a_2 + F \tan a_1$$

$$D_1 - D_4 = 2(E - F) \tan a_2 + 2 F \tan a_1$$

$$2 \tan a_1 = T_1$$

$$2 \tan a_2 = T_2$$

Then $$D_1 - D_4 = T_2 (E - F) + T_1 F$$

$$D_1 - D_4 = T_2 E - T_2 F + T_1 F$$

$$F (T_2 - T_1) = T_2 E - (D_1 - D_4)$$

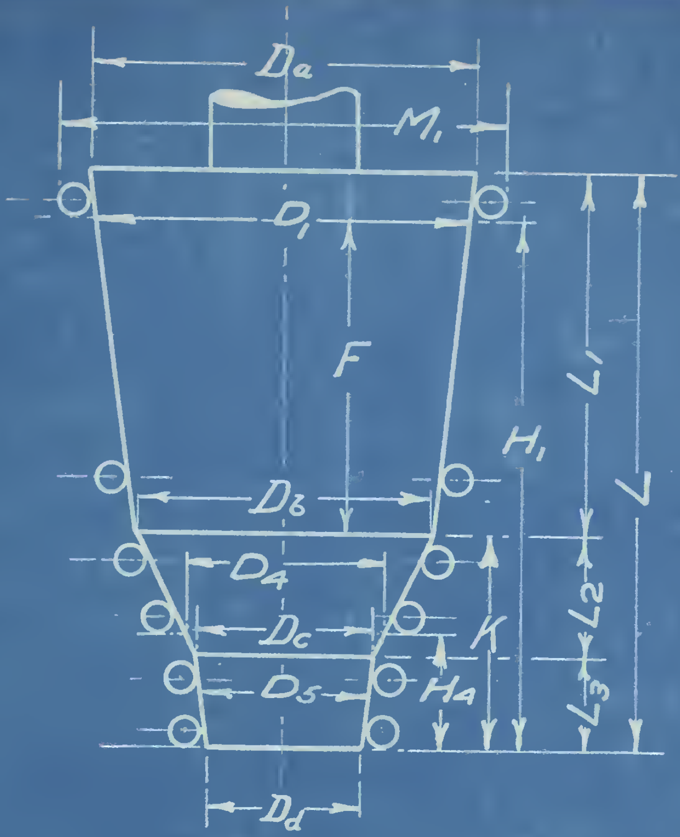
$$F = \frac{T_2 E - (D_1 - D_4)}{T_2 - T_1}$$

Summary.
Type: Plug having double taper
Reference: Fig. 3
Required Dimensions: D_a, D_b, D_c, T_1, T_2, L_1, L_2, L_3, L
Measure: M_1, M_2, M_3, M_4, H_1, H_2, H_3, H_4, G, L, D_c
Formulae: (1), (2), (8), (9), (10), (11), (12)

MEASUREMENT OF PLUG HAVING TRIPLE TAPER.

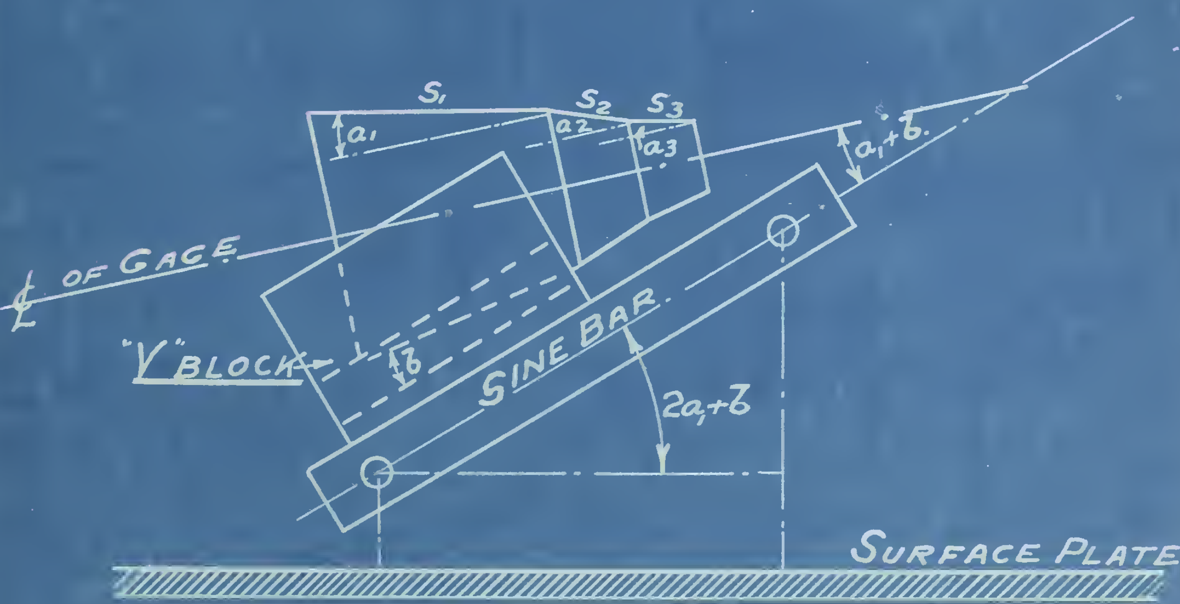
Some taper plug gages, such as checks for chamber gages for certain types of cartridge cases, have three different tapers. The method of inspecting such a gage is practically the same as that used on a plug having double taper, the principal differences being that it is necessary to make a greater number of measurements and more computation is involved. As indicated in Fig. 4 six measurements over cylinders placed at different heights are made and the tapers T_1, T_2, and T_3, and the diameters D_1, D_4, and D_d are determined. The dimension F is given by the expression,

$$F = \frac{(H_1 - H_4)\, T_2 - (D_1 - D_4)}{T_2 - T_1}$$



Da
M1
D1
F
L1
H1
L
Db
D4
L2
Dc
K
D5
HA
L3
Dd

Fig 4.



S1
S2
S3
a1
a2
a3
a1+b.
℄ OF GAGE
"V" BLOCK
b
SINE BAR.
2a1+b
SURFACE PLATE

Fig 5.

as previously explained. The diameter D_b is expressed by

(13) $$D_b = D_1 - FT_1$$

To complete the inspection the dimensions L_1, L_2, L_3, and D_c must be determined.

Let $$K = L_2 + L_3 = H_1 - F$$

$$D_d + L_3T_3 = D_b - (K - L_3)\,T_2$$

$$D_d + L_3T_3 = D_b - KT_2 + L_3T_2$$

$$L_3\,(T_3 - T_2) = D_b - D_d - KT_2$$

(14) $$L_3 = \frac{D_b - D_d - KT_2}{T_3 - T_2}$$

Similarly $$L_2 = \frac{D_b - D_d - KT_3}{T_2 - T_3}$$

(15) or $$L_2 = H_1 - F - L_3$$

(16) $$L_1 = L - (L_2 + L_3)$$

(17) $$D_c = D_b - L_2T_2$$

(18) or $$D_c = D_d + L_3T_3, \text{ and } D_d = D_c - L_3T_3$$

In case some of the tapered surfaces are quite short it is difficult to measure the angle of taper with any considerable degree of accuracy by the above method since the distance between the two positions of measurement is very short and a small error in measurement is multiplied in the determination of the taper. On most gages of this type one of the tapered surfaces is long and its angle of taper can be determined by the method previously described. The angles of the short tapers can then be determined by another method.

Assuming the angle a_1, Fig. 5 to be thus determined, the angle a_2 may be measured by means of a sine bar according to the following method:

The gage is clamped in a V-block with the surface T_1 bearing in the V and the V-block is aligned on the sine bar. The sine bar is adjusted to the position in which the element S_1 is parallel to the plane of the surface plate on which the set-up is mounted; this position is most readily determined by means of a height gage and test indicator. The angle $a_1 + b$, which the axis of the gage makes with the sine bar is thus determined.

The sine bar is then turned until the element S_2 indicates parallel to the surface plate and the angle c which the sine bar makes with the surface plate is measured. The angle a_2 is given by the formula,

$$a_2 = c - (a_1 + b) \quad (19)$$

and

$$\tan a_2 = T_2$$

In a similar manner the third angle of taper a_3 may be measured.

This method can be adapted for comparing the diameters of a quantity of similar taper gages with a master by providing a stop to locate all gages in the V-block in the same position.

Summary.
Type: Plug having triple taper
Reference: Fig. 4
Required Dimensions: D_a, D_b, D_c, D_d, T_1, T_2, T_3, L_1, L_2, L_3, L
Measure: M_1 to M_6, incl., H_1 to H_6, incl., G, L
Formulae: (1) or (19), (3), (4), (13), (14), (15), (16), (17), (18)

MEASUREMENT OF TAPER RING GAGES.

Tapered ring gages having one or more tapers are commonly checked by means of a tapered check plug of the same nominal dimensions as the ring. The taper of the ring is checked by the so-called contact method. A thin line of Prussian Blue is marked along an element of the plug and then the contact which the profile along this line makes with the ring is determined by turning the plug in the ring and examining the distribution of the Prussian Blue.

If the plug makes contact with the ring along its entire length, the diameter of the ring at any point is readily determined.

Let T = taper per inch of plug = taper of ring
D = diameter of plug at large end

Measure H_1 and H_2, Fig. 6

$$D_1 = D - TH_1$$

$$D_2 = D - TH_2$$

In case the ring does not properly fit the master check plug or if such a check is not available it is necessary to resort to a method of direct measurement. A single taper may

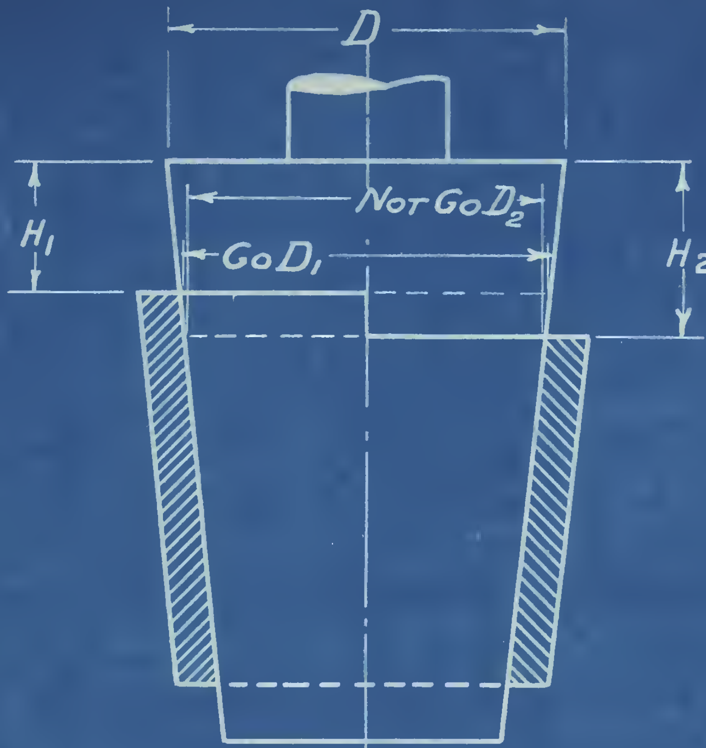


Fig. 6.

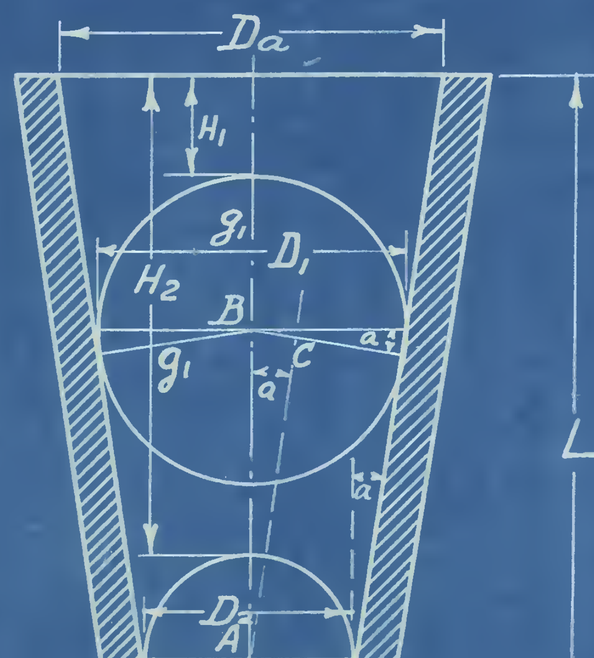


Fig. 7.

be measured by means of a depth micrometer and two steel balls which have been carefully selected for uniformity of diameter. The method is illustrated in Fig. 7. The balls are inserted singly in the ring as shown and the dimensions H_1 and H_2 are measured with a depth micrometer.

Let G_1 = diameter of maximum ball; g_1 = corresponding radius
G_2 = diameter of minimum ball; g_2 = corresponding radius

The distance between the centers of the balls

$$AB = (H_2 + g_2) - (H_1 + g_1)$$

$$g_1 - g_2 = BC$$

(20) $$\sin a = \frac{BC}{AB} = \frac{g_1 - g_2}{(H_2 + g_2) - (H_1 + g_1)}$$

(21) $2 \tan a = T$ = taper per inch
$D_1 = 2g_1 \sec a = G_1 \sec a$ = diameter of ring through center of large ball at depth $H_1 + g_1$ from top
$D_2 = 2g_2 \sec a = G_2 \sec a$ = diameter of ring through center of small ball at depth $H_2 + g_2$ from top

(22) $$D_a = D_1 + T\,(H_1 + g_1)$$

(23) $D_b = D_a - LT$, in which L = length of ring

In the case of a ring having a short taper the angle of taper may be determined by measuring a cast, made by pouring into the ring a fused mixture of about 90% sulphur and 10% graphite, by means of the sine bar.

The dimensions of rings having double tapers may also be determined by applying the methods described above.

Summary.
Type: Ring having single taper - ball method
Reference: Fig. 7
Required Dimensions: L, T, D_a, D_b
Measure: G_1, G_2, L, H_1, H_2
Formulae: (20), (21), (22), (23)

MEASUREMENT OF PROFILE GAGES HAVING TAPERED SURFACES.

The methods herein described for the measurement of taper gages may be applied with equal facility to the determination

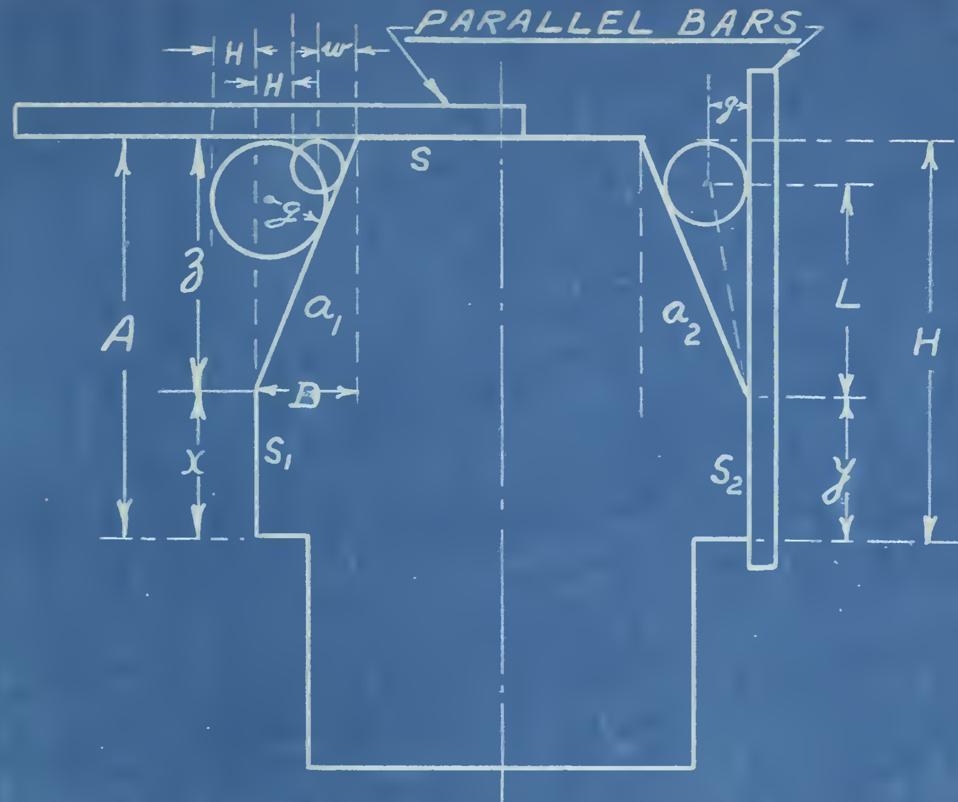


Fig. 8a.

Fig. 8b.

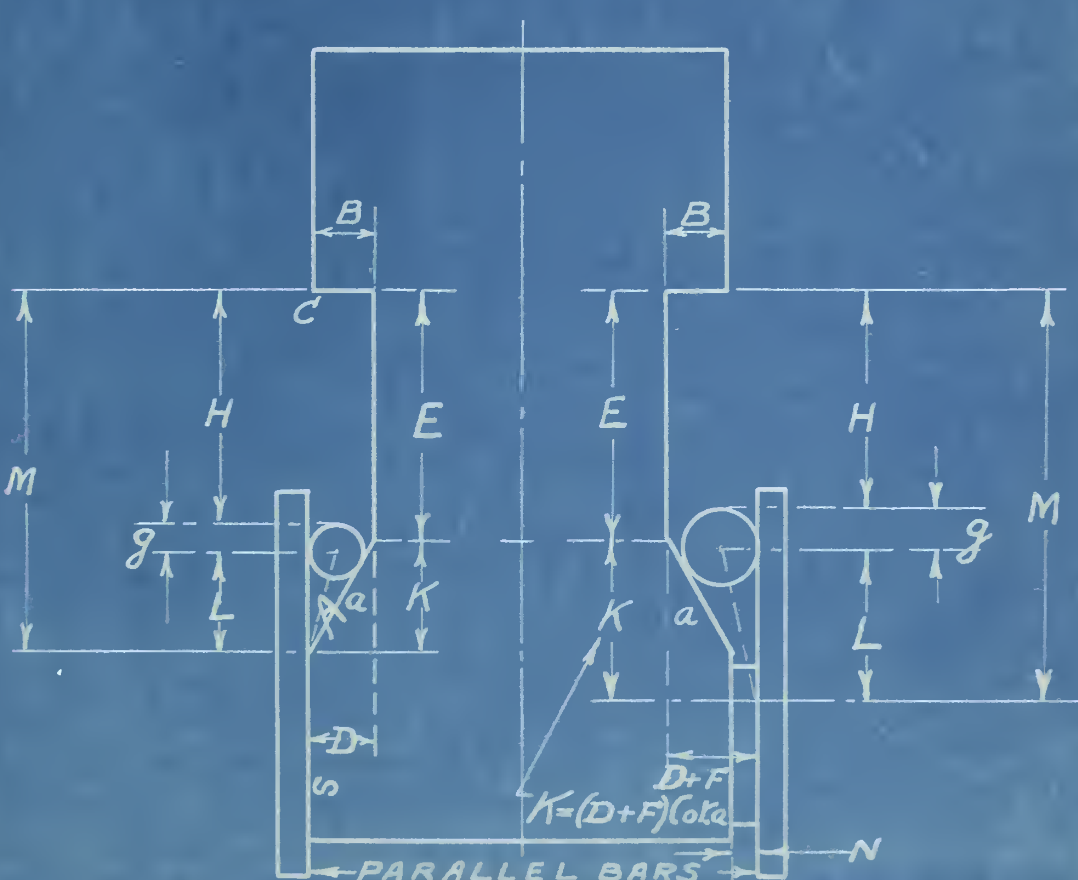


Fig 8c.

Fig 8d.

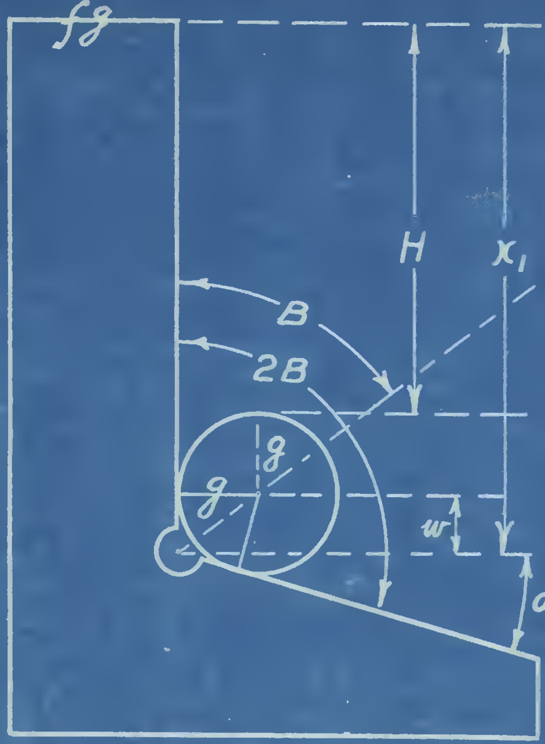


Fig. 9a.

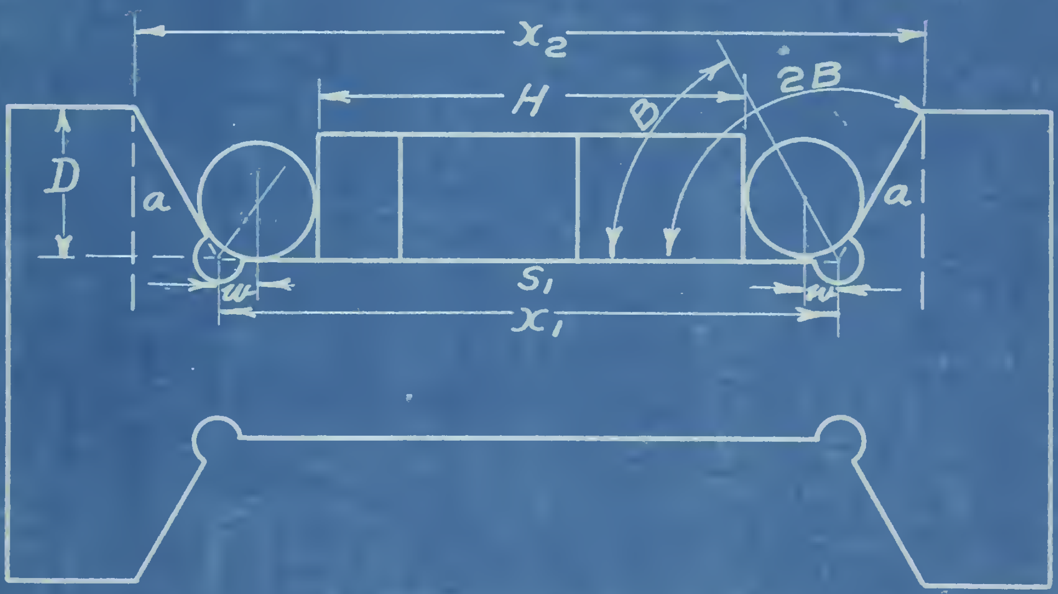


Fig. 9b.

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of such dimensions as angles between intersecting surfaces and distances between points of intersection on profile gages such as shown in Figs. 8 and 9.

To determine the dimension x, Fig. 8a, the dimension A is first determined by direct measurement. The angle a_1 is determined by sine bar measurement or by ascertaining the coordinates of two cylinders located at different points on the surface in accordance with the method described under "Measurement of a Single Taper". The gage is then clamped to an angle-iron with the surface S in a vertical position and a straight-edge is clamped against this surface. A cylinder is then placed in the angle as shown and the dimension H is measured. The depth D is then given by the expression,

$$D = g + g \cot 1/2\ (90° - a_1) \pm H$$
$$= g\ (1 + \cot 1/2\ (90° - a_1)) \pm H$$

in which the sign of H depends on whether the top of the cylinder is above or below the surface S_1 as shown by the two circles in the figure. The dimension x is then given by

$$x = A - D \cot a_1$$

A slightly different method illustrated in Fig. 8b may be used. After the angle a_2 has been determined the gage is clamped to an angle-iron with the surface S_2 vertical and a straight-edge is clamped to this surface. A ground cylinder is inserted and the dimension H is measured.

$$y = H - (L + g)$$
$$= H - g\ (1 + \cot 1/2\ a_2)$$

By following a similar procedure the dimension E on the gage shown in Fig. 8c may be determined. It is given by the formula

$$E = H + g\ (1 + \cot 1/2\ a) - D \cot a$$

In some cases it is necessary to insert a gage block between the surface S and the straight-edge as shown in Fig. 8d, in order that a cylinder of appreciable diameter may be used. If F is the thickness of the block

$$E = H + g\ (1 + \cot 1/2\ a) - (D + F) \cot a$$

Figs. 9a and 9b are further examples of gages to which these methods may be applied. In these cases

$$X_1 = H + g\ (1 + \cot 1/2\ (90° + a)) \text{ in Fig. 9a}$$
$$X_1 = H + G\ (1 + \cot 1/2\ (90° + a)) \text{ in Fig. 9b}$$
$$X_2 = X_1 + 2\ D \tan a.$$

BUREAU OF STANDARDS.

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