

DEPARTMENT OF COMMERCE  
BUREAU OF STANDARDS  
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THE SAFETY OF THEATRE PROSCENIUM CURTAINS

## I. Basis of Proposed Recommended Requirements for Theatre Curtains.

From time to time inquiry has been made as to what should be the minimum requirements for theatre proscenium curtains in order that reasonable safety might be assured to audiences in such places of assembly. Several fires in theatres equipped with so-called safety curtains and wherein many people have perished, have led to the conclusion that it is not sufficient to make a curtain of some incombustible material and be assured that it is adequate. Theatre owners and building officials alike when cognizant of the true conditions have sought to know what is a reasonable requirement. Some cities have required <sup>steel</sup> curtains exclusively for all but the very small theatres, others have written such requirements into their building laws and have not enforced them, but the majority of cities and towns, where any requirement is made, specify an asbestos cloth curtain overlapping the sides and top of the proscenium opening a certain amount. In some cases this curtain is required to be operated regularly. The strength of the curtain, its mounting and operating details are seldom specified. Examination of several of these installations by members of the Bureau staff led to the conclusion that under conditions that might easily occur in any theatre they would be entirely inadequate to secure even a moderate amount of protection to the audience. It was found that study of the subject both in Europe and America had proven to the investigators that the problem was a more serious one than it appears to have been considered by the designers of some of these devices. Tests of the materials in common use showed that they did not possess the strength at moderately high temperatures that should be required. Some of them have incorporated into them combustible materials which would lead to excessive smoke and glow when exposed to temperatures not in excess of 500 or 600 degrees Fahrenheit.

The structural strength of fabric and mounting in some cases does not have the necessary factor of safety at ordinary temperatures to provide a suitable margin above the stresses which are likely to occur at the beginning of stage fires. The ease with which many flexible curtains are stopped in their descent by even slight pressures indicates that as constructed and hung they are lacking in the necessary under balance and inertia to assure closure when subjected to wind pressures or air drafts. The friction factors where such curtains are pressed against the proscenium wall, which is usually unfinished, are very high.

In order to compare some of the better types of curtains and, if possible, develop a specification for the minimum requirement for such devices, a series of tests of various asbestos cloths and theatre cur-



tains has been made at the Bureau of Standards. Some of the better grades of commercial asbestos cloths were tested and also a few kinds which were developed in an effort to secure greater strength at high temperature.

Fire tests were made on sections of two <sup>rigid</sup> steel curtains, and both fire and operating tests on three asbestos cloth curtains and a fire test on a fourth. All curtains were tested on an 11 feet high by 14 1/2 to 16 feet wide opening except the steel curtain B, which was 6 feet high and 9 feet wide. All test curtain constructions were of such proportions in strength and size of members as would be used on approximately a 30 by 40 foot proscenium opening.

The relative efficiency of the types of curtains tested is indicated in the table. This refers to their resistance to heat transmission only. It will be observed that the ratio of temperature of the unexposed surface of the steel curtains to the temperature of the furnace at the end of fifteen minutes is approximately one-third of that of the best of the double asbestos cloth curtains, and the ratio at thirty minutes is approximately half that of the best asbestos curtain at fifteen minutes. See table, pages 4, 5, and 6.

The lower surface temperatures obtaining with curtain B as compared with curtain A can be ascribed in part to the smaller size tested which introduced greater cooling effects from the edges.

In several other factors than heat resistance curtains C, D, and E gave unfavorable results. The strength of curtain C, which was made of two sheets of <sup>plain</sup> asbestos cloth with pipe battens and standing cables for side guides, was almost negligible after the test, the sealing of the edges against the passage of smoke was poor, and the friction of the asbestos cloth against the brick wall or steel slide-ways was very great when the curtain was subjected to even slight wind pressures. Curtain D, which was made of a single asbestos cloth having monel metal wires and mounted in the same way as curtain C, had ample strength after the fire test, but had all the other defects of the double curtain of plain asbestos cloth, and in addition showed glow on the surface sooner. Curtain E was made of two sheets of asbestos cloth mounted on built-up tees at its edges and having a pipe batten at the top and a flexible batten at the bottom. The vertical tees at the sides had roller bearing trolleys operating in closed tracks so arranged that the tongue of the tee projecting into the closed track formed an effectual smoke seal at the sides. The <sup>monel metal reinforced</sup> cloth on the side exposed to the fire had been used in Curtain D and appeared to have sufficient strength to warrant its use in this test. The unexposed side was made of cloth having brass wires inserted in the yarn. This curtain did not have sufficient smoke seal at the bottom. The decorative paint which had one part of animal glue to nine parts of water began to give off dense smoke at two minutes after the fire was lighted. This would be very objection-

Dear Mother  
I received your letter of the 10th and was glad to hear from you. I am well and hope these few lines will find you the same.

I have not much news to write at present. I am still in the same place and doing the same work. I have not seen any of the old friends here, but I hope to see some of them soon.

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able with stage fires and had been characteristic of the two previous tests of cloth curtains. The general behavior of this design was better than the two previous ones. The cloth having the monel metal wires was stronger, even after being burnt the second time, than the burnt section of cloth with brass wire.

The curtain F had an outside frame of 3-inch structural channels with an interior frame work or lattice of 1 1/4 inch pipes. The covering on the fire side was of asbestos cloth having nickel wires inserted in the yarn. The unexposed face was covered with asbestos cloth having brass wire insertion. The paint used was composed of 4 parts of casein and 10 parts of silicate of soda (water glass, spec. grav. 1.38), in 100 parts water to which was added 20 parts pigment (about equal parts whiting and ochre). The face of the curtain started smoking at 1 minute 50 seconds. The smoke was light and bluish compared with the dense black smoke from the previous tests of asbestos cloth curtains. The decorative coat began to turn brown soon after the smoke started and at 6 minutes was brown over nearly the entire face. Not until 12 minutes was any glow visible on the curtain and that was in a small portion where the filling paint had not been brushed in well. As installed the design showed the defect of not having a close enough fit along the top edge of the curtain. The behavior of the curtain otherwise indicated that it would serve to give a reasonable protection to persons assembled in well designed and well ordered theatres and it is on the basis of such performance that the proposed minimum requirements are based.

The tests were initiated at the request of the Commissioners of the District of Columbia to supply information for use in the revision of the regulations for theatre proscenium curtains. Acknowledgements are due to Mr. Albert L. Harris, Municipal Architect of the District of Columbia, for materials and assistance in applying the decorative finish to the asbestos cloth curtains, <sup>and</sup> to the manufacturers of theatre curtains and asbestos cloth for samples of their materials and constructions furnished for the tests. Acknowledgements are also due to the Committee on Construction of Buildings of the National Board of Fire Underwriters for access to the results of strength tests, fire tests under load, and permeability tests of plain and metal reinforced asbestos cloths conducted for them at the Underwriters Laboratories in 1923, and to Mr. Ira H. Woolson, Consulting Engineer, for helpful suggestions in formulating the specifications.

The results of the tests made are being prepared for publication as a Technologic Paper, the general conclusions and specifications herewith given being intended to serve the purpose of information until such publication can issue.



## Fire-Test Data of Theatre Curtains

Curtain A. Seven and one-fourth inch thickness with steel face, 3/8 inch asbestos board back, channel frame with riveted horizontal girders.

Time, Min- utes.	Furnace Temperature		Surface Temperature		Ratio of Surface Temp. rise to Ave. Furnace Temp. above Initial.
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Ini- tial		37 (99)		23 (73)	room
:05	868 (1594)	675 (1247)	72 (162)	64 (147)	.102
:10	969 (1776)	844 (1551)	116 (241)	100 (212)	.137
:15	1044 (1911)	894 (1641)	184 (363)	160 (320)	.208
:20	1073 (1963)	916 (1681)	234 (453)	212 (414)	.266
:25	1157 (2115)	1012 (1854)	306 (583)	245 (473)	.293
:30	1239 (2262)	1070 (1958)	311 (592)	275 (527)	.315
Average		833 (1531)			.230

Curtain B. Five and one-half inch thickness with steel face; sectional 3/8 inch asbestos board back, pressed channel girders, welded construction.

Time, Min-	Furnace Temperature		Surface Temperature		Ratio of Surface Temp. rise to Ave. Furnace Temp. above Initial.
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Ini- tial		15 (59)		14 (57)	room
:05	666 (1231)	561 (1042)	29 (84)	22 (72)	.033
:10	969 (1812)	726 (1339)	87 (189)	64 (147)	.111
:15	1051 (1924)	805 (1481)	149 (300)	99 (210)	.154
:20	1129 (2064)	912 (1674)	206 (403)	136 (277)	.196
:25	1043 (1909)	902 (1656)	244 (471)	169 (336)	.230
:30	1101 (2014)	991 (1816)	265 (509)	201 (394)	.261
Average		743 (1369)			.164

1870-1880

1870-1880

1870-1880

1870-1880

1870-1880

1870-1880

1870-1880



## Fire-Test Data of Theatre Curtains

Curtain C. Double: plain asbestos cloth, weighing 2.6 lbs. per sq. yd. on each face, pipe battens, three 5/16 inch vertical cables between plies of cloth, spanning from batten to batten. Standing cable side guides.

Time, Min- utes.	Furnace Temperature		Surface Temperature		Ratio of Sur- face Temp. rise to Ave. Furnace Temp. above Initial
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Ini- tial		12 (54)		17 (63)	room
:02 1/2	712 (1314)	592 (1098)	154 (309)	112 (234)	.328
:05	818 (1504)	690 (1274)	334 (633)	250 (482)	.507
:07 1/2	870 (1598)	722 (1332)	424 (795)	350 (662)	.620
:10	865 (1589)	719 (1326)	427 (801)	377 (711)	.621
:12 1/2	879 (1614)	731 (1348)	427 (801)	370 (698)	.582
:15	927 (1701)	788 (1450)	484 (903)	400 (752)	.608
Average		642 (1186)			.544

Curtain D. Single: asbestos cloth weighing 4.9 lbs. per sq. yd. with monel metal wire reinforcement in yarn. Pipe battens, three 5/16 inch vertical cables attached to edges and center at back of curtain. Standing cable side guides.

Time, Min- utes.	Furnace Temperature		Surface Temperature		Ratio of Sur- face Temp. rise to Ave. Furnace Temp. above Initial.
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Ini- tial		16 (61)		11 (52)	room
:02 1/2	752 (1386)	483 (901)	342 (642)	174 (345)	.692
:05	782 (1440)	627 (1161)	444 (831)	324 (615)	.810
:07 1/2	702 (1296)	586 (1087)	441 (826)	331 (628)	.705
:10	879 (1614)	722 (1332)	531 (988)	432 (810)	.842
:12 1/2	1083 (1981)	820 (1508)	719 (1326)	525 (977)	.934
:15	1187 (2169)	887 (1629)	781 (1436)	555 (1031)	.908
Average		615 (1139)			.816

STATE OF TEXAS

County of ...

Know all men by these presents...

Table with 7 columns: Name, Address, City, State, Zip, Phone, and other details. Includes entries for 'John Doe' and 'Jane Smith'.

Witness my hand and seal of office...

Notary Public in and for the State of Texas...

Table with 7 columns: Name, Address, City, State, Zip, Phone, and other details. Includes entries for 'Robert Johnson' and 'Mary White'.

Fire-Test Data of Theatre Curtains

Curtain E. Double: 4.9 lb. asbestos cloth with monel metal wire on fire side, 3 1/4 lb. brass wire reinforced cloth on unexposed side. Rigid frame on sides and top, flexible bottom batten. Only marginal framework to keep plies of cloth separated. Rigid side guides with roller bearing trolleys.

Time, Min-utes.	Furnace Temperature		Surface Temperature		Ratio of Surface Temp. rise to Ave. Furnace Temp. above Initial
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Initial		18 (64)		10 (50)	room
:02 1/2	640 (1184)	494 (921)	146 (295)	99 (210)	.374
:05	947 (1737)	697 (1287)	397 (747)	231 (448)	.542
:07 1/2	1110 (2030)	820 (1508)	432 (810)	391 (736)	.734
:10	1131 (2123)	862 (1584)	483 (910)	447 (837)	.756
:12 1/2	1142 (2088)	873 (1603)	480 (896)	436 (820)	.657
:15	1159 (2082)	871 (1600)	482 (884)	429 (804)	.616
Average		698 (1288)			.610

Curtain F. Double: 2 3/4 lb. asbestos cloth with nickel wire reinforcement on the fire side, 3 lb. brass wire reinforced cloth on unexposed side. Rigid side guides. Rigid frame of 3-inch channels with interior frame of pipes to provide positive separation of plies of cloth and reduce the span of the cloth.

Time, Min-utes.	Furnace Temperature		Surface Temperature		Ratio of Surface Temp. rise to Ave. Furnace Temp. above Initial.
	Maximum °C. (°F.)	Average °C. (°F.)	Maximum °C. (°F.)	Average °C. (°F.)	
Initial		14 (57)		8 (46)	room
:02 1/2	816 (1501)	586 (1087)		68 (154)	.210
:05	881 (1616)	757 (1395)	174 (345)	127 (261)	.352
:07 1/2	915 (1679)	726 (1339)		183 (361)	.314
:10	937 (1719)	688 (1270)	330 (626)	239 (462)	.391
:12 1/2	954 (1749)	719 (1323)		300 (572)	.478
:15	967 (1773)	736 (1360)	402 (756)	353 (663)	.548
Average		642 (1188)			.366



## II. Recommended Requirements for Theatre Proscenium Curtains.

1. General Requirements: -

All proscenium openings shall have a curtain made of incombustible materials constructed and mounted to intercept hot gases, flame, and smoke and prevent glow from a severe fire on the stage from showing on the auditorium side for a sufficient length of time to permit the safe and orderly exit of all persons from the auditorium with a reasonable allowance of excess time. The period of protection shall not be taken less than 15 minutes. The curtain shall be opened and closed at least once ~~for~~ every time there is an audience in the theater. The closing of the curtain from the full open position shall be effected in less than one minute, but the last five feet of travel shall require not less than five seconds.

2. General Design: -

The curtain shall have a rigid framework of steel or other <sup>suitable</sup> metal covered either (a) with metal sheets and insulating materials or (b) with woven asbestos cloth having wires inserted or twisted into the yarns of the warp and filling, and in accordance with the details hereinafter specified. It shall have sufficient strength to resist a lateral pressure of ten (10) pounds per square foot of its area when in the closed position, with a factor of safety of not less than 2 on the ultimate strength of the construction, and the mounting and details shall be such as to insure ready and positive closure when subjected to pressure of five (5) pounds per square foot.

The design strength of tension members and cloth shall be based on center deflections of not to exceed one-tenth (1/10) of the span. In no case shall the maximum deflection cause a permanent set or bend in the curtain structure, nor shall it cause the curtain to rub against the wall at the edges of the proscenium opening.

The thickness of the curtain shall in no case be less than 3 inches, nor less than one one-hundred-twentieth (1/120) of its span. The width and height of the curtain shall be such that it shall overlap the sides and top of the proscenium opening at least 9 inches.

Provision shall be made for the expansion of the curtain and guides due to changes in temperature. An allowance of 1/16 inch per foot of length of steel members shall be made for such purpose.

The curtain shall be guided throughout its travel by rigid steel guides. Such guides shall be securely bolted in place and designed to form a stop between the curtain and the wall to prevent smoke and flame from passing around the edges of the curtain. Where the main curtain members carry the stresses from lateral pressure on the curtain as suspension tension members, the guides and their attachment to the building shall have adequate strength to safely carry the reactions from these tension members.



The top of the curtain shall have a smoke stop fitted to make it as nearly smoke tight as practicable. The bottom of the curtain shall have a yielding pad of incombustible material not less than three inches thick to form a seal against the floor.

The calculations for the strength of the curtain, curtain mountings and all the details thereof shall conform to generally accepted engineering methods and practice. The stresses in materials shall not exceed those prescribed in the local building regulations, or in the absence of any regulations on the subject they shall conform to engineering standards generally accepted as safe.

### 3. Curtain Covering: -

The metal curtain (a) shall have its front or auditorium side covered with well fitted metal plates. The stage side shall be covered with insulating boards made of asbestos or other suitable incombustible materials. Such insulating boards, if of asbestos, shall be at least three-eighths ( $3/8$ ) inch thick and weigh not less than two (2) pounds per square foot if of solid material, or not less than three-quarters ( $3/4$ ) inch thick and weigh not less than one and three-quarters ( $1\ 3/4$ ) pounds per square foot if of cellular construction. Other materials which have been proven the equivalent of the above specified asbestos boards in strength, fire resistance, and insulating properties may be used in lieu thereof. The covering materials shall be firmly attached to the curtain frame, and all joints and joinings thoroughly cemented with a fire resistive or furnace cement.

The asbestos cloth curtain (b) shall have its framework covered on both sides with a metal-reinforced close-woven asbestos cloth weighing not less than three and a quarter ( $3\ 1/4$ ) pounds per square yard. The marginal members of the frame shall have the cloth covering extended around them to form insulation of at least two thicknesses, or shall have equivalent insulation applied in other manner.

The covering cloth shall have incorporated into the yarn before weaving either monel metal, nickel, brass, chromel, nichrome, or other metal or alloy having not less strength at a temperature of 1700 degrees Fahrenheit and not less resistance to corrosion at ordinary temperature. Asbestos cloth made of long fibered blue Crocidolite asbestos is considered the equivalent of chrysotile asbestos cloth of the same weight having brass wire reinforcement. The wires may be either single or double but the tensile strength of wires in each strand of yarn shall be not less than seven (7) pounds as tested at ordinary temperatures and the strength of the yarn with the wires shall be not less than twelve (12) pounds when determined on a four inch length between the gripping jaws of the testing machine. The strength of the cloth in tension shall be not less than one hundred eighty (180) pounds per inch of width of warp and eighty-five (85) pounds per inch of width of filling when tested in strips one inch wide with four (4) inches length between the jaws of the testing machine. The head of the testing machine shall have a speed of travel not to exceed twelve inches per minute. The asbestos fiber of the yarns may contain cotton or other combustible fiber in amount not to

The first of the year was a very dry one, and the crops were much injured. The weather was very hot, and the ground was very hard. The crops were much injured, and the people were very poor.

The second of the year was a very wet one, and the crops were much injured. The weather was very cold, and the ground was very hard. The crops were much injured, and the people were very poor.

The third of the year was a very dry one, and the crops were much injured. The weather was very hot, and the ground was very hard. The crops were much injured, and the people were very poor.

The fourth of the year was a very wet one, and the crops were much injured. The weather was very cold, and the ground was very hard. The crops were much injured, and the people were very poor.

The fifth of the year was a very dry one, and the crops were much injured. The weather was very hot, and the ground was very hard. The crops were much injured, and the people were very poor.

The sixth of the year was a very wet one, and the crops were much injured. The weather was very cold, and the ground was very hard. The crops were much injured, and the people were very poor.



exceed four (4) per cent of the weight of the asbestos. The total carbon content of the cloth shall not exceed two and one-half (2 1/2) per cent of the total weight of fiber. A sample of cloth of not less than one-half square yard shall be submitted for inspection and tests.

The seams of the cloth shall have double rows of stitching of asbestos thread, having metal wires, of the same or greater strength than the yarns of the cloth.

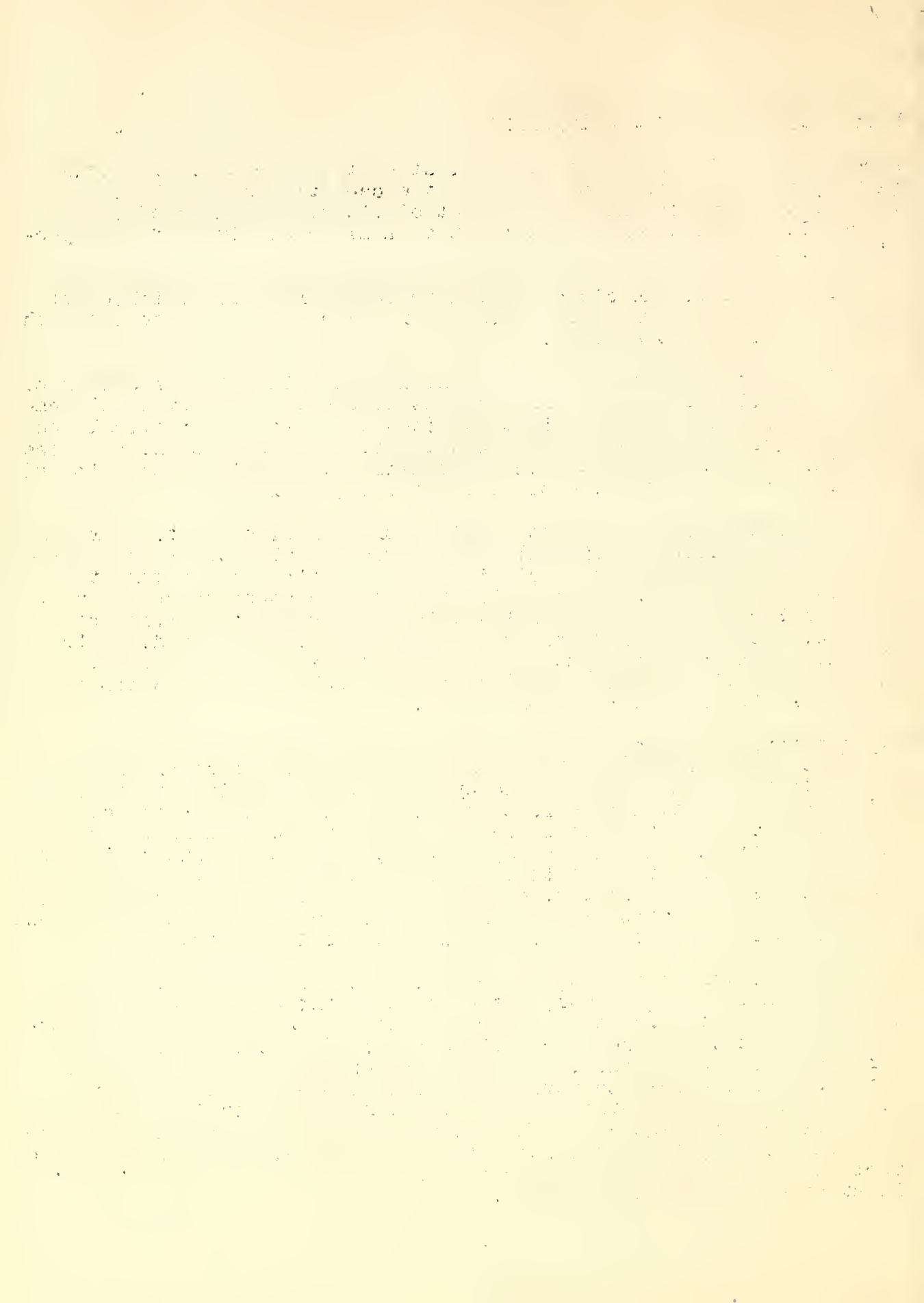
The asbestos cloth curtain shall have the interior framework of tubes, structural snapes, bars or rods of steel with horizontal members spaced not more than eight (8) feet apart and the vertical members not more than twelve (12) feet apart. The arrangement of these interior members shall be such as to secure a positive separation of the cloth on the two sides of at least two (2) inches.

After erection the asbestos cloth on both sides of the curtain shall be filled with a mineral paint having a silicate of soda binder to which may be added casein in the proportion of not more than four (4) parts casein to ten (10) parts of concentrated solution of sodium silicate. Any other paint which will completely fill the pores of the cloth and which shall be capable of withstanding fire without giving out more smoke than the above will be permitted. The paint shall be applied hot and brushed well into the cloth so as to make it practically smoke tight.

#### 4. Operating Equipment:-

The mechanism and devices for controlling the curtain shall be of simple design and positive in operation. The opening of the curtain may be by hand, hydraulic or electric power machine; the closing for emergency or automatic operation shall be the same as for ordinary operation and shall be by gravity, obtained by underbalancing the curtain with reference to the counterweights by not less than one pound per square foot of curtain. Any other method proposed for emergency or automatic closure shall be proven as at least equivalent to that outlined above in effectiveness and reliability.

In addition to the regular operating control station, which shall be on the stage floor, there shall be three emergency control stations, one of which shall be on the stage. At least three quick-acting heat-operated release devices for automatically closing the curtain shall be provided. The location of the emergency control stations and the release devices shall be subject to the approval of the proper legal authorities. The emergency control and the automatic closing equipment shall cause the opening of the ventilator over the stage while the curtain is being closed. The emergency control shall be used to close the curtain and open the ventilator at least once for each performance.



For hydraulic machines the water supply shall be taken from an elevated tank or from a suitable accumulator. When the water supply is taken from a sprinkler supply tank the connection shall be made at such a height as to leave sufficient water for sprinkler requirements. For electric machines the current supply shall be fused independently of the house supply. All parts of the electric power supply shall be enclosed and well protected against fire.

All machines and hoisting gear shall be designed in accordance with local elevator code, or in the absence of such regulations, the American Engineering Standards Committee Safety Code for Elevators requirements for passenger elevator machines and cables. Travel limit stops, and room for over travel shall be used on such machines. Lifting cables shall be not less than one-half (1/2) inch in diameter.

#### 5. Approval of Plans and Installations: -

Complete details of the proposed curtain shall be submitted to the proper legal authorities and approval obtained before the installation or erection of any proscenium curtain is begun. The completed installation shall be similarly approved after operating tests, before any theater is opened for public performances.

#### 6. Curtains of New Designs: -

Installations of curtains of other designs and materials than herein specified shall before acceptance be subjected to a fire test conducted in accordance with the Standard Specifications of the American Society for Testing Materials for Fire Tests of Materials and Construction; Serial Designation: C19-18, as applicable to non-bearing partitions with the following changes and exceptions: The test shall continue for a period of fifteen minutes, unless failure shall have occurred previously. The temperatures on the unexposed surface shall be measured by thermocouples with wires not over .02 inch diameter having junctions and adjacent wire so mounted as to indicate as accurately as possible the temperature of the unexposed surface. The temperature of the testing room adjacent to the unexposed surface shall be not lower than 10 degrees C. (50°F.) at the beginning of the test and shall be free from convection currents except such as are induced by the fire test. The average temperature on the unexposed side of the test curtain shall not exceed 260 degrees C. (500°F.) at 10 minutes nor 371 degrees C. (700°F.) at 15 minutes. The unexposed face of the curtain shall not glow within the test period nor shall there be any passage of smoke or flame through the curtain.

### III. Suggestions for the Improving of Existing Theatre Proscenium Curtains.

Whenever possible, present installations should be made to conform to the requirements for new installations. There are many defects in the commonly used asbestos cloth curtain installation which can be remedied. The principal ones are (1), the ineffectiveness of the single ply curtain as mounted on standing cables as a smoke barrier; (2), the low strength of the curtain structure compared with the strength it would require under even moderate pressures; (3), the

The first part of the paper is devoted to a study of the
 properties of the function  $f(x)$  defined by the
 equation  $f(x) = \sum_{n=0}^{\infty} a_n x^n$ . It is shown that
  $f(x)$  is analytic in the region  $|x| < 1$  and
 that it satisfies the functional equation  $f(x) = 1 + x f(x^2)$ .
 The coefficients  $a_n$  are shown to be given by the
 formula  $a_n = \frac{1}{2^n} \sum_{k=0}^{n-1} a_k$ .

In the second part of the paper, the function  $f(x)$  is
 extended to the region  $|x| > 1$ . It is shown that
  $f(x)$  has a branch point at  $x = 1$  and that it
 has a logarithmic singularity at  $x = 1$ . The
 asymptotic behavior of the coefficients  $a_n$  is
 shown to be  $a_n \sim \frac{1}{2^n} \log n$ .

The third part of the paper is devoted to a study of the
 function  $g(x) = \sum_{n=0}^{\infty} b_n x^n$ . It is shown that
  $g(x)$  is analytic in the region  $|x| < 1$  and
 that it satisfies the functional equation  $g(x) = 1 + x g(x^2)$ .
 The coefficients  $b_n$  are shown to be given by the
 formula  $b_n = \frac{1}{2^n} \sum_{k=0}^{n-1} b_k$ .

In the fourth part of the paper, the function  $g(x)$  is
 extended to the region  $|x| > 1$ . It is shown that
  $g(x)$  has a branch point at  $x = 1$  and that it
 has a logarithmic singularity at  $x = 1$ . The
 asymptotic behavior of the coefficients  $b_n$  is
 shown to be  $b_n \sim \frac{1}{2^n} \log n$ .

The fifth part of the paper is devoted to a study of the
 function  $h(x) = \sum_{n=0}^{\infty} c_n x^n$ . It is shown that
  $h(x)$  is analytic in the region  $|x| < 1$  and
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The seventh part of the paper is devoted to a study of the
 function  $i(x) = \sum_{n=0}^{\infty} d_n x^n$ . It is shown that
  $i(x)$  is analytic in the region  $|x| < 1$  and
 that it satisfies the functional equation  $i(x) = 1 + x i(x^2)$ .
 The coefficients  $d_n$  are shown to be given by the
 formula  $d_n = \frac{1}{2^n} \sum_{k=0}^{n-1} d_k$ .